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Liying Xu

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APPLICATION OF REAL OPTIONS TO VALUATION AND DECISION MAKING IN THE PETROLEUM E&P INDUSTRY

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APPLICATION OF REAL OPTIONS TO VALUATION AND DECISION MAKING IN THE PETROLEUM E&P INDUSTRY

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Soli Deo Gloria

To my husband Tian Qi

and

my daughter Wei (Maggie) Qi

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I receive my PhD degree at an age of 49 after about 10 years of study, covering my daughter's entire high school and most of her college time, in a university of another state from home, without any promised financial support from university, paying an out-state tuition plus commuting twice a month between school and home for many years. With the unconditional love, never-ceasing encouragement, and continuing support, from both my husband and my daughter, I have been sustained and strengthened day after day. Without the innumerable ways that my husband and my daughter have done for me, my dream, pioneering and completing an interdisciplinary PhD degree from an department ranked number one (#1) in the U.S., could not become true. While enjoying doing a great job in his career, supporting my whole family, including my daughter's college study at Brown University with double degrees plus one-year study-abroad in Oxford University, UK, my husband also enjoyed supporting my PhD study, both financially and much more beyond. With his extraordinary wisdom, knowledge, and understanding, my husband helped me tremendously with the programming on real options evaluation, an area much different from his own academic and career majors. My daughter was my first reader for most of the chapters of this dissertation, no matter when she was near, at home, or very far away, studying in Brown University or in Oxford University. I am also very thankful for my daughter for decently editing many of my application materials for scholarships and fellowships. If the PhD degree I received could be divided, more than half of it would have belonged to my husband and my daughter.

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Isaac Newton also state that "I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me." (Wikimedia Foundation, 2010b) Even though my dissertation covers disciplines of economics, finance, accounting, mathematics, and engineering, upon the completing of my PhD study, one fact that I feel so true is that the more I learn, the more I know that what I do not know is much more. Then I gratefully confess that the world is so wonderfully made. The knowledge of creating and sustaining the universe is immeasurable. It is not discouraging. It is amazing. So, I really enjoyed, with my best efforts, even though so limited, picking up the little "part" from the vast and endless sea of knowledge one by one, and putting them into the pool of "parts" of the discovered knowledge by giants and peers, being utilized for their designed purposes, and manifesting the glory of the knowledge of making and sustaining the universe. Again, thank you, each and every one who once supported, and will support me along this joyful and rewarding journey, including you, my readers.

APPLICATION OF REAL OPTIONS TO VALUATION AND DECISION MAKING IN THE PETROLEUM E&P INDUSTRY

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Co-Supervisor: Willem C. J. van Rensburg

This study is to establish a risk-neutral binomial lattice method to apply real options theory to valuation and decision making in the petroleum exploration and production ($E\&P$) industry with a specific focus on the switching time from primary to water flooding oil recovery. First, West Texas Intermediate (WTI) historical oil price evolution in the past 25 years is studied and modeled with the geometric Brownian motion (GBM) and one-factor mean reversion price models to capture the oil price uncertainty. Second, to conduct real options evaluation, specific reservoir simulations are designed and oil production profile for primary and water flooding oil recovery for a

synthetic onshore oil reservoir is generated using UTCHEM reservoir simulator. Third, a cash flow model from producing the oil reservoir is created under a concessionary fiscal system. Finally, the binomial lattice real options evaluation method is established to value the project with flexibility in the switching time from primary to water flooding oil recovery under uncertain oil prices. The research reaches seven conclusions: 1) for the GBM price model, the assumptions of constant drift rate and constant volatility do not hold for the historical WTI oil price; 2) one-factor mean reversion price model is a better model to fit the historical WTI oil prices than the GBM model; 3) the calibrated long run prices and mean reversion rates for the historical WTI oil prices reveal that the evolution of historical WTI oil prices from January 2, 1986 to May 28, 2010 was according to three price regimes with different long run prices, and that since 2003, the world economy has increased its tolerance to higher oil prices and to the higher price fluctuation from its long run price compared with that from 1986 to 2003; 4) the established real options evaluation method can be used to identify the best time to switch from primary to water flooding oil recovery using stochastic oil prices; 5) with the one-factor mean reversion oil price model and the most updated cost data, the real options evaluation method finds that the water flooding switching time is earlier than that from the traditional net present value (NPV) optimizing method; 6) the real options evaluation results reveal that most of time water flooding should start when oil prices are high, and should not start when oil prices are low; and 7) water flooding switching time is sensitive to oil price models and to the investment and operating costs.

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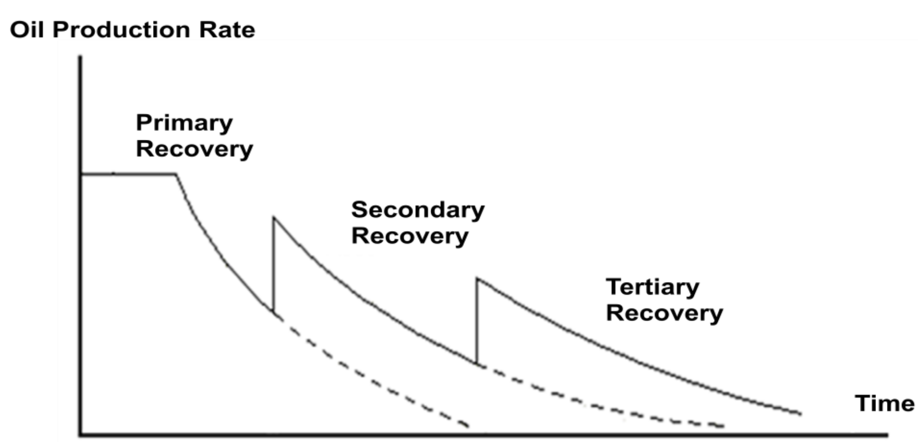
CHAPTER 1: INTRODUCTION

The petroleum industry is commonly referred to as the oil and gas industry. It is characterized by large investment and asset base. In addition to high level of technical complexity in exploration and production, the petroleum industry is very vulnerable to the fluctuation in oil and gas prices. It is also under complex regulations, taxation laws, and financial accounting rules.

The value of a petroleum company relies on its oil and gas reserves. However, the evaluation of producing an oil and gas reserve depends on many certain and uncertain factors. These factors include how much and when the total oil and gas is produced from the reserve, at what prices the oil and gas can be sold, and how to capitalize investment costs under the regulated accounting rules and taxation laws.

Taking only oil production into consideration, after an oil field is discovered and developed, there are three major stages in the production of the oil field, *i.e.* primary, secondary, and tertiary production, as shown in the following figure.



Statistically, for an oil reservoir, primary, secondary, and tertiary recovery produces about 10%, 40%, and 15% of the original oil in place ($OOIP$), respectively. In primary recovery, oil is produced by natural drives in the reservoir. Afterwards, external forces or chemicals need to be added to the reservoir to produce the remaining oil from the reservoir. So the cost of secondary recovery is more expensive than that of primary recovery; and generally speaking, the cost of tertiary recovery is higher than that of secondary recovery. Water flooding is the least expensive and most widely used secondary oil recovery method. Since the oil production rate declines with production time, switching from primary to secondary, and from secondary to tertiary recovery increases oil production rates. There is a cost and benefit tradeoff between production rate and production method. In addition, when the oil prices are high, more oil is supposed to be produced to meet the market demands and thus the petroleum companies can benefit from the high oil prices.

How should a petroleum company arrange the oil production according to specific reservoir conditions and the changing oil prices so that the total value which the company receives from producing the natural resource can be maximized? A sound method is desired to provide the answer to this question. Traditionally, net present value (NPV) is used to value the project with the deterministic oil prices. However, this method cannot capture the uncertainty of oil prices and value of flexibility in decision making, such as the flexibility to decide when to switch from one oil production option to another, contingent on uncertain factors such as oil prices.

The real options approach, built on the theory for pricing financial options, addresses this challenge more successfully than conventional techniques such as the NPV method. The real options method takes uncertainty in oil prices, oil production rate change, and managers' flexibility in decision making into consideration and discovers the best time for switching from one production option to another. Thus the maximum value of producing the oil reservoir is obtained.

With a synthetic petroleum company and a synthetic oil exploration and production ($E\&P$) project, this dissertation establishes a binomial lattice real options evaluation method for the petroleum $E\&P$ industry to perform project evaluations under uncertainty. With the unique industry needs oriented, the dissertation builds binomial lattices for two stochastic oil price models, $i.e.$, the geometric Brownian motion (GBM) and one-factor mean reversion models, generates oil production profile with different water flooding switching times through reservoir simulations, and creates binomial lattice cash flow models, which take fiscal regimes, financial accounting rules, and taxations laws into consideration. The dissertation integrates the established real options evaluation framework with the binomial lattices of stochastic oil price models, oil production profile, and binomial lattice cash flow to achieve the evaluation functions. This real options evaluation method is then applied to identify the best switching time at changing oil prices with the flexibility of switching time from primary to water flooding oil recovery, and to capture the maximum value of the project for producing the synthetic oil reservoir. The results from the real options evaluation method are compared with those from the traditional NPV evaluation method.

The research and results for this study are presented through nine chapters. Chapter 2 is literature review on the related subjects. In Chapter 3, the historical West Texas Intermediate (WTI) oil prices are analyzed and the parameters for the geometric Brownian motion (GBM) oil price model are calibrated using the historical WTI oil prices. In Chapter 4, the parameters for one-factor mean reversion oil price model are calibrated with the historical WTI oil prices in different price regimes. In Chapter 5, simulations and comparisons of the GBM and one-factor mean reversion price models for the historical WTI oil prices are conducted. For the purpose of real options evaluations, in Chapter 6, specific reservoir simulations are designed and conducted, and an oil production profile for primary and water flooding oil recovery is generated. Chapter 7 covers comprehensive topics on cash flow projections for the petroleum $E\&P$ industry, including theories and applications on depreciation, full cost and successful efforts accounting methods, fiscal regimes, and cost of capital. In Chapter 8, the theory on risk- neutral world and risk-neutral probability is presented; the oil price stochastic processes are reconstructed into the risk-neutral binomial lattices; and the binomial real options evaluation method is established for determining the switching time from primary to water flooding oil recovery and for evaluating the petroleum $E\&P$ project under oil price uncertainty and with the flexibility in water flooding switching time. In Chapter 9, the conclusions from this study and the recommendations for future work are reached.

CHAPTER 2: LITERATURE REVIEW

2.1 Brownian Motion - from under the Microscope to the Stock Market

Brownian motion may be a familiar term to scholars as well as laymen. However, it may not be well-known now about the long history that the discovery of the "Brownian motion" phenomenon, the development of the Brownian movement theory, the establishment of the Brownian motion mathematic model, and the application of Brownian motion in many disciplines had dramatically changed the world of mathematics, physics, chemistry, finance, and economics. Studying Brownian motion led to the discovery of atoms and molecules; it transformed the molecular-statistical theory of the world from a hypothesis to a well-confirmed reality (Masani, 1990) and influenced the philosophy of perceiving the world. Two Nobel Prize awards were related to Brownian motion. Today, standard Brownian motion is the fundamental building block for almost all models for stock prices and other commodity prices. Understanding Brownian motion plays an important role in constructing and utilizing price models. "Studying the development of a topic in science can be instructive" (Nelson, 2001). This chapter shows how Brownian motion had evolved from under the microscope to the stock market.

Nowadays, the term "Brownian motion" has two meanings. One is the physical phenomenon relating to the fundamental properties of fluids: small particles suspended in a fluid will experience a continuous random movement which can be observed under a microscope. The other meaning of Brownian motion is the mathematical models used to describe this kind of irregular motion. The mathematical formula of Brownian motion can also be used to describe many other phenomena in many disciplines such as the movement of stock prices.

2.1.1 Physical Brownian Motion

In the 19th century and early 20th century, Brownian motion was studied by many scientists and researchers as a physical phenomenon (Mandrekar *et al*, 1994). Physical Brownian motion is the natural continuous random fluctuations of small particles suspended in stationary fluids. Physical Brownian motion is caused by the fundamental physical property of fluids. It was named in honor of the Scottish botanist Robert Brown because he was the first to systematically and scientifically study this incessant irregular motion after many other researchers had previously observed this phenomenon (Mazo, 2002).

In 1827, Robert Brown established Brownian motion as an important phenomenon. He studied the "Brownian motion" by carrying out a series of detailed observations of the movement of pollen grains suspended in water under the microscope (Haw, 2002). Brown noticed that the suspended particles were in constant motion, neither slowing down nor stopping; and the motion appeared to be irregular (Haw, 2005). Brown also showed that inorganic grains demonstrated the same kind of motion, which ruled out the interpretation for these phenomena as being due to motion particles' living origin (Haw, 2002). Brown verified that this motion was not caused by external influences such as light or temperature (Haw, 2005).

Thirty years later, by the 1860s, theoretical physicists became interested in investigating Brownian motion and searching for explanations of the phenomenon. Further systematic investigation showed that Brownian motion could be described with the following characteristics (Nelson, 2001): 1) the irregular motion was composed of translations and rotations and the trajectory was without tangent; 2) a given particle was equally likely to move in any direction; 3) further motion was totally unrelated to past motion; 4) the motion never stopped; 5) the smaller the particles, the more active the irregular motion; and 6) the less viscous and the higher temperature the fluid, the more active the motion. One theory that was proposed to explain Brownian motion was the kinetic theory of matter, though the theory at the time was only a hypothesis. The molecular movements of fluid were considered the cause of the continuously irregular motion of suspended particles. If the kinetic theory of fluids was right, then the molecules of fluid would move incessantly, and the particles immersed in the fluid would receive random bombardment by the molecules of the fluid which would then cause the suspended particles to move irregularly and unceasingly (Brownian Motion, 2006).

In the mid-19th century, great achievements in physics on the law of thermodynamics were made to successfully explain many material behaviors by the concepts of energy and entropy on the macro scale. In the third quarter of the 19th century, the kinetic theory of matter was developed by J. C. Maxwell, L Boltzmann, and R. J. E. Clausius to explain the heat phenomena on the micro scale (Brownian Motion, 2006). According to the kinetic theory, matter is composed of many tiny particles (molecules or atoms). These tiny particles are in constant motion, colliding with each other and bouncing back and forth. Calculating the statistical behavior of such a collection of particles can explain many thermodynamic results (Brownian Motion, 2006). However, since at that time there were neither observations nor experimental proofs for the existence of molecules or atoms, many scientists still did not believe in the theory. In addition, apparent contradiction existed between kinetic theory and the second law of thermodynamics on the reversibility of energy. In the kinetic theory, the motion of single particles obeys Newton's reversible mechanics; contrarily, the second law of thermodynamics demonstrated that many process are irreversible (Haw, 2005). Where, then, does the irreversibility come from when matters made up of particles obeying the reversible Newton's mechanics? One hypothesis proposed to reconcile the paradox of the kinetic theory and the second law of thermodynamics was that single particles follow Newton's law and large amounts of particles demonstrate the second law statistically (Haw, 2005). Since the irregular motion of Brownian particles never ceases, Brownian motion apparently violates the second law. By the end of the 19th century, scientists, including Gouy, suggested that Brownian motion might offer a "natural laboratory" which would allow direct testing of the kinetic theory and the examination of how kinetic theory and the second law of thermodynamics could be reconciled (Haw, 2005). However, efforts made in testing the kinetic theory did not use the right measurement. The measurement used was the velocity of Brownian particles. Since the velocity of Brownian motion constantly changes its direction and magnitude, the true velocity of Brownian motion is not directly measurable (Perrin, 1910). Finally, in 1905, Einstein applied both the molecular theory of heat and the macroscopic theory of dissipation to the Brownian movement, and obtained the first testable theory of Brownian motion with the right measurement (Chalmers, 2005 and Perrin, 1910).

With statistical approach and considering osmotic pressure as the driving force for diffusion, Einstein derived that a Brownian particle would diffuse according to the following equation (Einstein, 1956):

$$E(D^2) = \frac{RT}{N}\frac{1}{6\pi\eta P}t, \qquad (2-1)$$

where $E(D^2)$ is the mean-square displacement of the particle, R is the ideal gas constant, T is the absolute temperature, N is the universal constant-Avogadro's constant, η is the viscosity of the liquid, P is the size of the particle, and t is displacement time of the Brownian particle. The displacement was defined as "the length of the rectilinear segment which separates the point of departure from the point of arrival" (Perrin, 1910). Equation (2-1) indicates that the displacement of a Brownian particle would not increase linearly with time. Einstein identified the mean-square displacements of suspended particles, rather than their velocities, as suitable observations and measurable quantities.

Einstein's theory in explaining Brownian motion laid the theoretical foundations for the discovery of atoms. Even though molecules and atoms were too small to be seen directly, Einstein's derivation of the laws governing Brownian motion demonstrated how experimentalists could use the Brownian motion phenomenon to prove the real existence of molecules and atoms (Haw, 2005 and Stachel *et al*., 1989).

Einstein's theory on Brownian motion allowed French physicist Jean-Baptiste Perrin and others scientists to prove the physical reality of molecules and atoms (Haw, 2002).

During 1905 to 1911, Jean Perrin was successful in verifying Einstein's analysis, confirming the square root of time law, establishing the method to determine the Avogadro's number, and proving that atoms do exist (Haw, 2002, Haw, 2005, and Perrin, 1910). Because of his success in experimentally proving Einstein's theory and his contribution to reveal the reality of atoms, Perrin received the Physics Nobel Prize in 1926 (Haw, 2002).

2.1.2 Mathematical Brownian Motion - Wiener Process

As stated above, in 1905, Einstein developed a theory to explain Brownian motion as a physical phenomenon. Subsequently, mathematicians became interested in studying the "mathematical properties" of the curve followed by the Brownian movement and in constructing the continuous nondifferentiable curves of Brownian particles.

Once Einstein's model is analyzed only from a purely mathematical point of view, it can be described as follows (Jerison *et al.*, 1997):

"1. Brownian particles travel in such a way that the behavior over two different time intervals is independent. Thus, there is no way to predict future behavior from past behavior.

2. The particle is equally likely to move in any direction and the distance traversed by a Brownian particle during a time interval is on average proportional to the square root of the time.

3. The trajectories of Brownian particles are continuous."

It was challenging to construct a distribution mathematically on the space of trajectories so that the above criteria could all be met. In the early 20th century, many mathematicians, including Levy and Kolmogorov, were working on this problem. However, none of them were able to mathematically rationalize Einstein's model on Brownian motion (Jerison *et al*., 1997).

Around 1920, motivated by the achievements of Einstein and others, American mathematician Norbert Wiener started to work on Brownian motion at MIT (Mandrekar *et al.*, 1994). Norbert Wiener solved the measure problem (called Wiener measure) and mathematically constructed the standard Brownian motion (also called mathematical Brownian motion). This mathematical model of standard Brownian motion is also called the Wiener process, in honor of Wiener's contribution. Now the terms "Wiener process" and "standard Brownian motion" are used interchangeably (Neftci, 1996). Wiener's mathematical model for physical Brownian motion is one of the major 20th century's achievements in mathematics (Mandrekar *et al.*, 1994). The Wiener process also plays an important role in the development of the stochastic process theory and modern probability theory by other mathematicians (Masani, 1990 and Jerison *et al.*, 1997).

Mathematically, the Wiener process is a continuous-time stochastic process W_t for $t \geq 0$ with $W_0 = 0$ and W_t being normally distributed with a mean of zero and a variance of t. For any $0 \leq s < t$, the increment $W_t - W_s$ is distributed normally with a mean of zero and a variance of $(t - s)$. The increments for any non-overlapping time intervals are independent. There are three important properties of the Wiener process:

Property 1: Wiener process is a Markov process.

A Markov process is a stochastic process in which the probability distribution of all future values at any time of the process depends only on its current value. The past values and the path of how past values were reached will not affect the future distribution. Any other current information will not affect the future path of the process either (Dixit and Pindyck, 1994).

Property 2: Wiener process has independent increments over different time intervals.

With this property, the distributions of dW for any two different time intervals are independent. That is, if $0 \leq s < t \leq u < v$, then $(W_t - W_s)$ and $(W_v - W_u)$ are independent random variables. Similar results hold for more than two non-overlapping intervals (Dixit and Pindyck, 1994).

Property 3: The change in the Wiener process is normally distributed (Dixit and Pindyck, 1994).

Within any time interval $dt,$ the mean of the change dW is zero and the variance of the change is dt. The variance of the change is proportional to the length of time interval. That is,

$E(dW) = 0$, and

$Var(dW) = dt.$

In other words,

$$dW \sim N\left(0, \sqrt{dt}\right), \qquad (2-2)$$

or, if $0 < s < t$, then

$$(W_t - W_s) \sim N\left(0, \sqrt{t-s}\,\right).$$

The Wiener process can also be described as

$$dW = \varepsilon\sqrt{dt}\,,$$

where $\varepsilon = N(0,\ 1)$ is the standard normal distribution with mean being equal to zero and standard deviation being equal to one.

With the above mathematical model of Brownian motion, Einstein's equation for Brownian motion can be expressed by the Wiener process. The exact position of a Brownian particle from now to time t is unknown. Einstein's theory gives a probability distribution of displacement D (position change). Eq. (2-1) can be written as

$$E(D^2) = \sigma^2 t\ ,$$

where

$$\sigma^2 = \frac{RT}{N}\frac{1}{6\pi\eta P}\,.$$

The following derivation shows that when the mean displacement of a Brownian particle is zero, and the variance of the displacement is $\sigma^2 t$.

Let $D = \sigma dW$

with $dW = \varepsilon\sqrt{dt}\,,$

where $\varepsilon = N(0, 1)$ is the standard normal distribution with a mean of zero and a variance of one. Then,

$$E(D) = E\left(\sigma\varepsilon\sqrt{dt}\right) = \sigma\sqrt{dt}\,E(\varepsilon) = 0,$$

$$\begin{aligned} Var(D) &= E(D^2) - E^2(D) \\ &= E(\sigma^2\varepsilon^2 dt) - 0 \\ &= \sigma^2 dt\, E(\varepsilon^2) \\ &= \sigma^2\, dt\,. \end{aligned}$$

That is, the distribution of the displacement of a Brownian particle is according to:

$$D = N\left(0,\ \sigma\sqrt{dt}\right). \qquad (2-3)$$

Comparing Eq. (2-2) with Eq. (2-3), it is observed that the mathematical Brownian motion (the Wiener process, or the standard Brown motion), is a specific physical Brownian motion when the value of $\left(\frac{RT}{N}\frac{1}{6\pi\eta P}\right)$ is equal to one in Einstein's equation for the physical Brown motion.

2.1.3 Brownian Motion in the Stock Market

The mathematical theory of Brownian motion has been applied to contexts extending far beyond the movement of particles in fluids which usually cannot be observed without a microscope. The research in the mid-20th century supports that prices on the stock market varies in a similar fashion to the particles in the physical Brownian motion. The Brownian motion theory has then been widely applied to the modeling of stock prices.

Stock prices have many characteristics in common with Brownian motion particles. Not only are the macro phenomena of the evolution of stock prices and the movement of Brownian particles similar to each other, but the driving forces for both phenomena are also comparable. On the "micro" scale, for physical Brownian motion, a Brownian particle moves irregularly by "hitting" from millions of molecules in fluid. Physical Brownian motion has three dimensions. The Brownian particles changes their positions in both X and Y directions with time. In stock markets, prices could be imagined as "Brownian particles" that are driven about by many traders. Stock prices rise when "hit" by buyers; they fall when “hit” by sellers. Such movement of stock prices is just like Brownian particles moving “up” and “down” in the Y direction caused by the molecules’ shock of fluid.

In 1958, Osborne first introduced the study of a logarithmic form of stock price and “explicitly tied the random walk hypothesis in with stock price change” (Cootner, 1964). It is the proportionate change in the value of stock, rather than the absolute stock value that investors are interested in. Osborne showed that “logarithms of common stock prices can be regarded as an ensemble of decisions in a statistical steady state, and that this ensemble of logarithms of prices, each varying with the time, has a close analogy with the ensemble of coordinates of a large number of molecules” (Osborne, 1959). Thus, the changing in the logarithms of stock price is a close analogy to the changing of molecules’ positions. While the exact position of a Brownian particle from now to time t is uncertain, the probability distribution of the position change (displacement D) of the Brownian particle can be modeled using the Wiener process. Even though the next “position” of the stock price is unknown, the probability distribution of the price change can also be derived from the Wiener process.

Not only were the similarity between the position change of Brownian particles and the price change in the stock market studied and understood, the differences in these two phenomena were also captured when stock prices were modeled with the Wiener process. Unlike Brownian particles which have the mean displacement of zero, stock price earns a certain return. Thus the mean change of stock price should be greater than zero. For example, when the mean price change per unit of time is assumed as μ, the stock price change can thus be modeled as

$$dLnP = \mu dt + \sigma dW = \mu dt + \sigma\varepsilon\sqrt{dt}$$

$$E(dLnP) = \mu dt$$

$$\begin{aligned} Var(dLnP) &= E(dLnP)^2 - E^2(dLnP) \\ &= E(\mu dt)^2 + E(\sigma^2\varepsilon^2 dt) + 2E\left[\mu(dt)\sigma\varepsilon\sqrt{dt}\right] - (\mu dt)^2 \\ &= \sigma^2 dt E(\varepsilon^2) \\ &= \sigma^2 dt\,. \end{aligned}$$

In the options pricing theory, the Wiener process is an important building block for modeling asset prices. In 1973, Fisher Black, Myron Scholes, and Robert Merton developed the theory for pricing stock options, known as the Black-Scholes model (Hull, 2000). In the Black-Scholes options pricing model, stock prices were modeled using Geometric Brownian motion (Brownian motion with drift) (Black and Scholes, 1973). The breakthrough in options pricing made by Black, Scholes, and Merton started a new era of derivative securities. Myron Scholes and Robert Merton were awarded the Nobel Prize in Economics in 1997 (Hull, 2000).

2.1.4 Summary

The history from discovery of the Brownian motion phenomenon to the application of Brownian motion theory bears many fruits in science. In 1828, Brown first established Brownian motion as a phenomenon to study. In 1905, Einstein developed the theory to explain the physical Brownian motion. From 1905 to 1911, Perrin verified Einstein's theory and revealed the reality of atoms and molecules. Perrin received the Nobel Prize for his contribution. In 1923, Wiener created a mathematical model for Brownian motion. Modern probability theory and stochastic process theory were developed based on Wiener's pioneering work. In 1973, Fisher Black, Myron Scholes, and Robert Merton used a geometric Brownian motion price model to develop options pricing for stocks. Myron Scholes and Robert Merton were awarded the Nobel Prize in 1997. Nowadays, the standard Brownian motion is the building block for almost all price models.

During almost two centuries after Robert Brown, it is gradually becoming clearer that, in nature, there is a balance between function and fluctuation, and between strict physical rules and random effects. Understanding Brownian fluctuations is vital to the understanding of many phenomena ranging from under the microscope to the stock market. Brownian motion continues to be of immeasurable importance in modern science and human life.

2.2 From Aboveground Financial Market to Underground Petroleum Reservoir - Real Options Evaluation in Petroleum *E*&*P* Industry

With the generation of models for the stochastic processes built on the fundamental building block of Brownian motion, the theory and application of option pricing have been developed. Bachelier (1900), a French mathematician, first derived a formula for pricing stock options with the assumption that stock prices evolves according to Brownian motion with zero drift, in his Ph.D. dissertation. In 1973, "Fischer Black and Myron Scholes presented the first completely satisfactory equilibrium option pricing model." (Cox, Ross, and Rubinstein, 1979) The Black-Scholes theory of option pricing (Black and Scholes, 1973) can be used for pricing options for common stocks, corporate bonds, and warrants. Later in the same year, Merton (1973) presented the mathematical understanding of the Black-Scheloes option pricing theory. The Black-Scholes formula is used for pricing European-style options with the assumption that the stochastic prices follow the geometric Brownian motion (GBM) process. The Black-Schelos theory of option pricing formed the basis for many subsequent academic studies.

Shortly after the development of the option pricing theory by Black, Scholes, and Merton, three other main methodologies for pricing options for financial assets were proposed: the binomial model, the Monte Carlo simulation, and the finite difference model.

Cox, Ross, and Rubinstein (1979) developed a discrete-time binomial model for valuing options. Based on the no arbitrage assumption, the binomial option pricing model is a mathematically simple, intuitive, and powerful method for pricing many complex options, such as American options. Cox, Ross, and Rubinstein also showed that the binomial model converges to the Black-Scholes model when the number of time periods increases to infinity and the length of each time period is infinitesimally short.

The Monte Carlo method was first proposed in 1977 by Phelim Boyle for European options (Boyle, 1977). Later Broadie and Glasserman (1996) presented a method to price Asian options through Monte Carlo simulation. Longstaff and Schwartz (2001) developed a Monte Carlo method to price American options. The Monte Carlo simulation method for option pricing is based on the risk neutral valuation; it can be used in pricing those options whose value is dependent on more than one stochastic variables or can be used in the jump-diffusion process and other complicated option pricing. The drawback of this method is the intensive computation involved.

The third method of pricing options is a numerical method called finite difference method. This method was first presented by Schwartz (1977). When the option value evolves as a function of time and underlying prices according to a partial differential equation (PDE), such as in the case of Black-Schelos PDE, then the PDE can be modeled in a discretized form with finite differences and the option value can be derived. This method is limited by the number of underlying variables.

Not only is options theory widely used for financial assets, but also for real assets as well. "Option pricing theory is relevant to almost every area of finance. For example, virtually all corporate securities can be interpreted as portfolios of puts and calls on the assets of the firm." (Cos, Ross, and Rubinstein, 1979). When option pricing theory is applied to real investment under uncertainty to capture the value of managerial flexibility, it is called real options. Real options application started from 1977 by Stewart Myers (Smith and McCardle, 1998). Since 1985, after Brennan and Schwartz (1985) presented a paper on how to value natural resource investments under uncertain output prices, academic scholars and industry researchers have been developing the real options evaluation for petroleum exploration and production ($E\&P$) industry.

A rich body of literature shows that petroleum exploration and production ($E\&P$) industry is a very important area of the application of real options evaluation. Siegel, Smith, and Smith (1985) demonstrated the insights of applying financial option pricing theory to the valuation of undeveloped petroleum properties based on the analogy between undeveloped oil reserves and stock call options. Lehman (1989) illustrated how to value oil field investments using option pricing theory with the dynamic programming approach. Gibson and Schwartz (1990) developed and tested the two-factor stochastic oil price model for the application to value real assets and decisions indexed on oil prices. Cortazar and Schwartz (1997) presented a no arbitrage model for the evaluation of an undeveloped oil field based on the mean reversion and risk adjusted oil prices. Smith and McCardle (1998) proposed an approach to integrate option pricing and decision analysis in valuing oil properties. Claeys and Walldner (1999) discussed the technologies on discovering real options in the exploration and development of oil field. Zettl (2000) compared different approaches in the real options evaluation for the petroleum $E\&P$ projects, including the continuous models, discrete models, and combining option pricing theory with Monte Carlo simulation. In 2001, real option evaluation of a satellite field in the North Sea (Galli, Jung, Armstrong, and Lhote, 2001) and farmout valuation using exotic real options (Rutherford, 2001) were presented. The value of flexibility in managing uncertainty in oil and gas investments using real options approach was proposed (Begg and Bratvold, 2002). In 2009, a real options evaluation model was developed for the petroleum exploration project while production rate was considered the key factor of uncertainty (De Abreu and Filho, 2009). Jafarizadeh and Bratvold (2009) analyzed the strength and weakness of the real option methods applied in petroleum $E\&P$ industry and pointed out that attention on the underlying assumptions of each valuation method is very important to prevent the systematic errors in real options evaluation.

Specifically, among the methods to solve real options problems related to uncertain oil prices, many discrete time approaches have been proposed and developed. Jailet, Ronn, and Tompaidis (2004) developed the framework, using trinomial lattice to approximately model the one-factor mean reversion price model, to value commodity-based swing options. Brandao and Dyer (2005) proposed a discrete time method to enhance the real options methodology developed by Copeland and Antikarov (2001). Hahn and Dyer (2008) generated recombining lattice approach to value managerial flexibility with underlying stochastic processes being mean reversion. The discrete time methods of real options evaluation can include multiple underlying uncertainties and provide greater flexibility in the modeling of real options valuation.

After the Black-Scholes option pricing theory was developed to value the options in financial market, the applications of option pricing to real assets were emerged and extended in many areas. In more than three decades, the real options evaluation in petroleum exploration and production ($E\&P$) industry has evolved from concepts towards practical evaluation and decision making methodology. Nowadays, the real options pricing and evaluation, especially the discrete time approach, is becoming a powerful tool for the petroleum $E\&P$ industry to value the underground petroleum reservoir.

CHAPTER 3: WEST TEXAS INTERMEDIATE (WTI) OIL PRICE ANALYSIS AND GEOMETRIC BROWNIAN MOTION OIL PRICE MODEL

3.1 ANALYSIS OF HISTORICAL WTI OIL PRICE BEHAVIOR

The oil prices for this analysis are the West Texas Intermediate (WTI) crude oil free on board (FOB) historical spot prices from January 2, 1986 to April 7, 2010. This period covers 24.27 actual years, around 292 months, with an actual total of 6121 trading days, excluding weekends and holidays. The actual total of trading days for the 20 years from January 2, 1986 to December 31, 2009 is 6056 days. Compared to the standard trading days of 252 days per year, which is 6,048 days for 20 years, the difference is eight days with an error of 0.13%. The data are from U. S. Energy Information Administration (EIA) (U. S. Energy Information Administration, 2010a).

WTI is a crude stream produced in Texas and southern Oklahoma. WTI is one of the most actively traded U.S. domestic crudes. It serves as the benchmark crude for the U. S. petroleum industry for the pricing of a number of other crude streams. The principal production area of WTI is Midland, Texas, with an approximate production rate of 400,000 bbl/day in the area. WTI is traded in the U.S. domestic spot market in Cushing, Oklahoma, the nexus of spot-market trading in the United States; it is also accessible to the international spot markets via pipelines.

Figure 3-1 shows the evolution of WTI daily spot prices from January 2, 1986 to April 7, 2010. The lowest oil price was about $10/bbl, which occurred in March 1986. There were several low price periods between 1986 and 1987, around 1989, around 1994, and between 1988 and 1989, when oil prices fluctuated between \$15/bbl and less than \$20/bbl. The highest oil price was more than \$140/bbl, which occurred in July 2008. From early 2002 to April 7, 2010, oil prices have been higher than \$20/bbl. Figure 3-2 shows the logarithms of WTI daily spot prices LnP_t . LnP_t was between 2.3 and 5.0.

In finance, $\frac{\Delta P_t+D_t+P_t}{P_t}$ is the gross holding period return during the time period of $[t, t+\Delta t]$ for an investment, where D_t is the dividend payment and $\frac{\Delta P_t+D_t}{P_t}$ is the net return. For the price itself, there is no dividend payment, and thus $\frac{\Delta P_t}{P_t}$ is a measurement of net return, or net price return.

Since $\lim_{\Delta P \to 0} \frac{\Delta LnP}{\Delta P} \approx \frac{1}{P}$, then $\frac{\Delta P_t}{P_t} \approx \Delta(LnP_t)$ when ΔP_t is small enough. $\Delta LnP_t = LnP_t - LnP_{t-1}$ is a measurement for net price return. $\Delta LnP_t = LnP_t - LnP_{t-1}$ is also called geometric price return; and $\frac{\Delta P_t}{P_t}$ is also called arithmetic price return. When prices are modeled with the logarithm form of prices, they are prevented from becoming negative values.

Figures 3-3 and 3-4 are the WTI historical daily spot price return in the forms of $(LnP_t - LnP_{t-1})$ and $(P_t - P_{t-1})/P_{t-1}$, respectively. Figures 3-3 and 3-4 look similar to each other, showing very strong randomness.

Table 3-1 is the summary statistics for the WTI daily price return in both the forms of $(LnP_t - LnP_{t-1})$ and $(P_t - P_{t-1})/P_{t-1}$, as well as the weekly and monthly price return in the form of $(LnP_t - LnP_{t-1})$. Figures 3-5, 3-6, and 3-7 are histograms of the price return $(LnP_t - LnP_{t-1})$ for WTI daily, weekly, and monthly oil prices, respectively. Figures 3-8, 3-9, and 3-10 are the Q-Q normality test plots for the price return $(LnP_t - LnP_{t-1})$ for WTI daily, weekly, and monthly oil prices, respectively. The straight lines in the plots illustrate the normal distribution. Figures 3-8, 3-9, and 3-10 show that the distributions of three kinds of $(LnP_t - LnP_{t-1})$ are disparate from normal distribution and the daily price data have the largest discrepancy from normal distribution. Table 3-1 shows that the distribution skewness for $(LnP_t - LnP_{t-1})$ of three kinds of WTI oil prices and $(P_t - P_{t-1})/P_{t-1}$ for daily prices is negative, indicating that the left tails of the distributions are heavier than the right tails. The kurtosis of $(LnP_t - LnP_{t-1})$ and $(P_t - P_{t-1})/P_{t-1}$ for daily prices is much greater than 3.0, indicating that the two distributions have sharper peaks, thinner shoulders, and fatter tails than normal distribution. This describes a leptokurtic distribution. The distribution of $(LnP_t - LnP_{t-1})$ for monthly prices has a kurtosis value of less than 3, which is termed platykurti, indicating less peaked and thinner tailed plot than the normal distribution.

3.2 Description of the Geometric Brownian Motion (GBM) Price Model

The geometric Brownian motion (GBM) price model is built on the Wiener process W by adjusting the variance of the increment of the Wiener process dW with a parameter σ and adding the drift term μdt as a deterministic part. That is, the GBM process is the sum of a deterministic term and a random term: Einstein's Brownian motion σdW. The

GBM price model can be expressed with the following formula: let P be the stock price at time t, then the price change dP in $[t, t+dt]$ is

$$dP = \mu P dt + \sigma P dW, \tag{3-1}$$

where both μ and σ are constants. W is the Wiener process with $dW = N(0,1)\sqrt{dt}$. From Eq. (3-1), price returns in both forms of $(P_t - P_{t-1})/P_{t-1}$ and $(LnP_t - LnP_{t-1})$ are derived as follows.

First, Eq. (3-1) can be rewritten as

$$\frac{dP}{P} = \mu dt + \sigma dW. \tag{3-2}$$

Over any time interval $[t, t+dt]$, $\frac{dP}{P}$ is normally distributed with

$$E\left(\frac{dP}{P}\right) = \mu dt,$$

$$Var\left(\frac{dP}{P}\right) = \sigma^2 dt.$$

That is

$$\frac{dP}{P} \sim N\left(\mu dt, \sigma\sqrt{dt}\right). \tag{3-3}$$

Secondly, let

$$X = LnP. \tag{3-4}$$

Applying Ito's Lemma to Eq. (3-4) (Hull, 2000):

$$dX = \frac{\partial X}{\partial t}dt + \frac{\partial X}{\partial P}dP + \frac{1}{2}\frac{\partial^2 X}{\partial P^2}(dP)^2.$$

From Eq. (3-1),

$$(dP)^2 = \sigma^2 P^2 dt,$$

then

$$dX = \left[\frac{\partial X}{\partial t} + \mu P \frac{\partial X}{\partial P} + \frac{1}{2}\sigma^2 P^2 \frac{\partial^2 X}{\partial P^2}\right] dt + \sigma P \frac{\partial X}{\partial P} dW.$$

Since

$$\frac{\partial X}{\partial t} = 0,$$

$$\frac{\partial X}{\partial P} = \frac{1}{P},$$

$$\frac{\partial^2 X}{\partial P^2} = -\frac{1}{P^2},$$

then

$$dX = dLnP = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dW, \tag{3-5}$$

where $= N(0,1)\sqrt{dt}$, σ and μ are considered constants within $[t, t + dt]$.

Lastly, from time 0 to time T, the change in LnP is

$$LnP_T - LnP_0 \sim N\left[\left(\mu - \frac{\sigma^2}{2}\right) T, \sigma\sqrt{T}\right].$$

That is

$$LnP_T \sim N\left[LnP_0 + \left(\mu - \frac{\sigma^2}{2}\right) T, \sigma\sqrt{T}\right]. \tag{3-6}$$

Equation (3-6) shows that given a price today, the price at time T is lognormally distributed. In the finite time T, the price P_T has the mean of

$$E(P_T) = P_0 Exp(\mu T), \tag{3-7}$$

and the variance of

$$Var(P_T) = P_0^2 Exp(2\mu T)[Exp(\sigma^2 T) - 1] \quad \text{(Hull, 2000)}. \tag{3-8}$$

3.3 Method of Parameter Estimations for the Geometric Brownian Motion Price Model

From Eq. (3-6), it is clear that as long as the parameters μ and σ are estimated, the price probability distribution in the future can be forecasted. The formulas for estimating the parameters in the GBM model using historical price data can be derived as follows:

$$dLnP = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dW,$$

$$(dLnP)^2 = \left[\left(\mu - \frac{\sigma^2}{2}\right)dt\right]^2 + (\sigma dW)^2 + 2\left(\mu - \frac{\sigma^2}{2}\right)dt(\sigma dW),$$

$$E(dLnP) = E\left[\left(\mu - \frac{\sigma^2}{2}\right)dt\right] + E[\sigma dW]$$

$$= \left(\mu - \frac{\sigma^2}{2}\right)dt + E\left(\sigma N(0,1)\sqrt{dt}\right)$$

$$= \left(\mu - \frac{\sigma^2}{2}\right)dt$$

$$= \psi dt,$$

$$\psi = \left(\mu - \frac{\sigma^2}{2}\right) = \frac{E(dLnP)}{dt}, \tag{3-9}$$

$$\mu = \psi + \frac{\sigma^2}{2}, \tag{3-10}$$

$$Var(dLnP) = E(dLnP)^2 - E^2(dLnP)$$

$$= \left[\left(\mu - \frac{\sigma^2}{2}\right)dt\right]^{\mathbf{2}} + \sigma^2(dt)E(N(0,1))^2 - \left[\left(\mu - \frac{\sigma^2}{2}\right)dt\right]^2,$$

$$= \sigma^2 dt\,,$$

then,

$$\sigma^2 = \frac{Var(dLnP)}{dt}, \tag{3-11}$$

or

$$\sigma = \sqrt{\frac{Var(dLnP)}{dt}}. \tag{3-12}$$

Equations (3-9), (3-10), and (3-12) are used to estimate the two parameters, i.e. drift rate μ and volatility σ, in the GBM price model.

3.4 Results of Parameter Estimations for the Geometric Brownian Motion Price Model Using Historical WTI Oil Price Data

Historical WTI daily, weekly, and monthly price data from January 2, 1986 to April 7, 2010 are used to calibrate and analyze the drift rate μ and volatility σ for the GBM oil price model. For daily price data, both monthly and yearly grouping sizes are used. When daily prices are grouped monthly, it is assumed that both the daily drift rate and the daily standard deviation are constants within each month. When daily prices are grouped yearly, the daily drift rate and the daily standard deviation within each year are assumed not to change. When daily drift rate is annualized, it is assumed that the daily drift rate can be scaled to annual drift rate with 252 trading days per year. Annualized standard deviation for price return in the form of logarithm price $(LnP_t - LnP_{t-1})$ is called volatility. The same concept applies to the weekly and monthly price data. Thus, annualized drift rate and volatility have the same base of comparison for daily, weekly, and monthly price data with different grouping sizes. Figures 3-11 and 3-15 are the annualized drift rate and volatility using daily oil prices grouped monthly. Figures 3-12 and 3-16 are the annualized drift rate and volatility using daily oil prices grouped yearly. Figures 3-13 and 3-17 are the annualized drift rate and volatility using weekly oil prices grouped yearly. Figures 3-14 and 3-18 are the annualized drift rate and volatility with monthly price data grouped yearly.

Figures 3-11 and 3-15 show that both the annualized drift rate and volatility from WTI daily oil prices change month by month. The highest monthly grouped annualized drift rate from daily oil prices is around $5/year, while the lowest is $-4/year. The majority of the monthly grouped annualized daily drift rate is between $-2/year and $2/year. The highest volatility from monthly grouped daily oil prices is 1.8, while the lowest is around 0.1; the majority is ranged between 0.2 and 0.5.

From Figures 3-12 and 3-16 it is observed that the yearly grouped annualized daily drift rate and volatility change year by year. The range for the yearly grouped annualized daily drift rate is between $-0.6/year and $0.8/year, much narrower than that of the aforementioned monthly grouped annualized daily drift rate, which ranges between $-4/year and $5/year. The range of the lowest and highest yearly grouped volatility from daily oil prices is between 0.2 and 0.69, much narrower than that of the lowest and highest monthly grouped volatility from daily prices, which ranges between 0.1 and 1.8. Thus, grouping size has a very significant impact on the annualized daily price drift rate and volatility.

Figures 3-13 and 3-14 show that the yearly grouped annualized drift rate from weekly and monthly price data has a very similar changing pattern and value range year by year, and both are very similar to the changing pattern and value range of the yearly grouped annualized daily price drift rate. In other words, when price data are grouped yearly, no matter daily, weekly, or monthly data are used, the annualized drift rate is very similar to each year.

Volatility from yearly grouped weekly and monthly prices changes year by year as it can be seen from Figures 3-17 and 3-18. Their changing pattern is very close to that of the volatility from daily prices grouped yearly.

The results from Figures 3-11 to 3-18 show that the assumptions of constant drift rate and constant volatility for the GBM price model do not hold for WTI historical daily, weekly, or monthly price data.

Tables 3-2 and 3-3 are the results of the annualized drift rate and volatility using WTI historical daily, weekly, monthly, and yearly price data in three periods: whole price data from January 2, 1986 to April 7, 2010; low oil price data from January 2, 1986 to December 30, 1999; and high oil price data from January 4, 2000 to April 7, 2010. Table 3-2 shows that for all the four kinds of price data, the period from January 4, 2000 to April 7, 2010 has the highest annualized drift rate; the period from January 2, 1986 to December 30, 1999 has the lowest annualized drift rate. Within the same period, yearly price data results in the highest annualized drift rate; next to the yearly data is the daily data; the weekly and monthly data have very close and lowest annualized drift rate. Table 3-3 shows that, unlike the annualized drift rate, for all of the four kinds of price return, the volatility in the three time periods is very close to each other. For example, the volatility with daily prices from January 4, 2000 to April 7, 2010 is 0.4277, while that in the time period of January 2, 1986 to December 30, 1999 is 0.4119. Within the same time period, yearly data result in the highest volatility; weekly and monthly prices result in the lowest volatility, and the volatility from weekly and monthly data are very close to each other.

3.5 Ergodicity and Stationarity Assumptions

Theoretically, the parameters in the price model should be estimated from an ensemble, which is the multiple time series data over the same time period for the same process. For example, to test the distribution of the displacement of Brownian particles in the physical Brownian motion from time zero to time t, the experiments can be repeated under the same conditions for as many times as designed. However, for the historical oil price data, there is only one realization of price for any given time t. From time series data, the time series average (first moment) and time series variance and covariance (second moment) can be calculated. Once the process is ergodic and stationary, the time series average will converge to the ensemble mean, and the time series variance and covariance will be converged to the ensemble variance and covariance. Under those ergodicity and stationary assumptions, the unknown mean and variance can be estimated by the sample mean and sample variance of one realization $\{x_t\}_1^T$ with the estimated mean $\bar{x}$ and variance s^2 as follows (Hamilton, 1994, Mills, 2003, and Enders, 1995):

$$\bar{x} = \frac{1}{T}\sum_{1}^{T} x_t,$$

$$s^2 = \frac{1}{T}\sum_{t=1}^{T} (x_t - \bar{x})^2.$$

Mills (2003) states that all time series are assumed to have an ergodicity property. Thus, the parameters in the GBM model can be estimated from one realization of the historical time series price data.

Table 3-1: Summary Statistics for the WTI Oil Price Return from January 2, 1986 to April 7, 2010

	$LnP_t - LnP_{t-1}$ WTI Daily Prices	$(P_t - P_{t-1})/P_{t-1}$ WTI Daily Prices	$LnP_t - LnP_{t-1}$ WTI Weekly Prices	$LnP_t - LnP_{t-1}$ WTI Monthly Prices
Mean	0.00020	0.00054	0.00092	0.00726
Variance	0.00070	0.00069	0.00207	0.01599
Std. Dev.	0.02637	0.02618	0.04550	0.12644
Skewness	-0.7857	-0.2332	-0.1859	-1.2695
Kurtosis	17.5638	13.0861	3.3576	1.6781
Median	0.00066	0.00066	0.00307	0.03229
Minimum	-0.40640	-0.33395	-0.19234	-0.33198
Maximum	0.19151	0.21107	0.25125	0.20408
Count	6120	6120	1265	34

(Source: U. S. Energy Information Administration, 2010a)

Table 3-2: Annualized Drift Rates of the WTI Oil Prices for the GBM Price Model Using Historical Price Data from January 2, 1986 to April 7, 2010

	January 2, 1986- April 7, 2010	January 2, 1986 - December 30, 1999	January 4, 2000 - April 7, 2010
Daily	0.13740	0.08539	0.20921
Weekly	0.10190	0.05262	0.17463
Monthly	0.10032	0.05465	0.16247
Yearly	0.17781	0.11859	0.20891

Table 3-3: Volatility of the WTI Oil Prices for the GBM Price Model Using Historical Oil Price Data from January 2, 1986 to April 7, 2010

	January 2, 1986 - April 7, 2010	January 2, 1986 - December 30, 1999	January 4, 2000 - April 7, 2010
Daily	0.4186	0.4119	0.4277
Weekly	0.3281	0.3196	0.3393
Monthly	0.3098	0.3012	0.3217
Yearly	0.4823	0.4457	0.5094

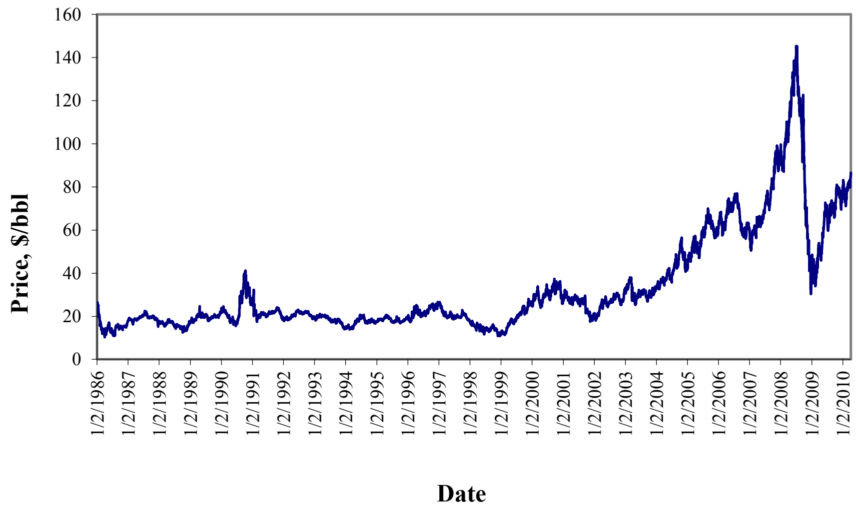


Figure 3-1: Evolution of WTI Daily Spot Prices (FOB) from January 2, 1986 to April 7, 2010
(Source: U. S. Energy Information Administration, 2010a)

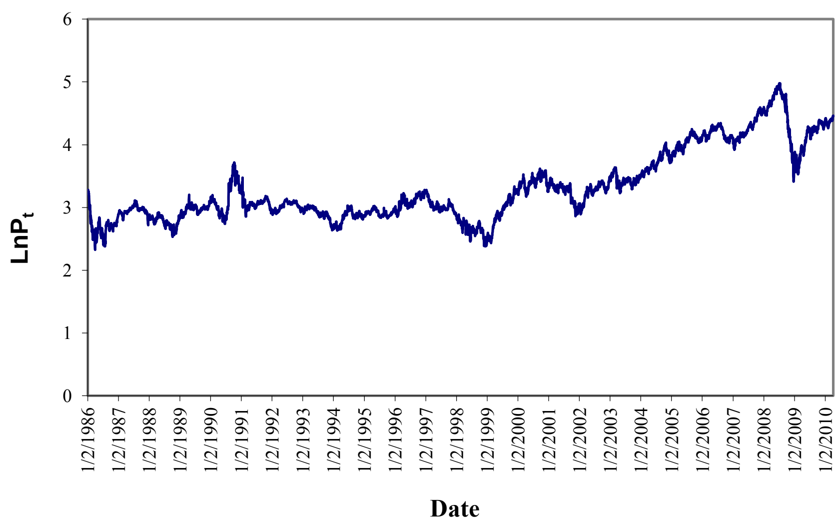


Figure 3-2: Evolution of WTI Daily Prices (LnP_t) from January 2, 1986 to April 7, 2010
(Source: U. S. Energy Information Administration, 2010a)

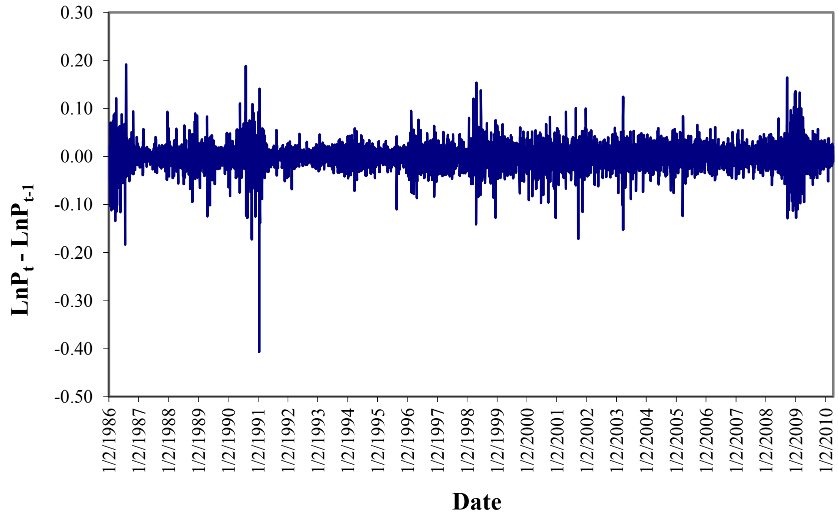


Figure 3-3: WTI Daily Price Return from January 2, 1986 to April 7, 2010 in the Form of $(LnP_t - LnP_{t-1})$
(Source: U. S. Energy Information Administration, 2010a)

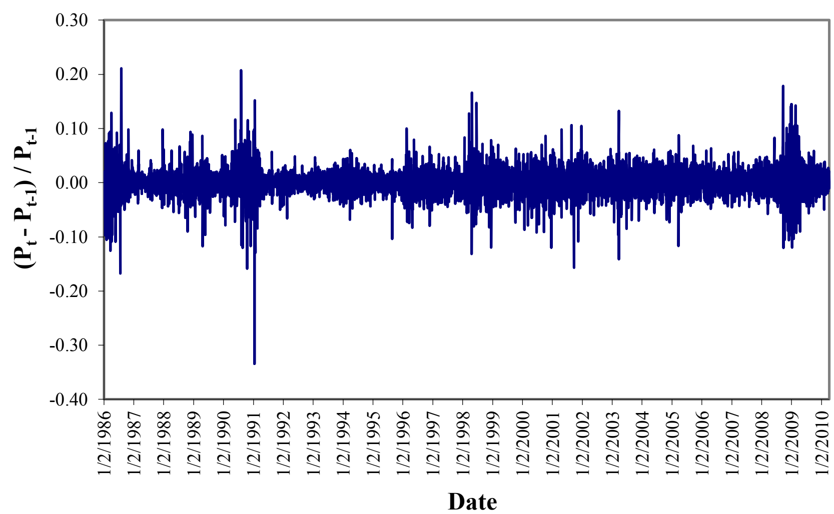


Figure 3-4: WTI Daily Price Return from January 2, 1986 to April 7, 2010 in the Form of $(P_t - P_{t-1})/P_{t-1}$
(Source: U. S. Energy Information Administration, 2010a)

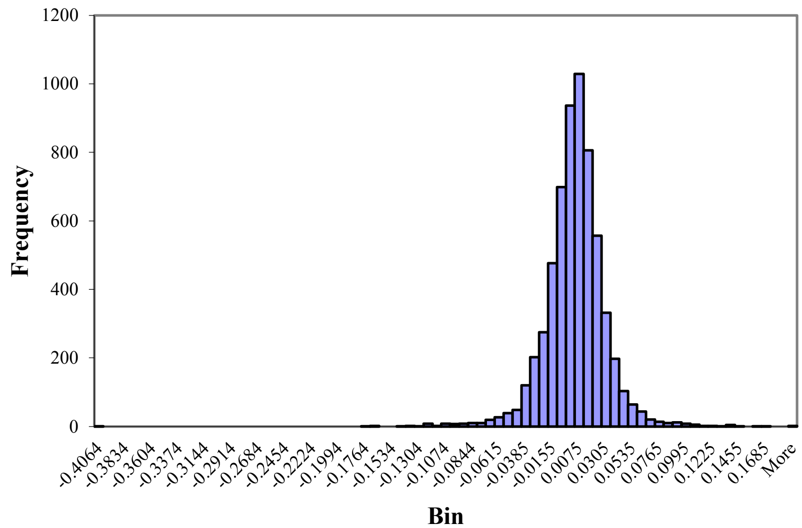


Figure 3-5: Histogram of $(LnP_t - LnP_{t-1})$for the WTI Daily Price Return from January 2, 1986 to April 7, 2010
(Source: U. S. Energy Information Administration, 2010a)

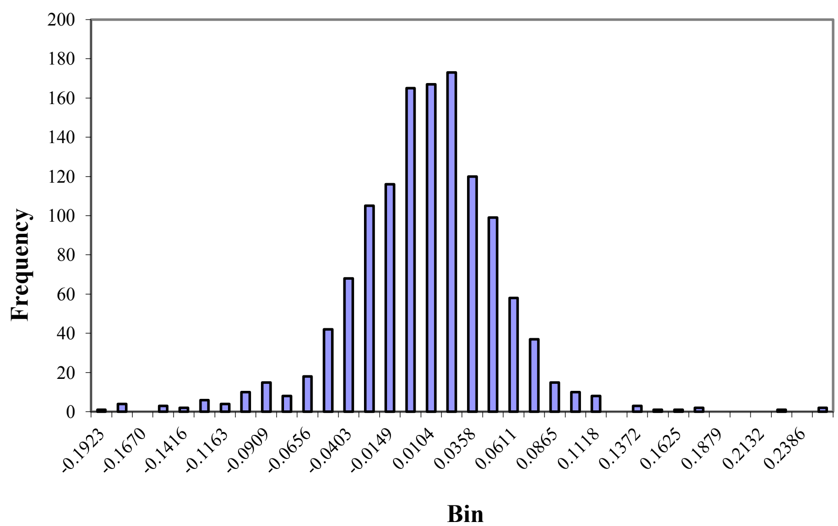


Figure 3-6: Histogram of $(LnP_t - LnP_{t-1})$ for the WTI Weekly Price Return from January 2, 1986 to April 7, 2010
(Source: U. S. Energy Information Administration, 2010a)

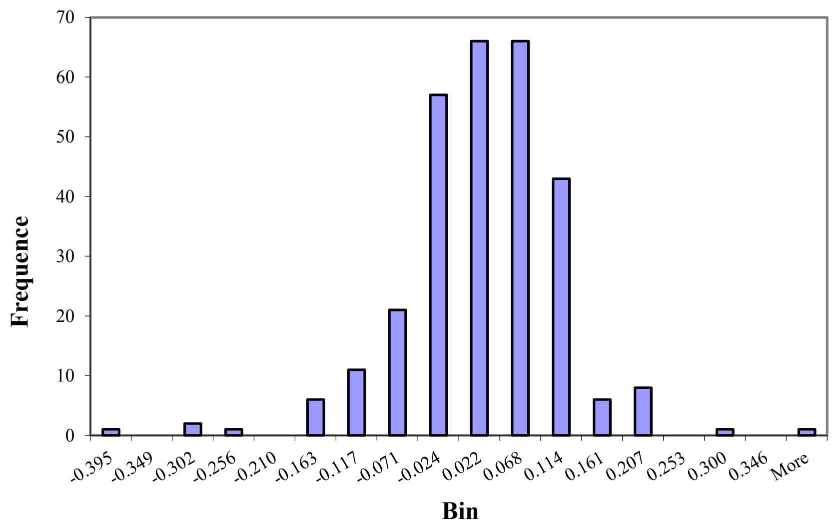


Figure 3-7: Histogram of $(LnP_t - LnP_{t-1})$ for the WTI Monthly Price Return from January 2, 1986 to April 7, 2010
(Source: U. S. Energy Information Administration, 2010a)

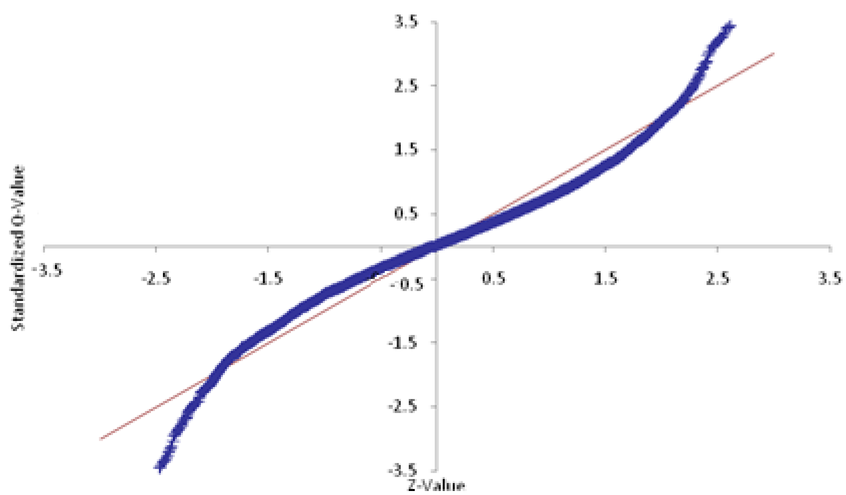


Figure 3-8: Q-Q Normality Plot of $(LnP_t - LnP_{t-1})$ for the WTI Daily Price Return from January 2, 1986 to April 7, 2010
(Source: U. S. Energy Information Administration, 2010a)

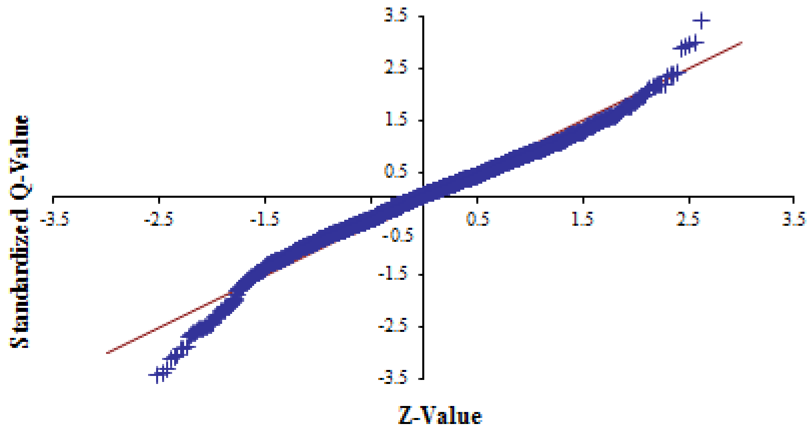


Figure 3-9: Q-Q Normality Plot of $(LnP_t - LnP_{t-1})$ for the WTI Weekly Price Return from January 2, 1986 to April 7, 2010 (Source: U. S. Energy Information Administration, 2010a)

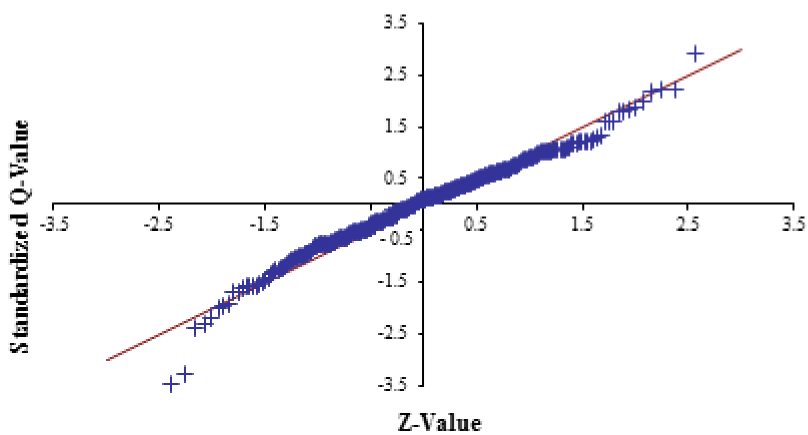


Figure 3-10: Q-Q Normal Plot of $(LnP_t - LnP_{t-1})$ for the WTI Monthly Price Return from January 2, 1986 to April 7, 2010 (Source: U. S. Energy Information Administration, 2010a)

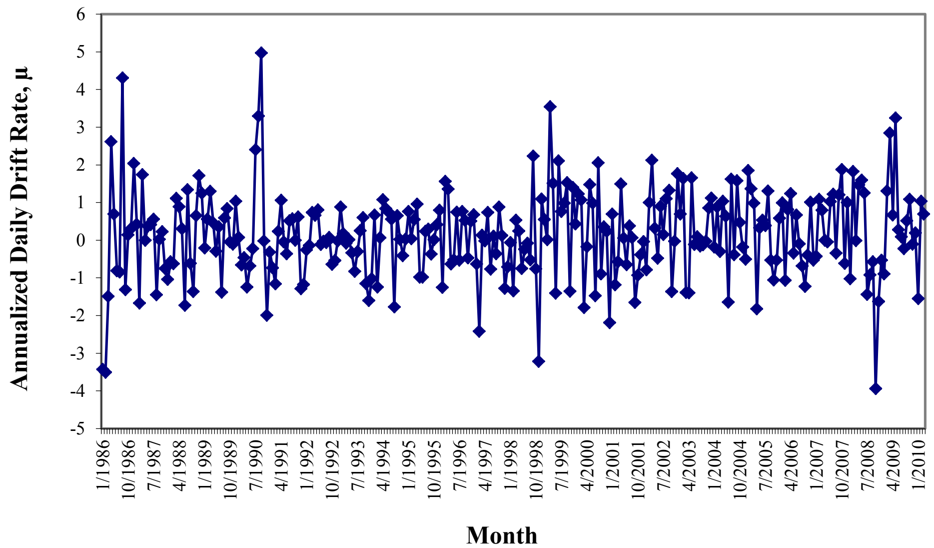


Figure 3-11: Annualized Daily Drift Rate (Grouped Monthly) for the GBM Price Model Using Historical WTI Oil Price Data from January 2, 1986 to April 7, 2010

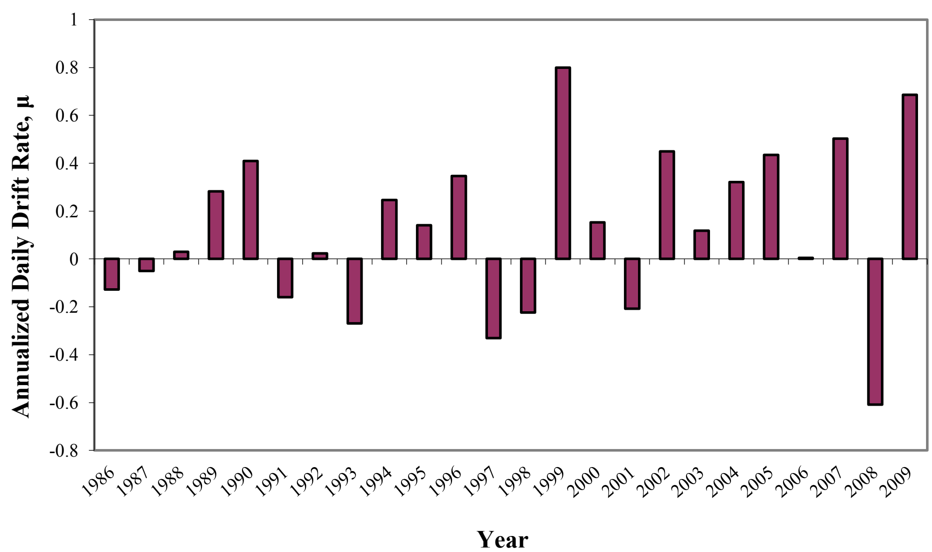


Figure 3-12: Annualized Daily Drift Rate (Grouped Yearly) for the GBM Price Model Using Historical WTI Oil Price Data from January 2, 1986 to April 7, 2010

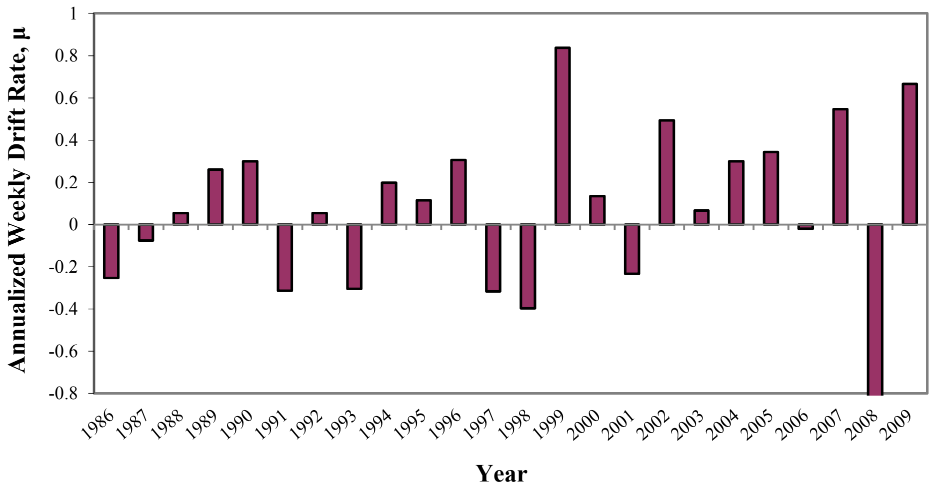


Figure 3-13: Annualized Weekly Drift Rate (Grouped Yearly) for the GBM Price Model Using Historical WTI Oil Price Data from January 2, 1986 to April 7, 2010

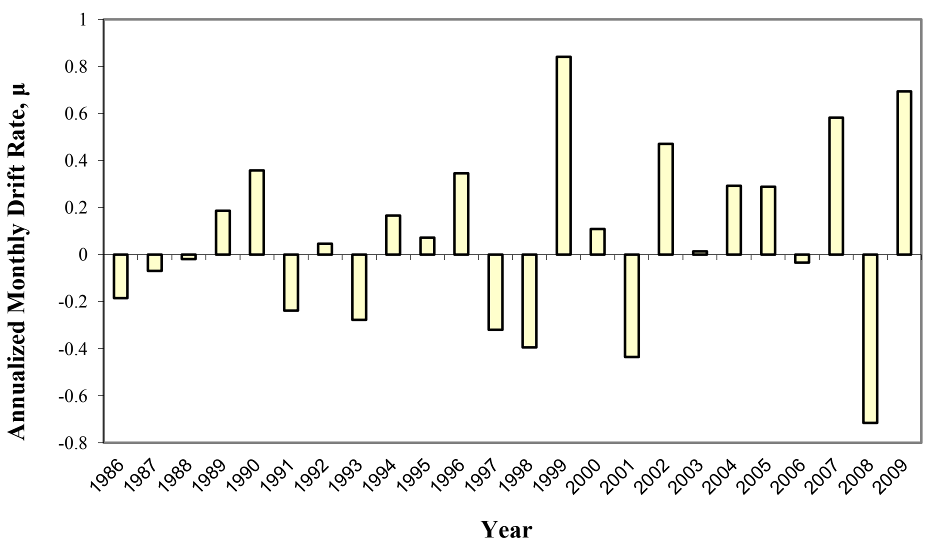


Figure 3-14: Annualized Monthly Drift Rate (Grouped Yearly) for the GBM Price Model Using Historical WTI Oil Price Data from January 2, 1986 to April 7, 2010

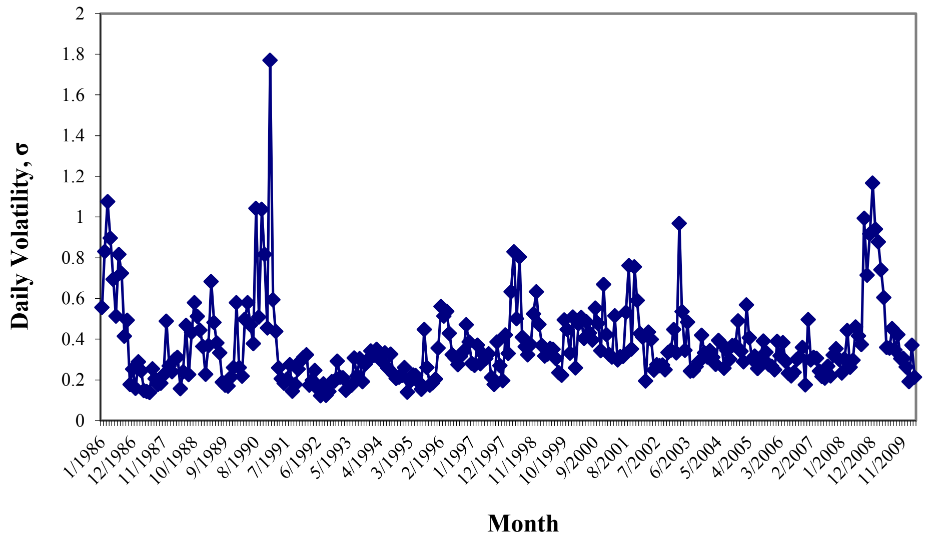


Figure 3-15: Volatility of WTI Daily Oil Prices (Grouped Monthly) for the GBM Price Model Using Historical Oil Price Data from January 2, 1986 to April 7, 2010

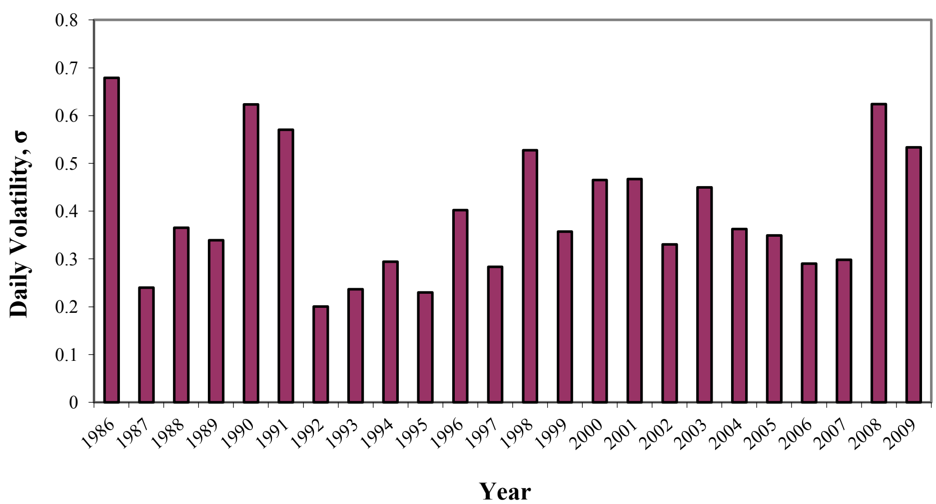


Figure 3-16: Volatility of WTI Daily Oil Prices (Grouped Yearly) for the GBM Price Model Using Historical Oil Price Data from January 2, 1986 to April 7, 2010

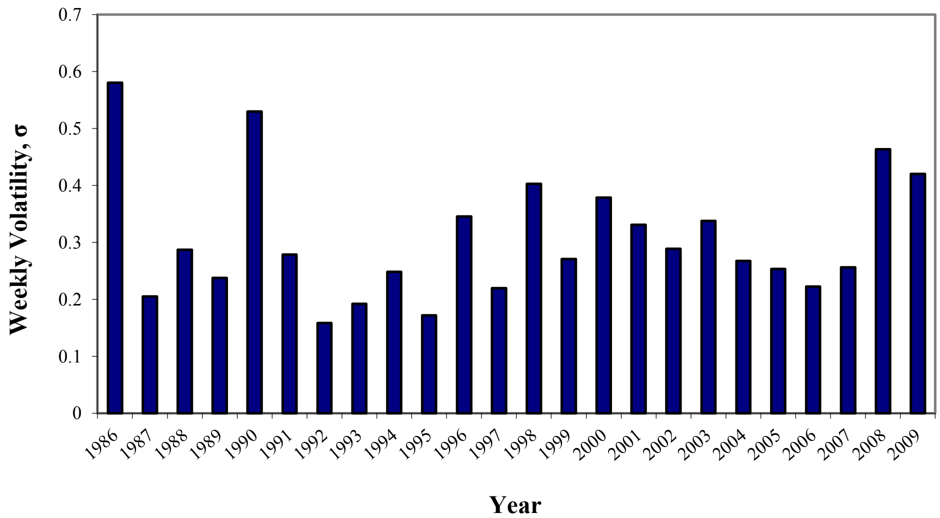


Figure 3-17: Volatility of WTI Weekly Oil Prices (Grouped Yearly) for the GBM Price Model Using Historical Oil Price Data from January 2, 1986 to April 7, 2010

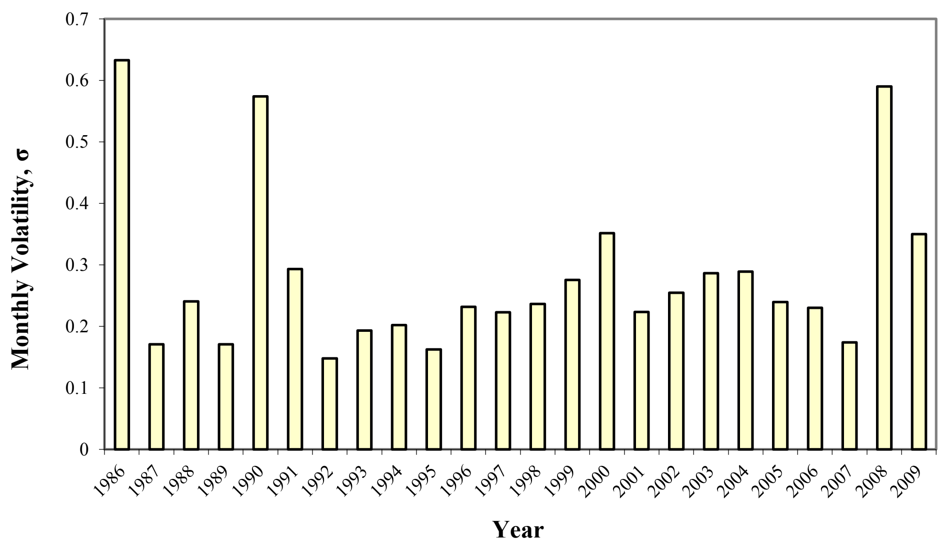


Figure 3-18: Volatility of WTI Monthly Oil Prices (Grouped Yearly) for the GBM Price Model Using Historical Oil Price Data from January 2, 1986 to April 7, 2010

CHAPTER 4: ONE-FACTOR MEAN REVERSION OIL PRICE MODEL

4.1 DESCRIPTION OF ONE-FACTOR MEAN REVERSION PRICE MODEL

When time series prices are modeled with the one-factor mean reversion stochastic process, several versions of models have been derived. In this dissertation, the following one-factor mean reversion price model is used, that is

$$dP = \eta(Ln\bar{p} - LnP)Pdt + \sigma PdW_t, \qquad (4-1)$$

where P is the price at time t,

dP is the price change from time t to time $(t + dt)$,

$\bar{p}$ is the long run price,

η is the mean reversion rate,

W_t is the Wiener process and $dW_t = N(0,1)\sqrt{dt}$,

σ is the volatility of the random process σdW_t.

Equation (4-1) can be rewritten as

$$\frac{dP}{P} = \eta(Ln\bar{p} - LnP)dt + \sigma dW_t\ . \qquad (4-2)$$

Equation (4-2) shows that price return in the time period $[t,\ t + dt]$ in a mean reversion process is determined by two parts: a deterministic one $[\eta(Ln\bar{p} - LnP)dt]$ and a random one $[\sigma dW_t]$. Taking the deterministic part into consideration, $\eta(Ln\bar{p} - LnP)$ is the unit time price return, and η should be positive. When P is higher than $\bar{p}$, price return $\frac{dP}{P}$ is negative, and price will be drawn down towards $\bar{p}$. However, when P is lower than

$\bar{p}$, price return $\frac{dP}{P}$ is positive, and price will increase till it reaches $\bar{p}$. If η is negative, price return $\frac{dP}{P}$ is positive when P is higher than $\bar{p}$, and negative when P is lower than $\bar{p}$. Thus, prices cannot converge to $\bar{p}$.

Now, let

$$X = LnP. \qquad (4-3)$$

Applying Ito's Lemma to Eq. (4-3):

$$dX = \frac{\partial X}{\partial t}dt + \frac{\partial X}{\partial P}dP + \frac{1}{2}\frac{\partial^2 X}{\partial P^2}(dP)^2, \qquad (4-4)$$

$$\frac{\partial X}{\partial t} = 0,$$

$$\frac{\partial X}{\partial P} = \frac{1}{P},$$

$$\frac{\partial^2 X}{\partial P^2} = -\frac{1}{P^2}.$$

Since $(dt)^2 \approx 0$, $(dt)^{\frac{3}{2}} \approx 0$, then

$$(dP)^2 = \sigma^2 P^2 dt, \qquad (4-5)$$

$$dX = \eta(Ln\bar{p} - LnP)dt + \sigma dW_t - \frac{1}{2}\sigma^2 dt,$$

$$dX = \eta\left(Ln\bar{p} - \frac{\sigma^2}{2\eta} - LnP\right)dt + \sigma dW_t,$$

$$dX = \eta(\bar{x} - X)dt + \sigma dW_t, \qquad (4-6)$$

where

$$\bar{x} = Ln\bar{p} - \frac{\sigma^2}{2\eta}. \qquad (4-7)$$

Equation (4-7) can be rewritten as

$$Ln\bar{p} = \bar{x} + \frac{\sigma^2}{2\eta}, \qquad (4-8)$$

$$\bar{p} = Exp\left(\bar{x} + \frac{\sigma^2}{2\eta}\right). \qquad (4-9)$$

Since $X_t = X(t)$, if at time t_0, price is P_0, and $X_0 = LnP_0$, then the logarithm of the price $X_t = LnP_t$ at time $t = t_0 + \Delta t$ can be derived as follows:

$$d(e^{\eta t}X) = \eta e^{\eta t}Xdt + e^{\eta t}dX, \qquad (4-10)$$

$$e^{\eta t}dX = d(e^{\eta t}X) - \eta e^{\eta t}Xdt. \qquad (4-11)$$

Multiplying both sides of Eq. (4-6) by a factor of $e^{\eta t}$ and substituting its left hand side with Eq. (4-11) lead to

$$\begin{aligned} d(e^{\eta t}X) - \eta e^{\eta t}Xdt &= e^{\eta t}[\eta(\bar{x} - X)dt + \sigma dW_t] \\ &= \eta e^{\eta t}\bar{x}\, dt - \eta e^{\eta t}Xdt + \sigma e^{\eta t}dW_t\,, \end{aligned}$$

and thus

$$d(e^{\eta t}X) = \eta e^{\eta t}\bar{x}dt + \sigma e^{\eta t}dW_t\,. \qquad (4-12)$$

Integrating both sides of Eq. (4-12) in the time interval of $[t_0, t = t_0 + \Delta t]$, that is,

$$\int_{t_0}^{t} d(e^{\eta s}X_s) = \eta\,\bar{x}\int_{t_0}^{t} e^{\eta s}\,ds + \sigma\int_{t_0}^{t} e^{\eta s}\,dW_s\,,$$

$$e^{\eta t}X_t - e^{\eta t_0}X_0 = \bar{x}\int_{t_0}^{t} d(e^{\eta s}) + \sigma\int_{t_0}^{t} e^{\eta s}\,dW_s\,,$$

$$e^{\eta t}X_t = e^{\eta t_0}X_0 + \bar{x}e^{\eta t} - \bar{x}e^{\eta t_0} + \sigma \int_{t_0}^{t} e^{\eta s}\, dW_s \,,$$

$$X_t = e^{-\eta(t-t_0)}X_0 + \bar{x} - e^{-\eta(t-t_0)}\bar{x} + \sigma \int_{t_0}^{t} e^{-\eta(t-s)} dW_s \,, \qquad (4-13)$$

or

$$X_t = e^{-\eta\Delta t}X_0 + (1 - e^{-\eta\Delta t})\bar{x} + \sigma \int_{t_0}^{t} e^{-\eta(t-s)} dW_s. \qquad (4-14)$$

According to the properties of Itô integral (Tavella, 2002 & Øksendal, 2003), the following equation holds:

$$E\left[\int_{t_0}^{t} f(W_s, s) dW_s\right] = 0\,.$$

Then

$$\mathrm{E}\left[\int_{t_0}^{t} e^{-\eta(t-s)} dW_s\right] = 0\,. \qquad (4-15)$$

From the definition of variance,

$$Var\left[\int_{t_0}^{t} e^{-\eta(t-s)}\, dW_s\right] = E\left[\int_{t_0}^{t} e^{-\eta(t-s)}\, dW_s\right]^2 - E^2\left[\int_{t_0}^{t} e^{-\eta(t-s)}\, dW_s\right].$$

According to Eq. (4-15),

$$Var\left[\int_{t_0}^{t} e^{-\eta(t-s)}\, dW_s\right] = E\left[\int_{t_0}^{t} e^{-\eta(t-s)}\, dW_s\right]^2 - 0 = E\left[\int_{t_0}^{t} e^{-\eta(t-s)}\, dW_s\right]^2.$$

$$(4-16)$$

According to Itô isometry (Steele, 2001 & Øksendal, 2003), the following equation holds:

$$E\left[\int_{t_0}^{t} f(W_s, s)dW_s\right]^2 = \mathrm{E}\left\{\int_{t_0}^{t} [f^2(W_s, s)]\, ds\right\}.$$

Then

$$E\left[\int_{t_0}^{t} e^{-\eta(t-s)}dW_s\right]^2 = E\left\{\int_{t_0}^{t}\left[e^{-\eta(t-s)}\right]^2 ds\right\} = \int_{t_0}^{t} E\left[e^{-\eta(t-s)}\right]^2 ds$$

$$= \int_{t_0}^{t}\left[e^{-\eta(t-s)}\right]^2 ds = \int_{t_0}^{t} e^{-2\eta(t-s)}ds$$

$$= \frac{1}{2\eta}\int_{t_0}^{t} d\left[e^{-2\eta(t-s)}\right].$$

That is

$$E\left[\int_{t_0}^{t} e^{-\eta(t-s)}dW_s\right]^2 = \frac{1}{2\eta}\left[1 - e^{-2\eta(t-t_0)}\right] = \frac{1}{2\eta}(1 - e^{-2\eta\Delta t}). \qquad (4-17)$$

From Eq. (4-17), Eq. (4-16) becomes

$$Var\left[\int_{t_0}^{t} e^{-\eta(t-s)}\, dW_s\right] = \frac{1}{2\eta}(1 - e^{-2\eta\Delta t})\,. \qquad (4-18)$$

Equations (4-15) and (4-18) show that $\int_{t_0}^{t} e^{-\eta(t-s)}dW_s$ is normally distributed with a mean of zero and a variance of $\frac{(1-e^{-2\eta\Delta t})}{2\eta}$, that is

$$\int_{t_0}^{t} e^{-\eta(t-s)}\, dW_s = N(0,1)\sqrt{\frac{(1-e^{-2\eta\Delta t})}{2\eta}}\ . \qquad (4-19)$$

Substituting Eq. (4-19) to Eq. (4-14), Eq. (4-14) becomes

$$X_t = e^{-\eta\Delta t}X_0 + (1-e^{-\eta\Delta t})\bar{x} + N(0,1)\sigma\sqrt{\frac{(1-e^{-2\eta\Delta t})}{2\eta}}\,. \qquad (4-20)$$

Equation (4-20) shows that X_t is normally distributed with a mean of $[e^{-\eta\Delta t}X_0 + (1-e^{-\eta\Delta t})\bar{x}]$ and a variance of $\frac{\sigma^2(1-e^{-2\eta\Delta t})}{2\eta}$, that is

$$E(X_t|X_0) = e^{-\eta\Delta t}X_0 + (1-e^{-\eta\Delta t})\bar{x}\,, \qquad (4-21)$$

or

$$E(X_t|X_0) = \bar{x} + (X_0-\bar{x})e^{-\eta\Delta t}, \qquad (4-22)$$

and

$$Var(X_t|X_0) = \frac{\sigma^2}{2\eta}(1-e^{-2\eta\Delta t}). \qquad (4-23)$$

Equation (4-21) shows that $E(X_t|X_0)$ is the weighted average of current price X_0 and long run price in the form of $\bar{x}$. When the mean reversion rate η is positive and fixed, if Δt is small, the weight of current price is high ($\Delta t \to 0,$ the weight of current price is close to 1); if Δt is large, the weight of the long run price is high ($\Delta t \to \infty,$ the weight of the long run price is close to 1); the higher the mean reversion rate, the higher the weight of long run price; and if $\eta \to 0,$ then $E(X_t|X_0) \to X_0$.

Equation (4-23) shows that $Var(X_t|X_0)$ is determined by the mean reversion rate η, volatility σ of random process σdW, and Δt. $Var(X_t|X_0)$ increases with the increase of

Δt when η and σ are fixed. When $\Delta t \to \infty$, $Var(X_t|X_0) \to \frac{\sigma^2}{2\eta}$. The higher the volatility σ of random process σdW, the higher the value of $Var(X_t|X_0)$ with the same mean reversion rate η and time interval Δt. When mean reversion rate $\eta \to \infty$, $Var(X_t|X_0) \to 0$, and price is kept close to the long run price, in the form of $\bar{x}$, irrespective of the current price X_0. When $\eta \to 0$, $Var(X_t|X_0) \to \sigma^2 \Delta t$. It becomes a Geometric Brownian Motion process.

According to Eq. (4-3), Eq. (4-7), and Eq. (4-20),

$$P_t = Exp\left[e^{-\eta\Delta t}X_0 + (1 - e^{-\eta\Delta t})\bar{x} + N(0,1)\sigma\sqrt{\frac{(1 - e^{-2\eta\Delta t})}{2\eta}}\right], \qquad (4-24)$$

or

$$P_t = Exp\left[e^{-\eta\Delta t}(LnP_0) + (1 - e^{-\eta\Delta t})\left(Ln\bar{p} - \frac{\sigma^2}{2\eta}\right) + N(0,1)\sigma\sqrt{\frac{(1 - e^{-2\eta\Delta t})}{2\eta}}\right], \qquad (4-25)$$

Since $X_t = LnP_t$ is normally distributed, P_t is log-normally distributed.

4.2 Theory on Parameter Estimations for the One-factor Mean Reversion Price Model Using Historical Time Series Price Data

Historical daily, weekly, monthly, or yearly price data may be used to estimate the parameters for the mean reversion price model. In a time series, for each time interval $[t-1, t]$, Eq. (4-20) can be rewritten as

$$X_t = e^{-\eta \Delta t} X_{t-1} + (1 - e^{-\eta \Delta t})\bar{x} + N(0,1)\sigma \sqrt{\frac{(1 - e^{-2\eta \Delta t})}{2\eta}}\,, \qquad (4-26)$$

where X_{t-1} is the logarithmic price at the beginning of a time interval; and X_t is the logarithmic price at the end of the same time interval, as well as the logarithmic price at the beginning of next time interval. Δt is the length of each time interval, which can be a day, a week, a month, or a year with the corresponding prices for Eq. (4-26) being called daily, weekly, monthly, or yearly prices. For the convenience of comparison, the unit for Δt is set to year. For daily data, $\Delta t = 1/252 \text{ year} = 0.003968 \text{ year}$; for weekly data, $\Delta t = 1/52 \text{ year} = 0.019231 \text{ year}$; and for monthly data, $\Delta t = 1/12 \text{ year} = 0.08333 \text{ year}$. Thus the parameters in Eq. (4-26), such as η and σ, are annualized values. In the following discussion, the values for η, σ and $\bar{x}$ are assumed constant along the time series.

Equation (4-26) can be further rewritten as

$$X_t = a + b\, X_{t-1} + \varepsilon_t\,, \qquad (4-27)$$

where

$$a = (1 - e^{-\eta \Delta t})\bar{x}\,, \qquad (4-28)$$

$$b = e^{-\eta \Delta t}\,, \qquad (4-29)$$

$$\varepsilon_t = N(0,1)\sigma \sqrt{\frac{(1 - e^{-2\eta \Delta t})}{2\eta}}\,. \qquad (4-30)$$

If repeated *n* pairs of data existed for X_{t-1} and X_t over the same time interval of $[t-1,\ t]$, then $\hat{a},\ \hat{b}$, and $\hat{\varepsilon}_t$ in Eq. (4-27) could be obtained with a linear regression of X_t on X_{t-1}. However, for time series data such as historical oil prices, there is only one realization of price at each time point. Based on the "time-summation" statistical mechanics, the values, X_t and X_{t-1}, in successive time intervals are permissible to be used for the regression according to Eq. (4-27).

Suppose that a time series consists of *n* successive time intervals with a data set of *n* available historical prices for X_t and X_{t-1}, that is

$$\begin{vmatrix} X_t^1 & X_{t-1}^1 \\ X_t^2 & X_{t-1}^2 \\ X_t^3 & X_{t-1}^3 \\ & \ddots \\ X_t^n & X_{t-1}^n \end{vmatrix}.$$

Applying the least square linear regression of X_t on X_{t-1} on the above data set, a set of parameters of $(\hat{a},\ \hat{b})$ is obtained. The data set $(\hat{a},\ \hat{b})$ satisfies Eq. (4-28), Eq. (4-29), and the following equation:

$$\hat{X}_t = \hat{a} + \hat{b}X_{t-1}. \qquad (4-31)$$

With the regressed data set $(\hat{a},\ \hat{b})$, the mean reversion parameters, η and $\bar{x}$, are obtained from Eq. (4-28) and Eq. (4-29), that is

$$\eta = -\frac{Ln\hat{b}}{\Delta t}, \qquad (4-32)$$

and

$$\bar{x} = \frac{\hat{a}}{1-\hat{b}}\ . \qquad (4-33)$$

Comparing Eq. (4-27) and Eq. (4-31), the following equation holds:

$$\varepsilon_t = X_t - \hat{X}_t\ , \qquad (4-34)$$

where ε_t is assumed to be identically, independently, and normally distributed with a mean of zero and a standard deviation of $sd(\varepsilon_t)$ in each successive time interval, that is

$$\varepsilon_t = N[0,\ sd(\varepsilon_t)]. \qquad (4-35)$$

Again, the distribution of ε_t is represented from the time series data, not from the repeated data over the same time interval, that is,

$$\begin{vmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \\ \varepsilon_t^3 \\ \cdots \\ \varepsilon_t^i \\ \cdots \\ \varepsilon_t^n \end{vmatrix} = \begin{vmatrix} X_t^1 - \hat{X}_t^1 \\ X_t^2 - \hat{X}_t^2 \\ X_t^3 - \hat{X}_t^3 \\ \cdots \\ X_t^i - \hat{X}_t^i \\ \cdots \\ X_t^n - \hat{X}_t^n \end{vmatrix}. \qquad (4-36)$$

The distribution of $[\varepsilon_t^1, \varepsilon_t^2,\ \varepsilon_t^3, \ldots, \varepsilon_t^i, \ldots, \varepsilon_t^n]$, which is assumed to be a normal distribution with a mean of zero and a standard deviation of $sd(\varepsilon_t)$, is estimated by the regression standard error $\widehat{sd}(\varepsilon_t)$, known as the residual standard deviation, from the time series regression results of X_t on X_{t-1}. Combining Eq. (4-30) and Eq. (4-35), the following equation is derived:

$$\varepsilon_t = N\left[0, \widehat{sd}(\varepsilon_t)\right] = N(0,1)\sigma\sqrt{\frac{(1-e^{-2\eta\Delta t})}{2\eta}},$$

or

$$\varepsilon_t = N\left[0, \widehat{sd}(\varepsilon_t)\right] = N\left[0, \sigma\sqrt{\frac{(1-e^{-2\eta\Delta t})}{2\eta}}\right]. \qquad (4-37)$$

Then,

$$\widehat{sd}(\varepsilon_t) = \sigma\sqrt{\frac{(1 - e^{-2\eta\Delta t})}{2\eta}}\ . \qquad (4-38)$$

That is

$$\sigma = \widehat{sd}(\varepsilon_t)\sqrt{\frac{2\eta}{(1 - e^{-2\eta\Delta t})}}\ . \qquad (4-39)$$

Substituting Eq. (4-32) into Eq. (4-39), the following equation holds:

$$\sigma = \widehat{sd}(\varepsilon_t)\sqrt{\frac{2Ln\hat{b}}{(\hat{b}^2 - 1)\Delta t}}\ . \qquad (4-40)$$

By substituting Eq. (4-32), Eq. (4-33), and Eq. (4-40) into Eq. (4-9), the equation for the long run price can be obtained from the regression results directly:

$$\bar{p} = Exp\left[\frac{\hat{a}}{1 - \hat{b}} + \frac{\widehat{sd}^2(\varepsilon_t)}{1 - \hat{b}^2}\right]. \qquad (4-41)$$

Substituting Eq. (4-28) and Eq. (4-29) into Eq. (4-21) over the time interval of $[t - 1,\ t]$, Eq. (4-21) becomes

$$E(X_t|X_{t-1}) = \hat{a} + \hat{b}X_{t-1}. \qquad (4-42)$$

Comparing Eq. (4-38) with Eq. (4-23) gives

$$Var(X_t|X_{t-1}) = \widehat{sd}^2(\varepsilon_t) = Var(\varepsilon_t). \qquad (4-43)$$

Then Eq. (4-20) becomes

$$X_t = \hat{a} + \hat{b}X_{t-1} + N(0,1)\widehat{sd}(\varepsilon_t). \qquad (4-44)$$

Now, re-writing Eq. (4-25) over each time interval $[t - 1,\ t]$, the equation for simulation and forecasting for the future price evolution can be derived as

$$P_t = Exp\left[e^{-\eta\Delta t}(LnP_{t-1}) + (1 - e^{-\eta\Delta t})\left(Ln\bar{p} - \frac{\sigma^2}{2\eta}\right) + N(0,1)\sigma\sqrt{\frac{(1 - e^{-2\eta\Delta t})}{2\eta}}\right],$$

$$(4-45)$$

or simply,

$$P_t = Exp\left[\hat{a} + \hat{b}(LnP_{t-1}) + N(0,1)\widehat{sd}(\varepsilon_t)\right]. \quad (4-46)$$

In summary, there are three steps to use historical time series prices to estimate the parameters for the one-factor mean reversion price model and to conduct future price forecasting. First, by running the linear regression of X_t on X_{t-1} over the time interval of $[t-1,\ t]$ with time series price data, $\hat{a}$, the intercept of the regression, $\hat{b}$, the slope of the regression, and $\widehat{sd}(\varepsilon_t)$, the residual standard deviation (standard error) of the regression, are obtained. Second, substituting $\hat{a}$, $\hat{b}$, and $\widehat{sd}(\varepsilon_t)$ into Equations (4-32), (4-33), (4-40), and (4-41), the values of the mean reversion parameters, $\eta,\ \bar{x},\ \sigma,\ and\ \bar{p}$, are calculated. Third, with η, $\bar{x}$, σ, $\bar{p}$, and Eq. (4-45), or with $\hat{a}$, $\hat{b}$, $\widehat{sd}(\varepsilon_t)$ and Eq. (4-46), simulation and forecasting for the future price evolution can be performed.

In most papers in the literature, the mean reversion parameter estimations are conducted through the regression results of $(LnP_t - LnP_{t-1})$ on LnP_{t-1}. However, for the historical oil prices studied in this chapter, the regression *R*-Square of $(LnP_t - LnP_{t-1})$ on LnP_{t-1} is very small, only 0.001612 for the monthly data, and the t-Statistic for the regression intercept and slope is unacceptably small. Therefore, in this study, the regression is conducted between LnP_t and LnP_{t-1}.

4.3 Results of the Parameter Estimations for the One-factor Mean Reversion Oil Price Model Using Historical *WTI* Oil Price Data

The historical West Texas Intermediate (*WTI*) oil price data are used to estimate the parameters for the one-factor mean reversion oil price model. The chosen historical *WTI* oil price data are from January 2, 1986 to May 28, 2010, including daily spot prices, weekly prices, monthly prices, and yearly prices. May 28, 2010 was a Friday and thus the daily, weekly, and monthly data have the same ending date for comparison. This study reveals many aspects of parameter estimations for the one-factor mean reversion oil price model when different chosen sets of historical *WTI* oil prices are used to calibrate the model parameters. The results of the mean reversion parameter estimations also give insights into the behavior of historical oil prices and future oil price movements. For the rest of this dissertation, one-factor mean reversion price model is simply called mean reversion price model.

Firstly, mean reversion parameter estimations are studied by two price regimes. Historical *WTI* oil prices show that roughly two price regimes exist from January 2, 1986 to May 28, 2010: one regime covers January 2, 1986 to December 30, 1999, called the B4-2K price regime; the other regime from January 4, 2000 to May 28, 2010, called AF-2K price regime. The former price regime was already completed, while the latter may still be ongoing. The results of the mean reversion parameter estimations and linear regressions in the two price regimes with daily, weekly, monthly, and yearly *WTI* oil price data are shown in Tables 4-1 through 4-4. As illustrated, the original linear regression results for the *WTI* weekly price data from 1986 to 1999 are shown in both Table 4-5 and Figure 4-1.

Table 4-1 shows that in the B4-2K price regime, the long run oil price $\bar{p}$ is around \$19.50/bbl and is very consistent no matter if the daily, weekly, monthly, and yearly prices are used.

Table 4-3 shows that in the AF-2K price regime, the long run oil price varies with different price data sets. It ranges from daily prices of \$73.31/bbl to monthly prices of \$84.84/bbl.

Table 4-2 shows that in the B4-2K price regime, the t-Statistic for the regression slope $\hat{b}$ with yearly price data is only 0.33, which means that the confidence level to reject the null hypothesis that $\hat{b}$ equals zero is less than 75%. When $\hat{b}$ is equal to zero, the mean reversion rate $\eta = -Ln\,\hat{b}/\Delta t$ will be of no value. In addition, the R^2 of this regression is only 0.0097, which is unacceptably small. Thus, the yearly price data should be avoided in calibrating the mean reversion parameters for the B4-2K oil price regime.

Table 4-4 shows that in the AF-2K price regime, the t-Statistic for the regression interception $\hat{a}$ is 1.39 for weekly price and 1.31 for monthly price. The confident level of rejecting the null hypothesis that $\hat{a}$ equals zero is around 90%. If the null hypothesis for $\hat{a}$ cannot be rejected, then, according to Eq. (4-33) and Eq. (4-9), $\bar{x} = 0$, and thus $\bar{p} = Exp(\sigma^2/2\eta)$. This will result in the long run price $\bar{p}$ being only around \$1.00/bbl for the monthly price data and it is very misleading.

Comparing the annualized mean reversion rate η shown in Tables 4-1 and 4-3, it is observed that the annualized mean reversion rate in the B4-2K price regime is much higher than that in the AF-2K price regime. For example, using weekly price data, the annualized mean reversion rate in B4-2K regime is five times higher than that in the AF-2K regime.

A very strong mean reversion in the B4-2K price regime indicates that during the period from 1986 to 1999, people were willing to keep oil prices around the long run price, approximately \$19.50/bbl. Whenever the oil price was disparate from its long run price, efforts would be made in order to re-attain its long run price. On the other hand, the much lower mean reversion rate in the AF-2K price regime suggests that throughout the period from 2000 to 2010, the world economy has had a much higher tolerance for the higher oil prices and higher price fluctuation from its long run price than before.

As shown in Tables 4-1 and 4-3, the variation of mean reversion volatility σ is from 31.06% to 41.29% per year in the B4-2K regime and from 29.04% to 42.79% per year in the AF-2K regime when different types of price data are used. Daily price data results in the highest volatility. When the same kind of price data is used, there is almost no difference in mean reversion volatility σ between two price regimes. This indicates that the difference in price change in two price regimes results mainly from the deterministic part – that is, the mean reversion rate η and the long run price $\bar{p}$ – not from the random part, the mean reversion volatility σ.

Based on the t-Statistic and the long run price $\bar{p}$, for the B4-2K price regime, daily, weekly, and monthly oil price data can all be used to calibrate the mean reversion parameters and reach the same long run price of about \$19.50/bbl. For the AF-2K price regime, daily, weekly, monthly, and yearly price data reach a long run price with a difference of \$11.53/bbl between \$73.31/bbl and \$84.84/bbl, as well as fairly low linear regression t-Statistic for the weekly and monthly data.

Secondly, the relationship of mean reversion parameter estimations between using short term historical prices, such as one year's price data which were suggested in some literature, and the long term historical prices, such as prices from an entire price regime, is studied. The daily spot oil price data are used for the analysis in an attempt to capture the exact behavior of historical oil price movements.

The parameter estimations and the regression results from the daily spot oil price data for each year from 1986 to 2010 are shown in Tables 4-6 and 4-7. Since the long run price pattern is very clear and well established for the B4-2K price regime, it is used as a reference to see how close the long run price pattern is from the short-term price data comparing to the realized long run price pattern. Subsequently, insight can be given as to whether or not the short-term price data should be used to estimate the long run price pattern. The analyses for the reports in Table 4-6 are shown in Figures 4-2 through 4-10. Both Table 4-6 and Figure 4-2 show that most of the years in the B4-2K price regime, the resulted long run price from one year's daily spot price data differs significantly from the actual long run price $\bar{p}$ of \$19.49/bbl. For example, the long run price calculated from the price data in 1993 is \$39.92/bbl, while from the data in 1998 is only \$14.06/bbl. For the AF-2K regime, as shown in Table 4-6 and Figure 4-3, the long run price calculated from the price data in 2007 is as high as \$253.86/bbl, while from the price data in the previous year 2006 is only \$66.26/bbl. Thus a conclusion can be drawn that it is not reliable to use just one year's price data to estimate the long run price $\bar{p}$. From other mean reversion parameters calculated from one year's daily spot price data, important phenomena are observed from Tables 4-6, 4-7, and Figures 4-4, 4-5:

1) The mean reversion rate changed dramatically from one year to another. For example, the mean reversion rate is only 2.6464 per year with the price data from 1990; it increases significantly to 28.2449 per year with the price data from 1991 and drops back to 5.2697 per year with next year's price data.

2) The mean reversion rate with each year's price data is much different from that with the complete data in the two price regimes. Most of time, one year's price data result in much higher mean reversion rate than that from the data in one price regime. However, there also exists several very small and even negative mean reversion rate with one year's data.

3) The estimated mean reversion volatility, or the annualized σ, from one year's price data, varies from 20.21% to 68.33% per year in the years from 1986 to 1999, while the mean reversion volatility is 41.29% per year in the B4-2K price regime, and from 29.45% to 62.53% since year 2000, while the mean reversion volatility is 42.79% in the AF-2K price regime.

4) Very small values of t-Statistic, such as 0.29, and even negative t-Statistic, such as -0.49, are observed in both the B4-2K and AF-2K price regimes.

The above results show that a short time, such as one year, is not long enough to establish the long run price pattern; short term oil price data, such as one year's price data, are not adequately representative in calibrating the mean reversion parameters, even in the time when long run price was already established, such as in many years in the B4-2K price regime.

Analogy to the relationship of local optimization and global optimization, where the local optimal value may or may not be the global optimal result, the so called long run price from one-year price data may be considered "local" long run price as contrasting with the "global" long run price covering a long perspective of price data among which the long run stable price pattern is established. The "local" long run price may or may not be the "global" long run price.

By examining the relationship between the historical oil prices and the long run price calibrated from these prices, it is observed that for a set of price data, most of the times, the calibrated long run price is between the highest and lowest prices among the prices in the data set. If the price movement has already undergone several cycles across a specific price "$\bar{p}$" from below and above, the long run price to which the mean reversion theory is applied will converge to that $\bar{p}$. For example, for the price data in 1986, as shown in Figure 4-6, there were many cycles of price evolution around \$15/bbl. The "long run" price from these price data converges to \$14.62/bbl. It is very sure that the actual long run price $\bar{p}$ of \$19.49/bbl for the B4-2K price regime cannot be calibrated from the price data in 1986. However, there are several cases when price movements within the data set are in one direction, but not cyclically, the calibrated "long run" price can be beyond the price range of the price data set. For example, from the price data in 1999, the estimated "long run" price is

\$29.94/bbl, while the highest price in 1999 was only \$28.03/bbl, as shown in Figure 4-7; and from the price data in 2007, the estimated "long run" price is as high as \$253. 86/bbl, while the highest price in 2007 was less than \$100/bbl, as shown in Figure 4-8. Convincing explanations are still not arrived yet why the calibrated "long run" price in 1993 is \$39.92/bbl while the highest price in 1993 was only \$21.05/bbl and the prices were moving downside along 1993, as shown in Figure 4-9; and why the "long run" price in 2008 was as high as \$212.36/bbl while the highest price in 2008 was less than \$150.00/bbl, and about half of the time, the prices in 2008 moved downside, as shown in Figure 4-10. It does be observed that in the above two cases, the values of the linear regression t-Statistic for intercept $\hat{a}$, and the mean reversion rate are both negative, as shown in Table 4-7. Obviously, \$39.92/bbl is much higher than the actual long run price $\bar{p}$ of \$19.49/bbl for the B4-2K price regime; and \$212.36/bbl should not be the long run price $\bar{p}$ for the AF-2K price regimes as can be observed right now. It is validated again that the price data in just one year is not recommendable to use for the parameter estimations for the mean reversion oil price model, not only because that the "local" long run price may not be the "global" long run price, but also because of the unreliability of the long run price calibrated from price data in just one year when a stable long run price pattern is not fully established yet.

Thirdly, it is studied on how long the historical oil price data should cover to reliably calibrate the mean reversion parameters. Are the more current the oil prices the better the results of parameter calibrating or vice versa? Are there any other data sets than the ones by two price regimes which can improve the parameter estimations for the mean reversion oil price model? Four kinds of WTI daily spot oil price sets, *i.e.* 1) full range price data from 1986 to 2010, 2) rolling group data sets, 3) backward data sets, and 4) forward data sets, are used to examine in detail the results of parameter estimations for the mean reversion price model. Following are the analyses on the results of mean reversion parameter estimations from the above four data sets:

1) Mean reversion parameter estimations with full range price data from January 2, 1986 to May 28, 2010

Table 4-8 reports the parameter estimations using the full range price data from January 2, 1986 to May 28, 2010. The resulted long run price $\bar{p}$ ranges widely from \$51.36/bbl, with daily prices, to \$97.66/bbl, with yearly prices. Table 4-9 shows the regression results using these full range price data as mentioned above. The t-Statistic of $\hat{a}$ is around 1.00 for the weekly, monthly, and yearly data. The confident level of rejecting the null hypothesis that $\hat{a}$ equals to zero is only 85%. Both the regression and the mean reversion parameter estimation results suggest that, since in reality, there existed clear price regimes, the full range historical oil price data which cover at least two price regimes are not adequate representations for the calibration of the mean reversion parameters.

2) Mean reversion parameter estimations with backward rolling group data

One way to use historical price data to calibrate model parameters is to use the rolling group data. The backward rolling group data only include a set of the most current price data. For example, three years backward group data always use the most update price data in the current three years, neglecting the data in the years earlier than three years. Table

4-10 through Table 4-21 are reports of the mean reversion parameter estimations and regressions for the three years', five years', and seven years' backward rolling group data in the B4-2K and AF-2K price regimes. Figure 4-11 through Figure 4-28 include the charts for analyzing the reports in Table 4-10 through 4-21. It is expected that when it is in one price regime with a same stable long run price, then: a) at the same current year, the long run price calibrated from different years' backward rolling group data would be the same. For example, at the end of 1999, the long run price calibrated from three years', five years', and seven years' backward rolling group data should reach the same value as the one for the B4-2K price regime; b) when approaching different "current" years, the long run price calibrated from the backward rolling data with the same time length should be the same. For example, the calibrated long run price from the data of 1999-1997 and the one from the data of 1998-1996 should come up with the same result. This way, the most current data of three years, or five years, or seven years can always be used to calibrate the mean reversion parameters. The B4-2K price regime can be used as a reference. If the above two expectations can be met, then the backward rolling group data can be used to identify the price regimes and to calibrate the mean reversion parameters. Otherwise, either the prices are not in the same regime or the backward rolling group data sets are not adequate for the purposes.

From Tables 4-10, 4-14, 4-18, Figures 4-11, 4-17, and 4-23, it is observed that for the B4-2K price regime, the three years' backward rolling group data give a long run price varying from \$16.45/bbl to \$23.09/bbl while the actual long run price $\bar{p}$ for the B4-2K price regime is \$19.49/bbl; the five years' backward rolling group data calibrate a long run price ranging from \$18.64/bbl to \$21.03/bbl; and seven years' backward rolling group data result in a long run price from \$18.49/bbl to \$20.90/bbl, which is very close to the actual long run price $\bar{p}$. The above results suggest that with the most updated price information each year, the longer the backward rolling group, the closer the estimated long run price to the actual long run price. The seven years' rolling group data give the most close long run price to the actual long run price for the B4-2K price regime. Short term backward rolling group data, such as the three years' data, may calibrate a long run price which differs from the actual long run price. It also reveals that the most current data are not the best for the calibration of the mean reversion parameters.

From Table 4-10, Figures 4-12, and 4-13, it is also observed that there were a lot of variations for the three years' backward rolling group data in mean reversion rate and volatility. For the five and seven years' rolling group data, some mean reversion rate is much higher than the rest, as shown in Tables 4-14, 4-18, Figures 4-18, and 4-24. The exceptionally high mean reversion rate occurred when there was very big price jump inside that rolling group data. Even though the long run price can be very close to the actual long run price for the B4-2K price regime with five years' or seven years' rolling group data, some mean reversion rate still does not represent that for the whole price regime. The above results suggest that even for the B4-2K, a very clearly indentified and completed mean reversion price regime, using backward rolling group data does not make the estimations of mean reversion parameters better than using the whole data set in the regime from 1986 to 1999 when taking all the three mean reversion parameters into consideration.

From Tables 4-12, 4-16, and 4-20, together with Figures 4-14, 4-20, and 4-26, it is observed that for the AF-2K price regime, there are wide variations in the calibrated long run price from each of the three types of backward rolling group data. For example, for the three year rolling group, the long run price from grouping prices of 2009-2007 is \$87.80/bbl, and is \$61.63/bbl from grouping prices of 2006-2004. At the end of 2007, the three years', five years', and seven years' backward rolling group data give a long run price of \$76.28/bbl, \$102.25/bbl, and \$159.59/bbl respectively. Above results suggest that the backward rolling group data do not give consistent mean reversion long run price for the AF-2K price regime.

The t-Statistic for the linear regression intercept $\hat{a}$ can be lower than 1.00 in all the three types of rolling group data regressions, as shown on Tables 4-13, 4-17, and 4-21.

For both the B4-2K and AF-2K price regimes, the three years', five years' and seven years' backward rolling group data do not give better results for the mean reversion parameter estimations than those from the complete data from each price regime. The most current data may not be the best for the mean reversion parameter estimations.

3) Mean reversion parameter estimations with the increasing number of backward years' price data

The price data sets of the increasing number of backward years always take the most current year in a price regime as the reference year and then include more and more previous years' price data to see the changes in long run price pattern. The most current year for the B4-2K price regime is 1999 and the most current year for the AF-2K price regime is 2010.

The results of mean reversion parameter estimations and regressions with price data of increasing number of backward years for the B4-2K price regime are reported in Tables 4-22 and 4-23, and for the AF-2K regime in Tables 4-24 and 4-25. The analysis charts for the above reported results are shown in Figures 4-29 through 4-34.

In the B4-2K price regime, after about four backward years from 1999 to 1996, the calibrated long run price reached a stable value close to the actual long run price for the B4-2K price regime. Afterwards, including more backward prices year by year till 1986, the calibrated long run price ranged from $$19.45/bbl to $20.81/bbl, very small variation to the actual long run price of $19.49/bbl for the same price regime. The mean reversion rate increased year by year from 1999 to 1991 when increasing more and more backward prices. After about nine backward years since 1999, the calibrated long run mean reversion rate reached a stable value close to that for the same price regime. The calibrated mean reversion volatility fluctuated around the actual long run mean reversion volatility when more and more backward price data were included year by year from 1999 to 1990. After about ten years from 1999 to 1990, the calibrated mean reversion volatility reached a stable value close to that in the B4-2K price regime. The results again suggest that: a) for the purpose of estimating mean reversion parameters, most current data may not reflect the long run price pattern and the thus may not be of the best representative data to use; b) in the B4-2K price regime, after including ten years' backward prices from 1999 to 1990, the long run price pattern became stable, including long run price, mean reversion rate, and mean reversion volatility, and afterwards, including more backward years' data in the same price regime did not change long run price pattern.

For the AF-2K price regime, as shown in Table 4-24, Figures 4-32, 4-33, and 4-34, staring from 2010, including more and more backward years' data the calibrated mean reversion rate decreased and more and more closer to the long run mean reversion rate of this price regime; the calibrated long run price fluctuated and more and more closer to the long run price of this price regime; the calibrated mean reversion volatility first increased, and then decreased and more and more closer to the mean reversion rate for this price regimes. Upon about 2004, the three long run parameters became very close to the long run parameters in the AF-2K price regime.

The above results repeatedly show that for the purpose of mean reversion parameter estimations, the price data in the most current years may not be the best data to use. In different price regimes, the price data in backward years need to be included to reach a stable long run price pattern according to the three mean reversion parameters. Since after the backward time is longer enough, including more and more backward price data, the calibrated mean reversion parameters will be closer and closer to the actual long run mean reversion parameters in the same price regime, covering the full data in a price regime to calibrate mean reversion parameters is a better and simple choice.

4) Mean reversion parameter estimations with increasing number of forward years' price data

The results of mean reversion parameter estimations with the increasing number of forward years' price data provide insights on how long the long run price pattern is developed and established. The starting time for the B4-2K price regime is year 1986; the starting time for the AF-2K price regime is year 2000. Tables 4-26 and 4-27 contain results for the mean reversion parameter estimations and the regressions with the price data of increasing number of forward years from 1986 to 2004. Table 4-26, Figures 4-35, 4-36, and 4-37 show that for the B4-2K price regime, in about five years, the long run price pattern was established as can be observed by long run price, mean reversion rate , and mean reversion volatility. During those five years, daily spot oil prices evolved several complete cycles of higher and lower than the long run price $\bar{p}$, as shown on Figure 4-38. After 1990 till 1999, the three mean reversion parameters were very stable while oil prices repeated the cycles. The *t*-Statistic shown in Table 4-27 reveals that the confident level to reject the null hypotheses that $\hat{a}$ is equal to zero and $\hat{b}$ is equal to zero is higher than 99%.

The clear and gradual changes in mean reversion rate, mean reversion volatility, and long run price with price data of 1986-2000 through 1986-2004 in Table 4-26 indicate that year 2000 is the switch or transaction year from one price regime to the other. This result is consistent with what is shown in Figure 3-1 on the evolution of oil prices and the switch time from one price regime to the other.

Tables 4-28, 4-29, Figures 4-39, 4-40, and 4-41 show the mean reversion parameter estimation and the regression results with the price data of increasing number of forward years in the AF-2K price regime starting from year 2000. Unlike what happened in the B4-2K price regime, when there were many complete price evolution cycles across the same long run price from below and above, for the AF-2K price regime, there was no clear and 'dominant" long run price around which there were complete price cycles from below and above, as shown on Figure 4-42. It is observed from Table 4-28 and Figure 4-39 that in the AF-2K price regime, for the first four years starting from year 2000, with the increasing number of forward years, the long run price was very stable and was around \$30.00/bbl. After 2003, the long run price changed with more forward years' price data being included. It first jumped to the highest of \$115.12/bbl with price data between 2000 and 2007, and then fluctuated back to \$73.31/bbl when more forward years' price data were included.

With the above observation, the AF-2K price regime is further classified as two separate regimes: a short sub-regime which covers the time period of 2000-2003 called 2K-2.3K price regime; a long one called AF-2.3K price regime from 2004 to 2010. After leaving the B4-2K price regime, with a short stable time from 2000 to 2003, oil prices were evolving into a new long run price pattern since 2004.

Tables 4-30 and 4-31 contain the results of the mean reversion parameter estimations and regressions with the complete price data for the entire 2K-2.3K price regime including daily, weekly, and monthly data. The long run price for the 2K-2.3K price regime is \$29.11/bbl, \$29.25/bbl, and \$29.35/bbl with daily, weekly, monthly data respectively. The annualized mean reversion rate is between 2.1402 and 4.3989, with daily data having the highest mean reversion rate. The highest mean reversion volatility is 43.47% per year with daily price data. The lowest mean reversion volatility is 29.91% with monthly data. All the t-Statistics for the regression intercept are equal to or higher than 2.00 for the daily, weekly, and monthly data, which means that the confidence level to reject the null hypothesis that the regression intercept equals to zero is higher than 97.5%. Figure 4-43 shows the daily spot price evolution from 2000 to 2003. The prices were repeating the complete cycles around the long run price of around $29.00/bbl throughout the entire price regime.

Tables 4-32 and 4-33 are the results of mean reversion parameter estimations and regressions for the increasing number of the forward years from 2004 to 2010 for the AF-2.3K price regime. Figures 4-44, 4-45, and 4-46 show that it took about five years from 2004 to 2007 to establish the new long run price pattern with a long run price ranging from $73.41/bbl to $79.66/bbl, which is much higher than the previous long run prices in the B4-2K and 2K-2.3K price regimes, a mean reversion rate of around 1.00 per year, and a mean reversion volatility of around 40% per year. Figure 4-47 shows the daily price evolution and the long run price for the AF-2.3K price regime. There are several complete cycles around long run price of about $76.00/bbl from below and above. The t-Statistic in Table 4-33 suggests that the confidence level of rejecting the null hypothesis that the regression intercept equals to zero is higher than 95% among all the regressions with price data in the increasing number of forward years from 2004 to 2010, and higher than 99% with the full range data for the AF-2.3K price regime.

Tables 4-34 and 4-35 present the mean reversion parameter estimation and regression results with the daily, weekly, monthly, and yearly price data from January 5, 2004 to May 28, 2010. The long run price for the AF-2.3K price regime is from $76.36/bbl to $78.00/bbl with different data sets. The long run mean reversion rate is from 0.7483 per year to 1.2242 per year. The mean reversion volatility is between 32.81% and 42.51% per year. The t-Statistic is equal to or higher than 1.86 for the regression intercept and slope. Above results suggest that the long run price pattern for the AF-2.3K price regime is well established.

Table 4-36 gives the summary reports of the mean reversion parameters, regression t-Statistic, and average price for the B4-2K, 2K-2.3K, and AF-2.3K price regimes with daily, weekly, monthly, and yearly (if applicable) price data. Figures 4-48, 4-49, and 4-50 further demonstrate the results in Table 4-36. Not only can the historical WTI oil price data be used to calibrate the mean reversion model parameters, but also the mean reversion model can be used to describe the historical oil price behavior. It is very clear that from January 2, 1986 to May 28, 2010, *WTI* oil prices have undergone three stages. The low price stage covered time from January 2, 1986 to December 30, 1999, with an average price of $19.09/bbl, the lowest price of $10.25/bbl, and the highest price of $41.07/bbl. The characteristic mean reversion long run price for this stage was $19.50/bbl. The transaction stage lasted from January 4, 2000 to December 31, 2003 with an average price of $28.41/bbl, the lowest price of $17.50/bbl, and the highest price of $37.96/bbl. The characteristic mean reversion long run price for this stage is $29.24/bbl. The high price ongoing stage was since January 5, 2004 with an average price of $67.23/bbl, the lowest price of $30.28/bbl, the highest price of $145.31/bbl. The characteristic mean reversion long run price for this stage is $77.06/bbl. The AF-2.3K stage has the lowest mean reversion rate among the three stages, as shown in Figure 4-49. The mean reversion volatility is very close in the three stages when the same type of the oil price data is used, such as daily data, as shown on Figure 4-50.

It is observed that when the price data cover more than two price regimes, the long run price from daily, weekly, and monthly data can have significant difference. And the regression t-Statistic can be very low, as shown for the full range data from 1986 to 2010 or AF-2K price regime. When the price data are classified into the right price regimes, then in each price regime, the long run price is very consistent no matter daily, weekly, monthly, or yearly (if applicable) price data are used. And the regression t-Statistic for both the regression slope and intercept is equal to or higher than 1.86 and the confident level to reject the null hypothesis for both the regression intercept and slope is equal to or higher than 97%.

The existence of different price regimes in the historical oil price evolution can have significant impact on modeling oil price movements using mean reversion model and may reveal the limitation by using historical oil prices alone to calibrate the parameters for the mean reversion price model and to conduct the price forecasting based on those calibrated parameters. For example, if it was now in the middle of the 1990s, and when previous years' data, even could be as long as ten years, were used to calibrate the model parameters and to forecast future prices, it would be very good upon the time around year 2000, but not very good for the forecasting afterwards. When it were at the beginning of year 2000, it would be very likely to reach the conclusion that the long run price would converge to about $20/bbl. This conclusion would be very misleading as what would happen to oil prices since year 2000.

From the mean reversion oil price evolution point of view, there may be three possibilities for the future oil price movements: 1) Future prices may still be in the AF-2.3K price regime, and repeat the cycles of moving up and down across the same long run price of about $77.00/bbl. Including more future price data may reconfirm the long run price pattern. And the oil prices may keep the fluctuation momentum with a long run price of $77.00/bbl, a daily price mean reversion rate of 1.2242 per year, and a daily price mean reversion volatility of 42.51% per year. 2) Since the AF-2.3K price regime is an ongoing regime, variation in long run price and other mean reversion parameters may be observed when more price data are included. 3) If future oil prices are in another price regime, then another long run price pattern will be developed with a different set of long run price, mean reversion rate, and volatility.

4.4 SUMMARY

From the above study on how to use historical price data to estimate the mean reversion parameters, it is concluded that:

Parameter estimations for the mean reversion price model using historical prices are based on a solid mathematical theory. In this chapter, not only the theory is used to estimate the parameters for the mean reversion oil price model with historical oil prices, but also the resulted mean reversion model parameters reveals the historical oil price evolution from another perspective, gives insight on how different the world economy is in responding to the changes of oil prices in different periods of time from 1986 to 2010, and proposes the possibilities on how future oil prices may be evolving according to the mean reversion price theory.

To meaningfully apply mean reversion theory to historical oil prices, to correctly calibrate the mean reversion parameters from the historical oil price data, and to extend the theory and the calibrated parameters to the future oil price movement need to have the knowledge of historical oil price evolution, to understand the limitation in calibrating mean reversion parameters with historical oil prices, and to select the right sets of price data to conduct the parameter estimations.

With the estimated mean reversion parameters, the mean reversion oil price model reveals that, the evolution of historical WTI oil prices from January 2, 1986 to May 28, 2010 is classified into three price regimes: 1) B4-2K regime (1986 to 1999) with a long run price of \$19.50/bbl, which is almost the same as the average price of \$19.09/bbl in this price regime; 2) 2K-2.3K price regime (2000-2003) with a long run price of \$29.24/bbl, which is very close to its average price of \$28.41/bbl; and 3) AF-2.3K regime (2004-2010) with a long rune price of about \$77.00/bbl, about \$10/bbl higher than the average price from 2004 to 2010, which is \$67.23/bbl. Since the 2K-2.3K price regime only lasted for 3 years, the

2K-2.3K and the AF-2.3K regimes can be considered as AF-2K regime with a long run price of \$73.31/bbl from daily price data.

The long run price and mean reversion rate reveal that in the AF-2.3K price regime, the world economy has increased its tolerance to the higher oil prices and to the higher price fluctuation from its long run price comparing with that in the B4-2K and 2K-2.3K price regimes.

The relatively stable mean reversion volatility, which is from 0.4129 per year to 0.4347 per year among three price regimes with daily price data, over the period from 1986 to 2010, reveals that the difference in oil price changes from 1986 to 2010 are mainly caused by the deterministic force, that is, mean reversion rate and long run price, not by the random effect- mean reversion volatility.

Limitations of applying mean reversion theory to historical oil prices include: 1) The results of the calibrated mean reversion parameters from historical oil prices are very data dependant. Meaningful parameter estimations for the mean reversion price model demand that the historical oil price data used are within the same price regime, cover enough length of time, evolve in complete cycles, and are with right type of data. 2) Extending parameters to future oil prices involves the risk that the future oil prices may not be in the same price regime as the one the mean reversion parameters are calibrated from. 3) Very low values of t-Statistic may occur for the regression intercept and slope, which may cause the unreliability of the calibrated parameters.

The existence of oil price regimes implies that: 1) The correct mean reversion parameters are calibrated regime by regime. The parameters calibrated from one price regime can be much different from those in another price regime. That is, the results from one price regime may not be representative for another regime and are not appropriate to be applied to another regime. 2) Extending mean reversion parameters to the future oil price movement needs to make regime assumptions for future oil prices and this extending may not be always applicable. When it is by the end of one price regime, no matter how well the mean reversion parameters are estimated from the previous price regime, the application of the parameters from previous price regimes to the future price movement may be very misleading if future prices will be in a much different price pattern from before. In history, the mean reversion parameters from 1986 to 1999 were much different from those in the days afterwards. By the end of 1999, meaningful mean reversion parameters could be estimated with oil prices from 1986 to 1999. However, the calibrated long run price and mean reversion rate cannot be applied to the price evolution after 1999 because of the switch of the oil price regime since the year of 2000.

To identify the price regime, the increasing number of forward years' price data is used, together with the original historical oil prices, to reveal the establishment of the long run price pattern, to see the changing of mean reversion parameters, and to identify the oil price regimes. The original historical oil prices alone are used to roughly distinguish the price regimes, but not precisely.

It takes time for a long run price pattern to be established. The long run price pattern is developed after there are several complete price cycles around the same long run price from below and above. The price data that are not evolving in cycles may give wrong mean reversion parameters.

One year's oil price data are not long enough to establish a stable long run price pattern. Thus one year's price data should be avoided to use either to identify the price regimes or to calibrate the mean reversion parameters.

Prices evolving in one direction, or without complete cycles, should be avoided to use to calibrate the mean reversion parameters, even after the price regime has already be identified and the data are all inside the same price regime.

The most current oil price data may not capture the long run price pattern and may not be representative to calibrate the mean reversion parameters.

Full range historical WTI oil price data from 1986 to 2010, which cover several price regimes, are not recommended to use for calibrating the mean reversion parameters. Such utilizing of the historical WTI oil price data may cause difference in the mean reversion parameters with different types of data, *i.e.*, daily, weekly, monthly, or yearly (if applicable) price data, all of which cover the same range of time, low t-Statistic for the regression intercept and slope, and thus inconsistent and unreliable estimated results for mean reversion parameters together with the incorrect description about the price evolution from 1986 to 2010.

For the historical WTI oil prices from 1986 to 2010, within the three identified price regimes, *i.e.*, B4-2K (1986-1999), 2K-2.3K (2000-2003), and AF-2.3K (2004-2010), the daily, weekly, monthly, and yearly (if applicable) data give very close mean reversion parameters, *i.e.*, long run price, annualized mean reversion rate, and mean reversion volatility, and very reliable t-Statistic for the regression intercept and slope; while overlapped price regimes result in several very low values of regression t-Statistic and significant difference in long run price, mean reversion rate, and mean reversion volatility when different types of price data are used.

While daily, weekly, and monthly oil price data can all be used to calibrate the mean reversion parameters within the same price regime and can give the same or close mean reversion parameters, the yearly oil price data should be avoided to use for such a purpose, because of the unacceptably low values of the regression R-square and t-Statistic for the B4-2K price regime.

When very low values of the regression t-Statistic occur, it may be the case that the length the price data cover is not appropriate, either too short, or too long, or the type of the price data is not suitable, such as yearly price data.

There are three steps involved in the mean reversion parameter estimations: First, use increasing number of forward years' data and original price data to identify the price regimes. Second, use the full range data within the price regime, or use as many available price data as possible in the price regime, to calibrate the mean reversion parameters so that the completed long run price evolving information within the price regime is captured.

Third, use different types of price data, such as daily, weekly, or monthly data, to check if the calibrated results for mean reversion parameters are consistent and the regression t-Statistic is reliable.

There may be three possibilities for the future oil price movements in and after 2010: 1) Future prices may still be in the AF-2.3K price regime and keep the fluctuation momentum with a long run price of about \$77.00/bbl, a daily price mean reversion rate of 1.2242 per year, and a daily price mean reversion volatility of 42.51% per year. Future prices may be repeating the cycles of moving up and down across the same long run price of about \$77.00/bbl. 2) Since the AF-2.3K price regime may be an ongoing regime, variation in long run price and other mean reversion parameters may be observed when more price data are included. 3) Future prices may be evolving to another price regime and another long run price pattern may be developed several years later with a different set of mean reversion parameters from those with the oil price data from 2004 to 2010.

Table 4-1: Mean Reversion Model Parameter Estimations for *WTI* Oil Prices in B4-2K Price Regime (Price Data from January 2, 1986 to December 30, 1999)

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
Daily Price	2.2813	0.4129	19.49
Weekly Price	1.3876	0.3220	19.45
Monthly Price	1.2828	0.3106	19.55
Yearly Price	2.4211	0.3214	19.67

Table 4-2: Mean Reversion Model Linear Regression Results for *WTI* Oil Prices in B4-2K Price Regime (Price Data from January 2, 1986 to December 30, 1999)

Regression Results	$\hat{a}$	$\hat{b}$	*t*-Stat for $\hat{a}$	*t*-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
Daily Price	0.0264088	0.99098794	3.99	440.15	0.025893	0.982	3549
Weekly Price	0.0771645	0.97366783	3.10	114.66	0.044067	0.948	730
Monthly Price	0.2976138	0.89861488	2.88	25.53	0.085080	0.798	167
Yearly Price	2.6948236	0.08882144	3.39	0.33	0.145460	0.0097	13

Table 4-3: Mean Reversion Model Parameter Estimations for *WTI* Oil Prices in AF-2K Price Regime (Price Data from January 4, 2000 to May 28, 2010)

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
Daily Price	0.4275	0.4279	73.31
Weekly Price	0.2822	0.3402	79.74
Monthly Price	0.2525	0.3251	84.84
Yearly Price	0.2646	0.2904	79.61

Table 4-4: Mean Reversion Model Linear Regression Results for *WTI* Oil Prices in AF-2K Price Regime (Price Data from January 4, 2000 to May 28, 2010)

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
Daily Price	0.0069158	0.99830517	1.62	907.17	0.026931	0.997	2608
Weekly Price	0.0225891	0.99458808	1.39	236.04	0.047022	0.990	543
Monthly Price	0.0880924	0.97918127	1.31	56.32	0.092870	0.963	125
Yearly Price	0.9806203	0.76750650	1.65	4.82	0.255880	0.744	10

Table 4-5: Summary Output for the Linear Regression Results of LnP_t on LnP_{t-1} for *WTI* Weekly Oil Prices from 1986 to 1999

Regression Statistics	
Multiple R	0.973410717
R Square	0.947528424
Adjusted R Square	0.947456348
Standard Error	0.044066658
Observations	730

ANOVA

	df	*SS*	*MS*	*F*
Regression	1	25.52817361	25.52817361	13146.1784
Residual	728	1.41368159	0.00194187	
Total	729	26.9418552		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.077164509	0.024929167	3.095350433	0.002041142
X Variable	0.973667831	0.008492021	114.6567852	0

Table 4-6: *WTI* Oil Price Mean Reversion Model Parameter Estimations from Daily Price Data Each Year from 1986 to 2010

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
1986	11.7587	0.6833	14.62
1987	5.2160	0.2425	19.02
1988	8.9154	0.3744	16.07
1989	17.8866	0.3459	19.92
1990	2.6464	0.6280	28.13
1991	28.2449	0.5888	21.31
1992	5.2697	0.2021	20.68
1993	-0.3754	0.2381	39.92
1994	6.0085	0.2966	17.89
1995	11.8210	0.2340	18.60
1996	7.2687	0.4065	23.12
1997	7.8710	0.2856	19.67
1998	11.8264	0.5343	14.06
1999	1.7519	0.3589	29.94
2000	11.8769	0.4712	30.61
2001	3.6014	0.4705	24.45
2002	4.3307	0.3364	29.17
2003	14.8023	0.4578	31.27
2004	5.0214	0.3663	44.14
2005	7.2499	0.3526	59.49
2006	5.9892	0.2945	66.26
2007	0.3932	0.2988	253.86
2008	-0.7018	0.6253	212.36
2009	2.6986	0.5369	78.73
2010	13.3366	0.3406	78.14

Table 4-7: Linear Regression Results for *WTI* Oil Price Mean Reversion Model Parameter Estimations from Daily Price Data Each Year from 1986 to 2010

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
1986	0.1213806	0.95441044	3.00	63.68	0.042057	0.942	250
1987	0.0602248	0.97951450	1.40	67.30	0.015122	0.947	254
1988	0.0962534	0.96523995	2.10	58.37	0.023172	0.930	257
1989	0.2047656	0.93148170	3.11	42.03	0.021040	0.874	257
1990	0.0340810	0.98955340	1.15	105.69	0.039355	0.978	257
1991	0.3237312	0.89397001	4.13	35.01	0.035105	0.828	256
1992	0.0626079	0.97930548	1.68	79.29	0.012601	0.961	257
1993	-0.0056084	1.00149064	-0.20	101.57	0.015009	0.977	250
1994	0.0677799	0.97643890	2.06	84.42	0.018464	0.966	252
1995	0.1338421	0.95417438	2.37	49.16	0.014402	0.907	251
1996	0.0889690	0.97156808	1.85	62.50	0.025245	0.939	254
1997	0.0914576	0.96924870	2.29	73.36	0.017715	0.956	252
1998	0.1206487	0.95415386	2.45	51.70	0.032881	0.915	251
1999	0.0232954	0.99307207	1.39	173.83	0.022529	0.992	251
2000	0.1570761	0.95396300	2.48	51.32	0.028995	0.914	250
2001	0.0449245	0.98581046	1.09	77.73	0.029430	0.961	250
2002	0.0572527	0.98296159	1.73	96.68	0.021013	0.974	250
2003	0.1959920	0.94295276	2.68	44.34	0.028015	0.888	250
2004	0.0744571	0.98027084	1.89	92.35	0.022846	0.972	249
2005	0.1156265	0.97164047	2.35	79.69	0.021898	0.962	251
2006	0.0983224	0.97651349	1.71	71.00	0.018333	0.953	249
2007	0.0084559	0.99844080	0.29	146.53	0.018810	0.988	252
2008	-0.0157189	1.00278862	-0.49	142.06	0.039443	0.988	253
2009	0.0459374	0.98934831	1.25	110.26	0.033638	0.980	252
2010	0.2244411	0.94845321	1.55	28.55	0.020901	0.891	102

Table 4-8: Mean Reversion Model Parameter Estimations for *WTI* Oil Prices (Price Data from January 2, 1986 to May 28, 2010)

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
Daily Price	0.2107	0.4188	51.36
Weekly Price	0.1117	0.3285	64.64
Monthly Price	0.0907	0.3115	79.91
Yearly Price	0.0715	0.2435	97.66

Table 4-9: Mean Reversion Model Linear Regression Results for *WTI* Oil Prices (Price Data from January 2, 1986 to May 28, 2010)

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
Daily Price	0.0029447	0.99916411	1.48	1686.34	0.026368	0.998	6157
Weekly Price	0.0079101	0.99785397	1.04	443.40	0.045500	0.994	1273
Monthly Price	0.0289651	0.99246919	0.93	106.77	0.089569	0.975	292
Yearly Price	0.2876333	0.93097466	0.93	9.96	0.235031	0.825	23

Table 4-10: *WTI* Oil Price Mean Reversion Model Parameter Estimations from Daily Price Data in Three Backward Rolling Years from 1999 to 1986

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
1999-1997	1.6321	0.4044	18.57
1998-1996	1.2502	0.4173	17.63
1997-1995	4.1361	0.3159	20.50
1996-1994	2.6401	0.3185	20.97
1995-1993	3.9394	0.2574	18.10
1994-1992	2.2559	0.2481	18.67
1993-1991	6.4555	0.3804	19.61
1992-1990	4.1477	0.5057	22.34
1991-1989	4.5088	0.5302	22.38
1990-1988	1.6465	0.4636	23.09
1989-1987	3.3341	0.3229	18.79
1988-1986	5.3687	0.4673	16.45

Table 4-11: *WTI* Oil Price Linear Regression Results for Mean Reversion Model Parameter Estimations from Daily Price Data in Three Backward Rolling Years from 1999 to 1986

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
1999-1997	0.0185384	0.99354426	1.56	241.18	0.025395	0.987	754
1998-1996	0.0138553	0.99505113	1.05	221.50	0.026221	0.985	757
1997-1995	0.0489767	0.98372070	2.48	149.95	0.019741	0.968	757
1996-1994	0.0315139	0.98957822	2.02	186.78	0.019957	0.979	757
1995-1993	0.0447880	0.98448891	2.42	153.60	0.016091	0.969	753
1994-1992	0.0259657	0.99108784	1.83	203.90	0.015558	0.982	759
1993-1991	0.0749853	0.97470845	3.08	120.39	0.023660	0.950	763
1992-1990	0.0502050	0.98367543	2.48	150.19	0.031597	0.967	770
1991-1989	0.0545625	0.98226690	2.68	148.49	0.033101	0.966	770
1990-1988	0.0200195	0.99348766	1.48	217.97	0.029111	0.984	771
1989-1987	0.0383450	0.98685658	2.14	159.46	0.020209	0.971	768
1988-1986	0.0585971	0.97892078	3.20	150.20	0.029125	0.967	761

Table 4-12: *WTI* Oil Price Mean Reversion Model Parameter Estimation from Daily Price Data in Three Backward Rolling Years from 2010 to 2000

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
2010-2008	1.3675	0.5497	78.53
2009-2007	1.2750	0.5069	87.80
2008-2006	0.6988	0.4360	75.22
2007-2005	1.8186	0.3154	76.28
2006-2004	1.8673	0.3374	61.63
2005-2003	0.9005	0.3912	58.09
2004-2002	1.9433	0.3867	38.12
2003-2001	3.7658	0.4234	28.60
2002-2000	4.1221	0.4289	28.28

Table 4-13: *WTI* Oil Price Linear Regression Results for Mean Reversion Model Parameter Estimations from Daily Price Data in Three Backward Rolling Years from 2010 to 2000

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
2010-2008	0.0230164	0.99458814	1.30	243.97	0.034532	0.990	607
2009-2007	0.0220756	0.99495332	1.43	278.40	0.031853	0.990	757
2008-2006	0.0115868	0.99723098	0.75	279.61	0.027429	0.990	754
2007-2005	0.0309699	0.99280943	1.67	223.40	0.019795	0.985	752
2006-2004	0.0301986	0.99261761	2.22	290.26	0.021174	0.991	749
2005-2003	0.0141859	0.99643302	1.14	298.18	0.024596	0.992	750
2004-2002	0.0276722	0.99231817	2.01	250.40	0.024267	0.988	749
2003-2001	0.0493887	0.98516734	2.31	152.78	0.026475	0.969	750
2002-2000	0.0538638	0.98377535	2.45	147.78	0.026797	0.967	750

Table 4-14: *WTI* Oil Price Mean Reversion Model Parameter Estimations from Daily Price Data in Five Backward Rolling Years from 1999 to 1986

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
1999-1995	1.5086	0.3759	20.51
1998-1994	1.8208	0.3643	18.64
1997-1993	2.7385	0.2977	19.38
1996-1992	2.0436	0.2833	20.18
1995-1991	5.4410	0.3389	19.06
1994-1990	2.7264	0.4275	20.49
1993-1989	3.2194	0.4341	21.03
1992-1988	2.9375	0.4511	20.98
1991-1987	2.9081	0.4554	20.60
1990-1986	2.1259	0.4818	19.61

Table 4-15: *WTI* Oil Price Linear Regression Results for Mean Reversion Model Parameter Estimations from Daily Price Data in Five Backward Rolling Years from 1999 to 1986

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
1999-1995	0.0177502	0.99403140	1.82	298.39	0.023607	0.986	1259
1998-1994	0.0208002	0.99280049	1.94	269.86	0.022868	0.983	1260
1997-1993	0.0318651	0.98919169	2.58	237.27	0.018654	0.978	1259
1996-1992	0.0241096	0.99192313	2.01	244.76	0.017773	0.979	1264
1995-1991	0.0627377	0.97864028	4.16	191.51	0.021121	0.967	1266
1994-1990	0.0321380	0.98923932	2.59	239.81	0.026785	0.978	1272
1993-1989	0.0382953	0.98730595	2.67	209.11	0.027170	0.972	1277
1992-1988	0.0348720	0.98841109	2.78	236.75	0.028254	0.978	1284
1991-1987	0.0343025	0.98852614	2.73	235.02	0.028525	0.977	1281
1990-1986	0.0245427	0.99159918	2.23	263.58	0.030224	0.982	1275

Table 4-16: *WTI* Oil Price Mean Reversion Model Parameter Estimations from Daily Price Data in Five Backward Rolling Years from 2010 to 2000

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
2010-2006	1.3768	0.4527	80.66
2009-2005	1.4788	0.4428	79.43
2008-2004	1.0057	0.4065	73.41
2007-2003	0.4131	0.3556	102.25
2006-2002	0.7746	0.3621	60.29
2005-2001	0.4930	0.3974	56.53
2004-2000	1.7956	0.4208	33.78

Table 4-17: *WTI* Oil Price Linear Regression Results for Mean Reversion Model Parameter Estimations from Daily Price Data in Five Backward Rolling Years from 2010 to 2000

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
2010-2006	0.0235148	0.99455150	1.78	322.43	0.028437	0.989	1108
2009-2005	0.0252100	0.99414883	2.13	356.06	0.027814	0.990	1257
2008-2004	0.0167836	0.99601716	1.89	466.92	0.025553	0.994	1254
2007-2003	0.0073280	0.99836224	0.98	526.45	0.022383	0.996	1251
2006-2002	0.0123205	0.99693104	1.88	568.25	0.022774	0.996	1249
2005-2001	0.0075726	0.99804562	0.98	457.38	0.025008	0.994	1250
2004-2000	0.0246417	0.99289977	2.01	277.08	0.026411	0.984	1249

Table 4-18: *WTI* Oil Price Mean Reversion Model Parameter Estimations from Daily Price Data in Seven Backward Rolling Years from 1999 to 1986

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
1999-1993	1.6398	0.3487	19.53
1998-1992	1.6197	0.3294	18.49
1997-1991	4.5668	0.3420	19.67
1996-1990	2.7601	0.4020	20.90
1995-1989	3.0971	0.3936	20.42
1994-1988	2.6424	0.4081	20.06
1993-1987	2.5944	0.4029	20.06
1992-1986	2.7770	0.4678	19.59

Table 4-19: *WTI* Oil Price Linear Regression Results for Mean Reversion Model Parameter Estimations from Daily Price Data in Seven Backward Rolling Years from 1999 to 1986

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
1999-1993	0.0190344	0.99351408	2.24	341.48	0.021894	0.985	1761
1998-1992	0.0184737	0.99359334	2.04	321.89	0.020685	0.983	1767
1997-1991	0.0532721	0.98204109	4.27	235.11	0.021353	0.969	1772
1996-1990	0.0327943	0.98910682	3.04	275.84	0.025188	0.977	1777
1995-1989	0.0365385	0.98778498	3.34	269.87	0.024640	0.976	1780
1994-1988	0.0309500	0.98956917	3.07	291.86	0.025571	0.979	1786
1993-1987	0.0303918	0.98975764	2.88	280.04	0.025249	0.978	1788
1992-1986	0.0321747	0.98904071	3.20	290.80	0.029310	0.979	1788

Table 4-20: *WTI* Oil Price Mean Reversion Model Parameter Estimations from Daily Price Data in Seven Backward Rolling Years from 2010 to 2000

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
2010-2004	1.2242	0.4251	76.36
2009-2003	0.7330	0.4333	77.63
2008-2002	0.6457	0.4035	68.43
2007-2001	0.1888	0.3708	159.59
2006-2000	0.5159	0.3947	54.71

Table 4-21: *WTI* Oil Price Linear Regression Results for Mean Reversion Model Parameter Estimations from Daily Price Data in Seven Backward Rolling Years from 2010 to 2000

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
2010-2004	0.0206525	0.99515391	2.37	476.16	0.026715	0.993	1608
2009-2003	0.0122684	0.99709549	1.81	598.47	0.027258	0.995	1756
2008-2002	0.0104923	0.99744081	2.06	772.07	0.025383	0.997	1754
2007-2001	0.0035265	0.99925103	0.71	755.19	0.023352	0.997	1751
2006-2000	0.0078761	0.99795484	1.34	616.34	0.024839	0.995	1749

Table 4-22: *WTI* Oil Price Mean Reversion Model Parameter Estimations from Daily Price Data in Increasing Number of Backward Years from 1999 to 1986

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
1999	1.7519	0.3589	29.94
1999-1998	0.9477	0.4527	22.33
1999-1997	1.6321	0.4044	18.57
1999-1996	1.3963	0.4040	20.81
1999-1995	1.5086	0.3759	20.51
1999-1994	1.7252	0.3637	20.21
1999-1993	1.6398	0.3487	19.53
1999-1992	1.6351	0.3335	19.71
1999-1991	2.4433	0.3689	19.37
1999-1990	2.0414	0.4024	20.31
1999-1989	2.1773	0.3971	20.44
1999-1988	2.1055	0.3949	20.12
1999-1987	2.1371	0.3853	20.00
1999-1986	2.2813	0.4129	19.45

Table 4-23: *WTI* Oil Price Linear Regression Results for Mean Reversion Model Parameter Estimations from Daily Price Data in Increasing Number of Backward Years from 1999 to 1986

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
1999	0.0232954	0.99307207	1.39	173.83	0.022529	0.992	251
1999-1998	0.0112517	0.99624650	0.74	183.83	0.028466	0.985	502
1999-1997	0.0185384	0.99354426	1.56	241.18	0.025395	0.987	754
1999-1996	0.0164505	0.99447441	1.56	276.02	0.025380	0.987	1008
1999-1995	0.0177502	0.99403140	1.82	298.39	0.023607	0.986	1259
1999-1994	0.0202506	0.99317720	2.24	320.59	0.022830	0.986	1511
1999-1993	0.0190344	0.99351408	2.24	341.48	0.021894	0.985	1761
1999-1992	0.0190615	0.99353246	2.42	368.95	0.020939	0.985	2018
1999-1991	0.0283302	0.99035105	3.40	350.57	0.023126	0.982	2274
1999-1990	0.0239737	0.99193179	3.14	385.22	0.025249	0.983	2531
1999-1989	0.0256484	0.99139714	3.42	392.15	0.024909	0.982	2788
1999-1988	0.0246670	0.99167966	3.51	416.48	0.024774	0.983	3045
1999-1987	0.0250055	0.99155526	3.66	429.03	0.024166	0.982	3299
1999-1986	0.0264088	0.99098794	3.99	440.15	0.025893	0.982	3549

Table 4-24: *WTI* Oil Price Mean Reversion Model Parameter Estimations from Daily Price Data in Increasing Number of Backward Years from 2010 to 2000

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
2010	13.3366	0.3406	78.14
2010-2009	2.9344	0.4868	76.67
2010-2008	1.3675	0.5497	78.53
2010-2007	1.3460	0.4895	85.11
2010-2006	1.3768	0.4527	80.66
2010-2005	1.5355	0.4356	78.53
2010-2004	1.2242	0.4251	76.36
2010-2003	0.7583	0.4284	76.29
2010-2002	0.6790	0.4183	75.44
2010-2001	0.4325	0.4238	77.27
2010-2000	0.4275	0.4279	73.31

Table 4-25: *WTI* Oil Price Linear Regression Results for Mean Reversion Model Parameter Estimations from Daily Price Data in Increasing Number of Backward Years from 2010 to 2000

Regression Results	$\hat{a}$	$\hat{b}$	*t*-Stat for $\hat{a}$	*t*-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
2010	0.2244411	0.94845321	1.55	28.55	0.020901	0.891	102
2010-2009	0.0497699	0.98842318	1.73	144.00	0.030489	0.983	354
2010-2008	0.0230164	0.99458814	1.30	243.97	0.034532	0.990	607
2010-2007	0.0231993	0.99467281	1.56	289.62	0.030755	0.990	859
2010-2006	0.0235148	0.99455150	1.78	322.43	0.028437	0.989	1108
2010-2005	0.0261306	0.99392546	2.27	366.09	0.027355	0.990	1359
2010-2004	0.0206525	0.99515391	2.37	476.16	0.026715	0.993	1608
2010-2003	0.0126593	0.99699552	1.92	617.25	0.026948	0.995	1858
2010-2002	0.0112874	0.99730899	2.22	781.20	0.026315	0.997	2108
2010-2001	0.0070987	0.99828517	1.59	873.35	0.026675	0.997	2358
2010-2000	0.0069158	0.99830517	1.62	907.17	0.026931	0.997	2608

Table 4-26: *WTI* Oil Price Mean Reversion Model Parameter Estimations from Daily Price Data in Increasing Number of Forward Years from 1986 to 2004

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
1986	11.7587	0.6833	14.62
1986-1987	5.0148	0.5093	16.57
1986-1988	5.3687	0.4673	16.45
1986-1989	4.1814	0.4387	17.48
1986-1990	2.1259	0.4818	19.61
1986-1991	2.8192	0.4989	19.39
1986-1992	2.7770	0.4678	19.59
1986-1993	2.6378	0.4461	19.20
1986-1994	2.7476	0.4321	19.16
1986-1995	2.8115	0.4166	19.15
1986-1996	2.6335	0.4154	19.66
1986-1997	2.7672	0.4062	19.51
1986-1998	2.3515	0.4166	18.97
1986-1999	2.2813	0.4129	19.45
1986-2000	1.7304	0.4163	20.36
1986-2001	1.7148	0.4196	20.53
1986-2002	1.5012	0.4152	21.34
1986-2003	1.3288	0.4171	22.09
1986-2004	0.8771	0.4144	24.17

Table 4-27: *WTI* Oil Price Linear Regression Results for Mean Reversion Model Parameter Estimations from Daily Price Data in Increasing Number of Forward Years from 1986 to 2004

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
1986	0.1213806	0.95441044	3.00	63.68	0.042057	0.942	250
1986-1987	0.0548088	0.98029688	2.55	128.88	0.031764	0.971	504
1986-1988	0.0585971	0.97892078	3.20	150.20	0.029125	0.967	761
1986-1989	0.0467068	0.98354423	3.07	184.44	0.027406	0.971	1018
1986-1990	0.0245427	0.99159918	2.23	263.58	0.030224	0.982	1275
1986-1991	0.0324911	0.98887512	2.99	267.84	0.031252	0.979	1531
1986-1992	0.0321747	0.98904071	3.20	290.79	0.029310	0.979	1788
1986-1993	0.0303760	0.98958718	3.23	310.46	0.027954	0.979	2038
1986-1994	0.0316493	0.98915599	3.60	330.85	0.027070	0.980	2290
1986-1995	0.0324151	0.98890520	3.84	344.47	0.026096	0.979	2541
1986-1996	0.0306279	0.98960385	3.81	362.88	0.026030	0.979	2795
1986-1997	0.0321181	0.98907898	4.14	377.47	0.025446	0.979	3047
1986-1998	0.0269897	0.99071197	3.80	409.71	0.026123	0.981	3298
1986-1999	0.0264088	0.99098794	3.99	440.15	0.025893	0.982	3549
1986-2000	0.0202783	0.99315697	3.58	521.30	0.026136	0.986	3799
1986-2001	0.0201436	0.99321841	3.73	550.03	0.026345	0.987	4049
1986-2002	0.0178378	0.99406042	3.49	584.44	0.026077	0.988	4299
1986-2003	0.0159330	0.99474067	3.37	637.41	0.026206	0.989	4549
1986-2004	0.0107266	0.99652549	2.69	765.74	0.026058	0.992	4798

Table 4-28: *WTI* Oil Price Mean Reversion Model Parameter Estimations from Daily Price Data in Increasing Number of Forward Years from 2000 to 2010

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
2000	11.8769	0.4712	30.61
2000-2001	3.7274	0.4685	27.75
2000-2002	4.1221	0.4289	28.28
2000-2003	4.3989	0.4347	29.11
2000-2004	1.7956	0.4208	33.78
2000-2005	0.6531	0.4094	47.74
2000-2006	0.5159	0.3947	54.71
2000-2007	0.2273	0.3838	115.13
2000-2008	0.4382	0.4182	62.12
2000-2009	0.4137	0.4313	75.12
2000-2010	0.4275	0.4279	73.31

Table 4-29: *WTI* Oil Price Linear Regression Results for Mean Reversion Model Parameter Estimations from Daily Price Data in Increasing Number of Forward Years from 2000 to 2010

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
2000	0.1570761	0.95396300	2.48	51.32	0.028995	0.914	250
2000-2001	0.0483604	0.98531760	1.65	111.75	0.029294	0.962	500
2000-2001	0.0538638	0.98377535	2.45	147.78	0.026797	0.967	750
2000-2003	0.0579654	0.98269542	2.93	166.07	0.027145	0.965	1000
2000-2004	0.0246417	0.99289977	2.01	277.08	0.026411	0.984	1249
2000-2005	0.0096731	0.99741183	1.25	453.54	0.025757	0.993	1500
2000-2006	0.0078761	0.99795484	1.34	616.34	0.024839	0.995	1749
2000-2007	0.0039866	0.99909847	0.81	758.96	0.024166	0.997	2001
2000-2008	0.0068264	0.99826281	1.57	876.13	0.026318	0.997	2254
2000-2009	0.0067156	0.99835973	1.53	877.37	0.027147	0.997	2506
2000-2010	0.0069158	0.99830517	1.62	907.17	0.026931	0.997	2608

Table 4-30: Mean Reversion Model Parameter Estimations for *WTI* Oil Prices in 2K-2.3K Price Regime (Price Data from January 4, 2000 to December 31, 2003)

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
Daily Price	4.3989	0.4347	29.11
Weekly Price	2.6455	0.3392	29.25
Monthly Price	2.1402	0.2991	29.35

Table 4-31: Mean Reversion Model Linear Regression Results for *WTI* Oil Prices in 2K-2.3K Price Regime (Price Data from January 4, 2000 to December 31, 2003)

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
Daily Price	0.0579654	0.98269542	2.93	166.07	0.027145	0.965	1000
Weekly Price	0.1663726	0.95039726	2.26	43.09	0.045861	0.900	208
Monthly Price	0.5485866	0.83664642	1.99	10.12	0.079192	0.690	48

Table 4-32: *WTI* Oil Price Mean Reversion Model Parameter Estimations from Daily Price Data in Increasing Number of Forward Years from 2004 to 2010

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
2004	5.0214	0.3663	44.14
2004-2005	2.1937	0.3581	57.25
2004-2006	1.8673	0.3374	61.63
2004-2007	0.9956	0.3280	79.66
2004-2008	1.0057	0.4065	73.41
2004-2009	1.1881	0.4306	77.16
2004-2010	1.2242	0.4251	76.36

Table 4-33: *WTI* Oil Price Linear Regression Results for Mean Reversion Model Parameter Estimations from Daily Price Data in Increasing Number of Forward Years from 2004 to 2010

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
2004	0.0744571	0.98027084	1.89	92.35	0.022846	0.972	249
2004-2005	0.0348276	0.99133271	1.80	198.19	0.022460	0.987	500
2004-2006	0.0301986	0.99261761	2.22	290.26	0.021174	0.991	749
2004-2007	0.0170476	0.99605716	1.60	378.79	0.020618	0.993	1001
2004-2008	0.0167836	0.99601716	1.89	466.92	0.025553	0.994	1254
2004-2009	0.0200738	0.99529652	2.25	462.45	0.027064	0.993	1506
2004-2010	0.0206525	0.99515391	2.37	476.16	0.026715	0.993	1608

Table 4-34: Mean Reversion Model Parameter Estimations for *WTI* Oil Prices in AF-2.3K Price Regime (Price Data from January 5, 2004 to May 28, 2010)

Mean Reversion Parameters	η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl
Daily Price	1.2242	0.4251	76.36
Weekly Price	0.9136	0.3441	77.51
Monthly Price	0.9235	0.3512	78.00
Yearly Price	0.7483	0.3281	76.37

Table 4-35: Mean Reversion Model Linear Regression Results for *WTI* Oil Prices in AF-2.3K Price Regime (Price Data from January 5, 2004 to May 28, 2010)

Regression Results	$\hat{a}$	$\hat{b}$	t-Stat for $\hat{a}$	t-Stat for $\hat{b}$	$\widehat{sd}(\varepsilon_t)$	R^2	Obs.
Daily Price	0.0206525	0.99515391	2.37	476.16	0.026715	0.993	1608
Weekly Price	0.0746393	0.98258355	2.23	122.21	0.047305	0.978	335
Monthly Price	0.3177621	0.92592903	2.23	26.98	0.097613	0.907	77
Yearly Price	2.2462850	0.47315882	2.17	1.86	0.236270	0.463	6

Table 4-36: Summary of the Parameter Estimations for the Mean Reversion Price Model and Average Prices for *WTI* Historical Oil Prices in the Three Price Regimes

Mean Reversion Parameters		η Annualized	σ Annualized	Long Run Price $\bar{p}$ \$/bbl	Average Price	*t*-Stat for $\hat{a}$	*t*-Stat for $\hat{b}$
BF-2K Price Regime (1986-1999)	Daily Price	2.2813	0.4129	19.49	19.09	3.99	440.15
	Weekly Price	1.3876	0.3220	19.45	19.09	3.10	114.66
	Monthly Price	1.2828	0.3106	19.55	19.09	2.88	25.53
	Yearly Price	2.4211	0.3214	19.67	19.09	3.39	0.33
2K-2.3K Price Regime (2000-2003)	Daily Price	4.3989	0.4347	29.11	28.41	2.93	166.07
	Weekly Price	2.6455	0.3392	29.25	28.41	2.26	43.09
	Monthly Price	2.1402	0.2991	29.35	28.41	1.99	10.12
	Yearly Price	NA	NA	NA	NA	NA	NA
AF-2.3K Price Regime (2004-2010)	Daily Price	1.2242	0.4251	76.36	67.23	2.37	476.16
	Weekly Price	0.9136	0.3441	77.51	67.23	2.23	122.21
	Monthly Price	0.9235	0.3512	78.00	67.23	2.23	26.98
	Yearly Price	0.7483	0.3281	76.37	67.23	2.17	1.86

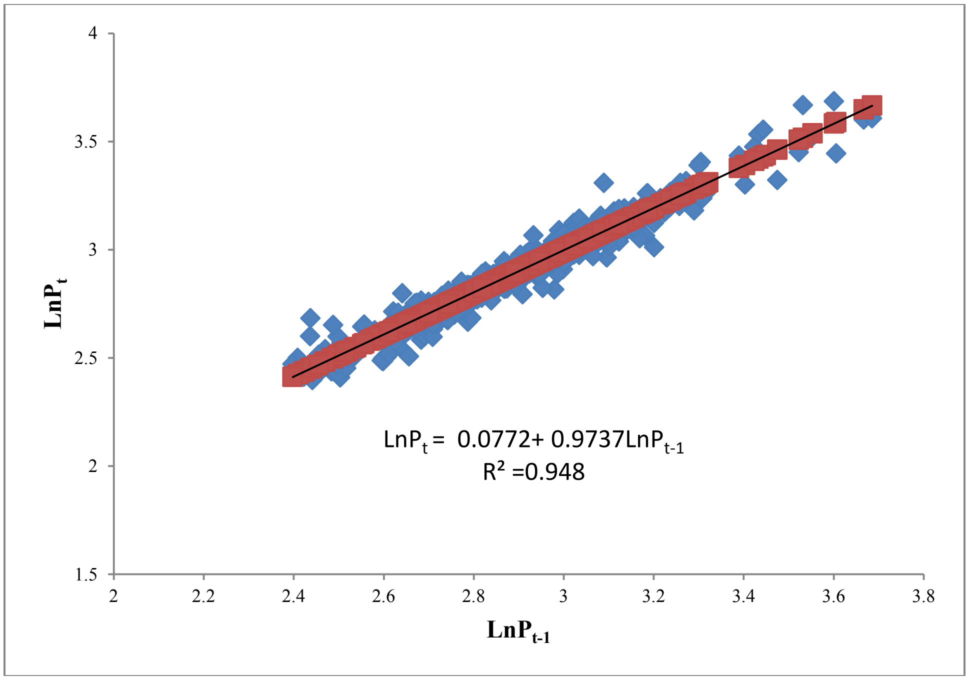


Figure 4-1: Linear Regression Results for *WTI* Weekly Oil Prices from 1986 to 1999

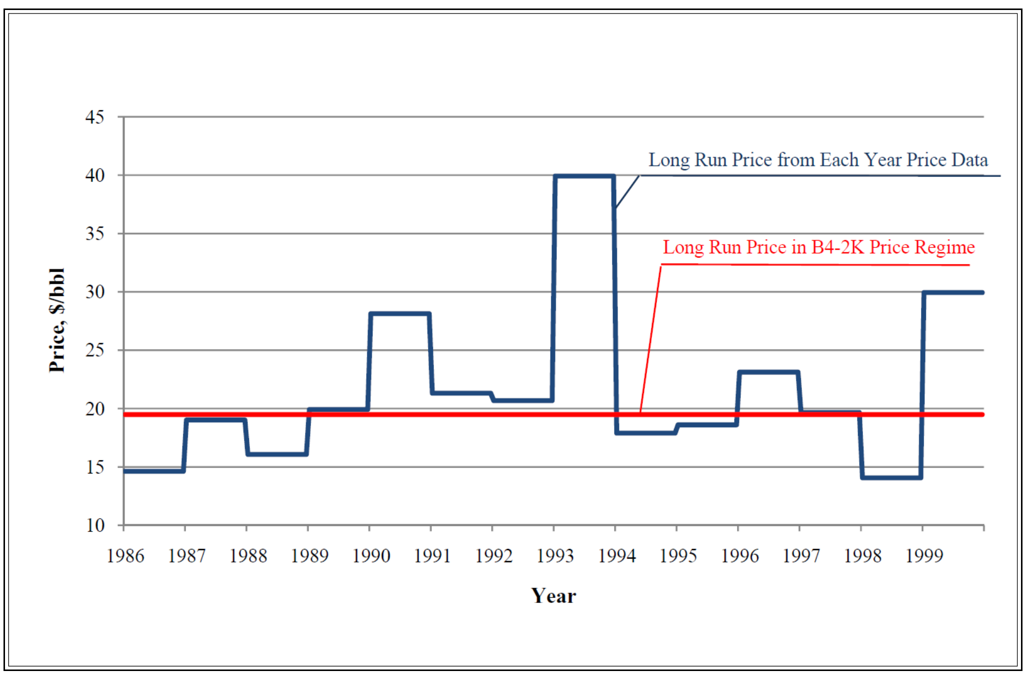


Figure 4-2: *WTI* Oil Price Mean Reversion Long Run Price from Daily Spot Price Data Each Year from 1986 to 1999

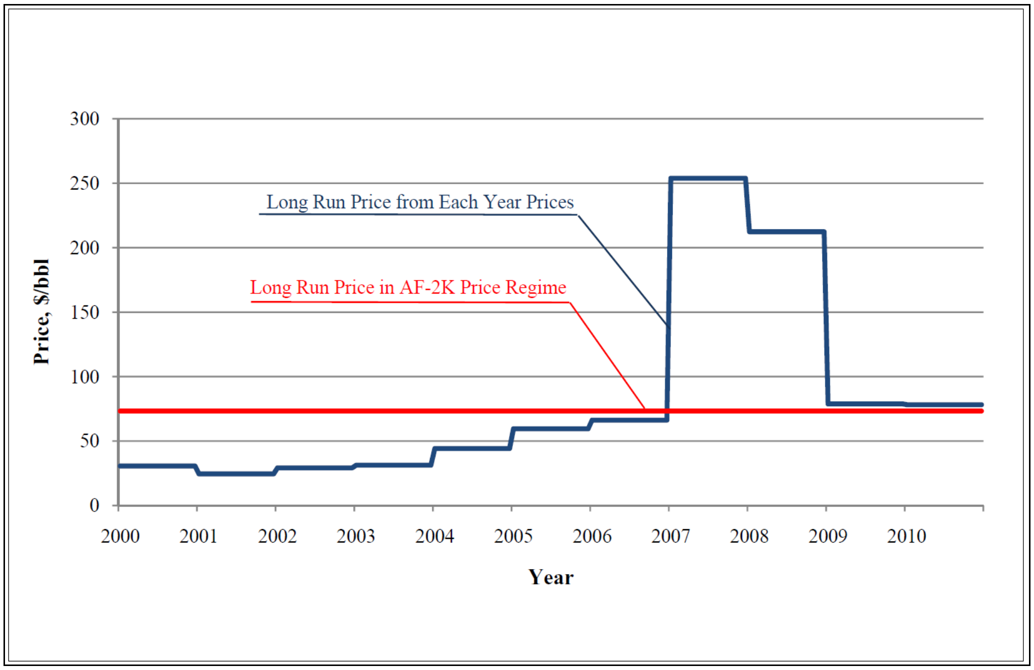


Figure 4-3: *WTI* Oil Price Mean Revision Long Run Price from Daily Spot Price Data Each Year from 2000 to 2010

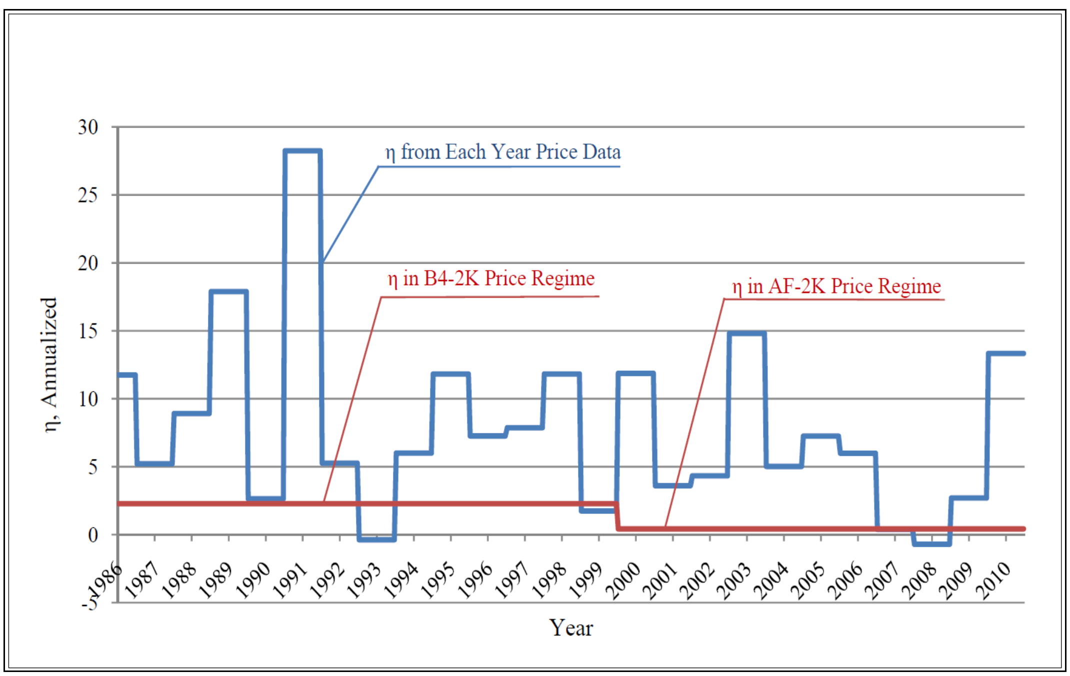


Figure 4-4: *WTI* Oil Price Mean Reversion Rate from Daily Spot Price Data Each Year from 1986 to 2010

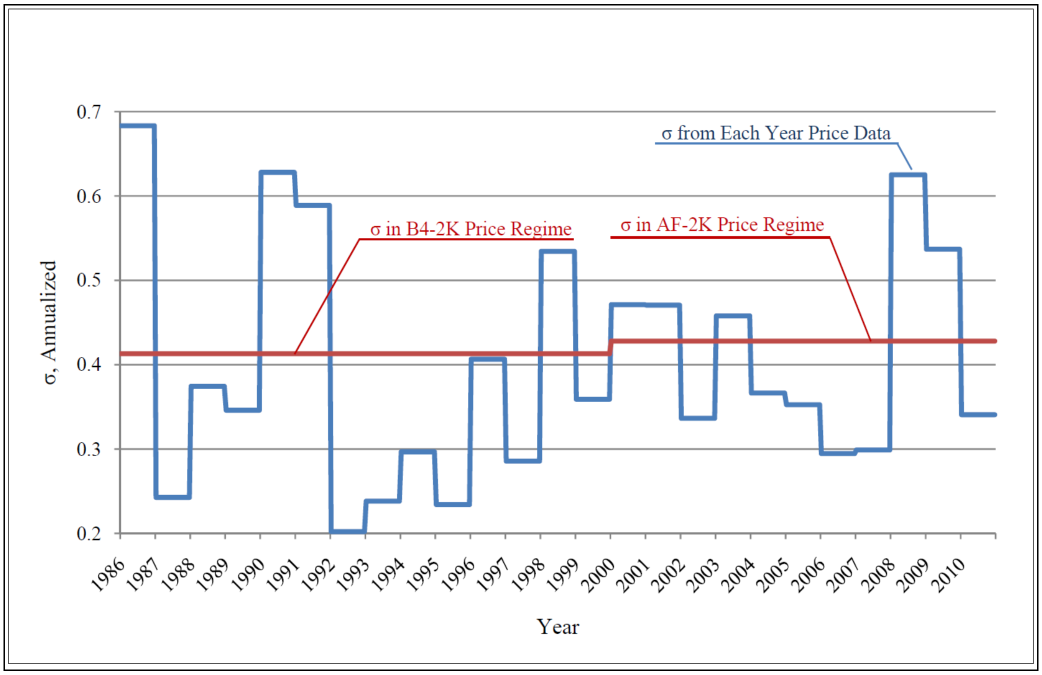


Figure 4-5: *WTI* Oil Price Mean Reversion Volatility from Daily Spot Price Data Each Year from 1986 to 2010

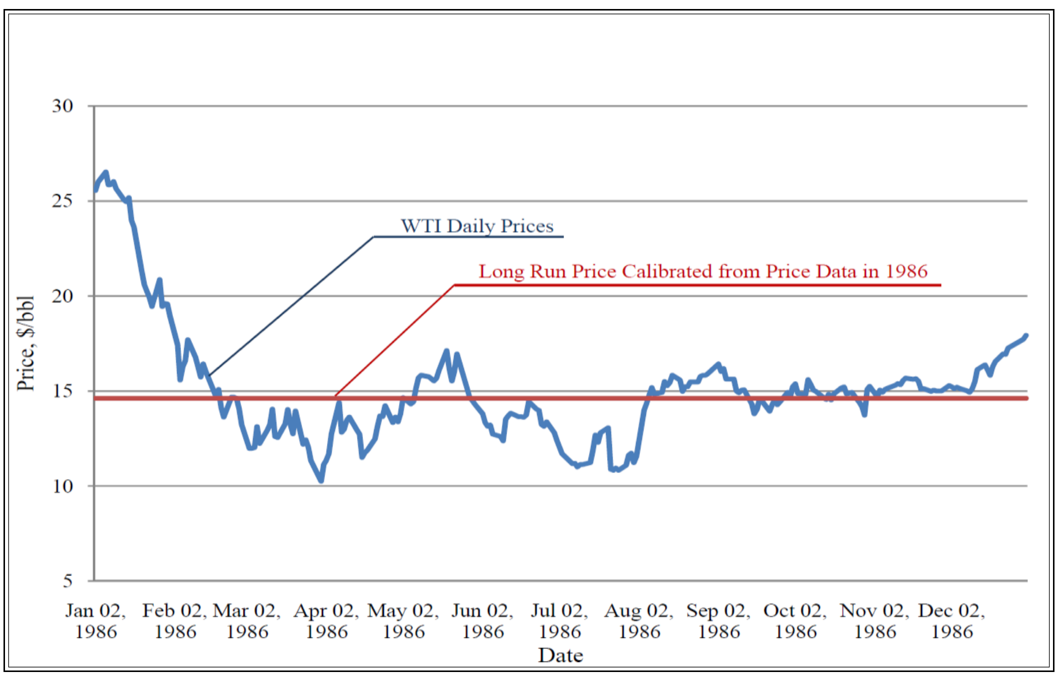


Figure 4-6: *WTI* Daily Spot Oil Prices in 1986 and the Long Run Price Calibrated from the Price Data in 1986

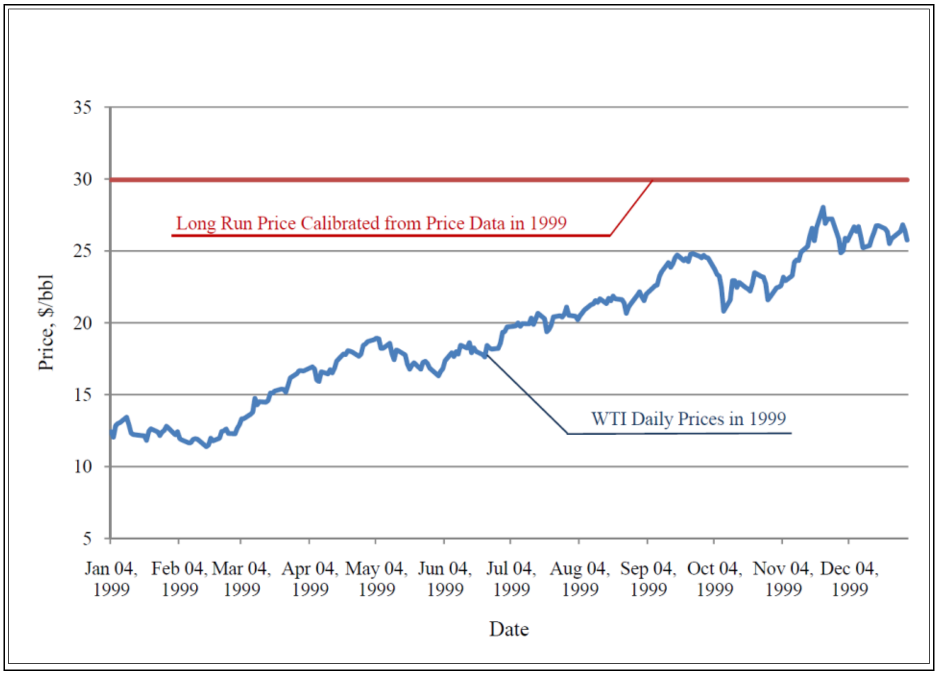


Figure 4-7: *WTI* Daily Spot Oil Prices in 1999 and the Long Run Price Calibrated from the Price Data in 1999

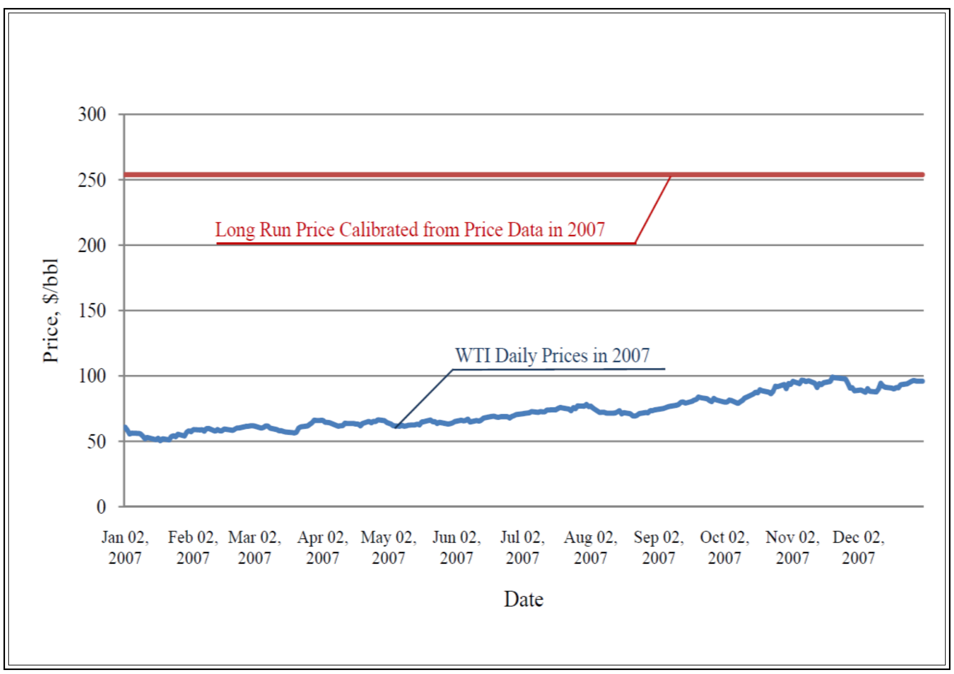


Figure 4-8: *WTI* Daily Spot Oil Prices in 2007 and the Long Run Price Calibrated from the Price Data in 2007

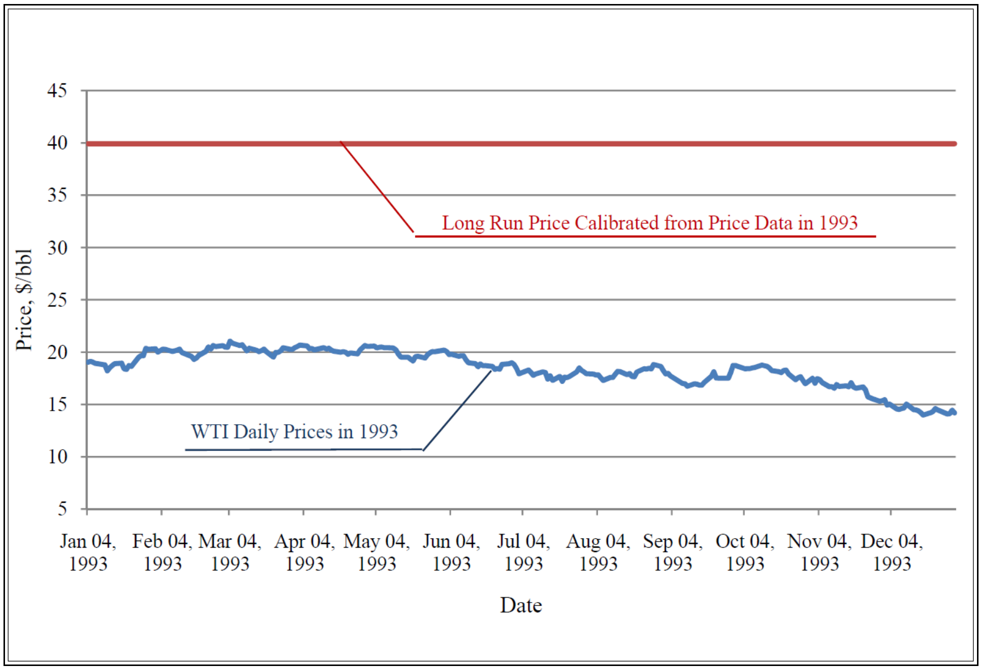


Figure 4-9: *WTI* Daily Spot Oil Prices in 1993 and the Long Run Price Calibrated from the Price Data in 1993

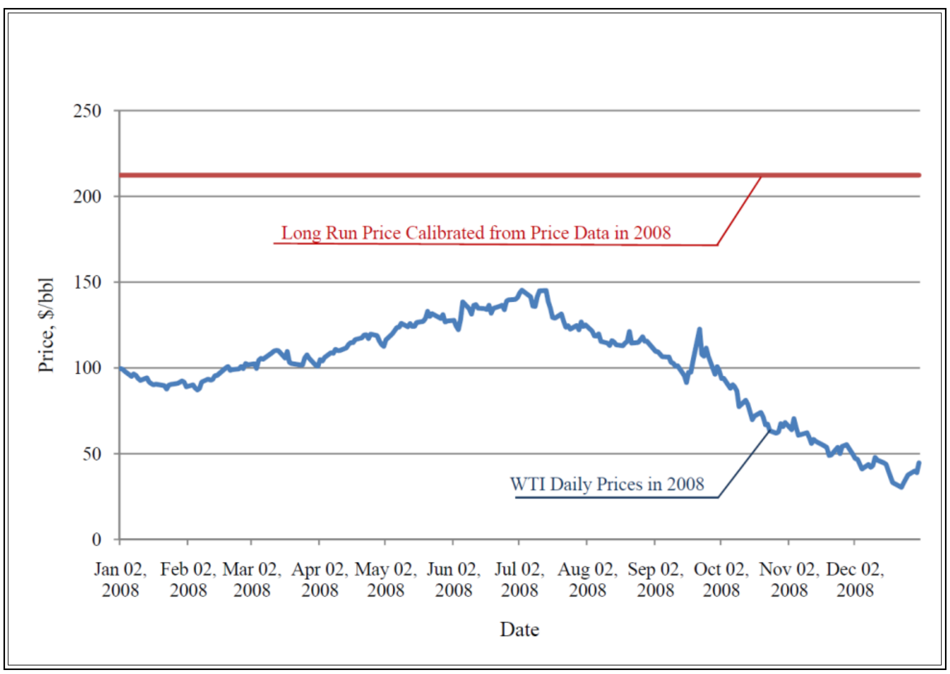


Figure 4-10: *WTI* Daily Spot Oil Prices in 2008 and the Long Run Price Calibrated from the Price Data in 2008

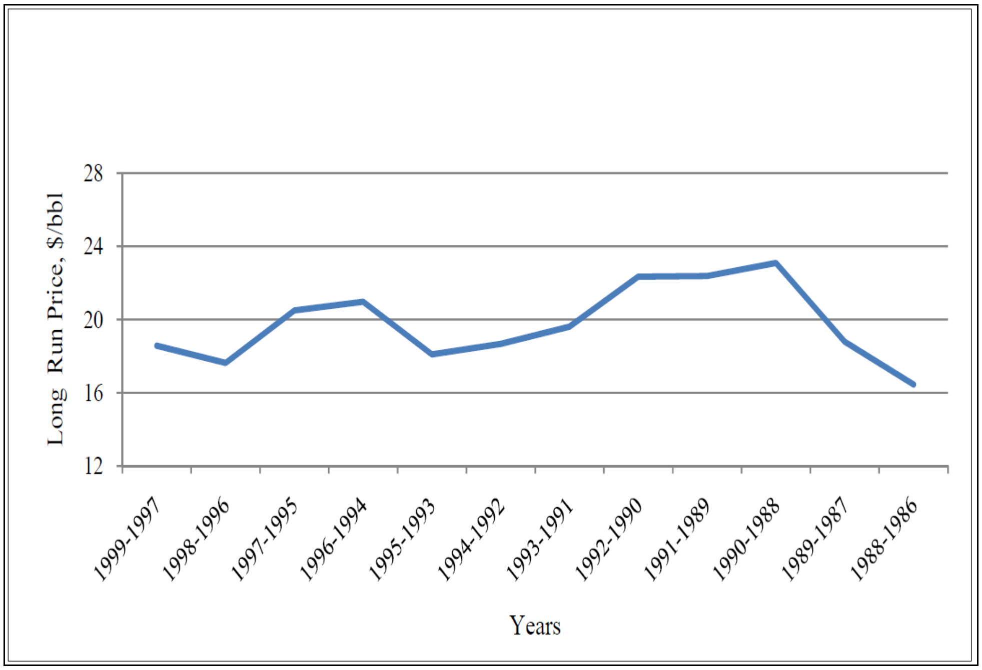


Figure 4-11: *WTI* Oil Price Mean Reversion Model Long Run Price from Daily Spot Price Data in Three Backward Rolling Years from 1999 to 1986

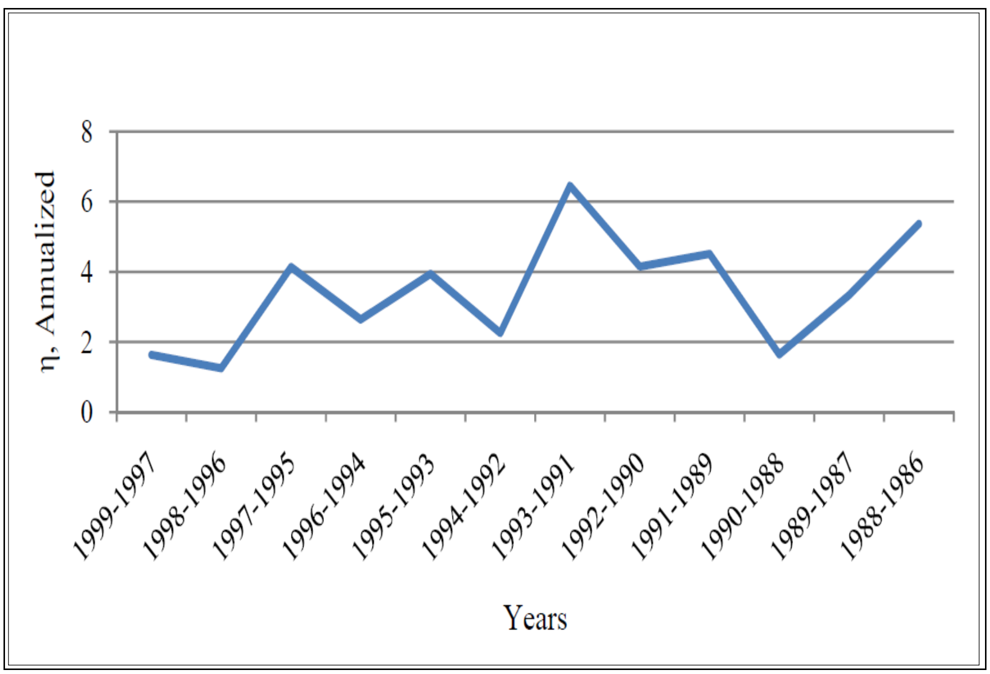


Figure 4-12: *WTI* Oil Price Mean Reversion Rate from Daily Spot Price Data in Three Backward Rolling Years from 1999 to 1986

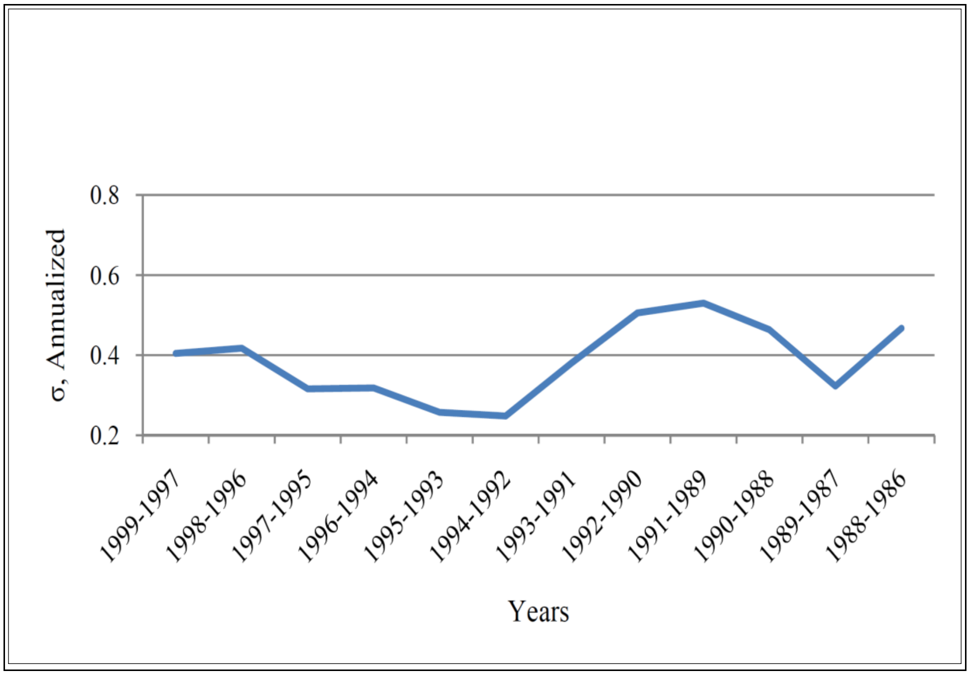


Figure 4-13: *WTI* Oil Price Mean Reversion Volatility from Daily Spot Price Data in Three Backward Rolling Years from 1999 to 1986

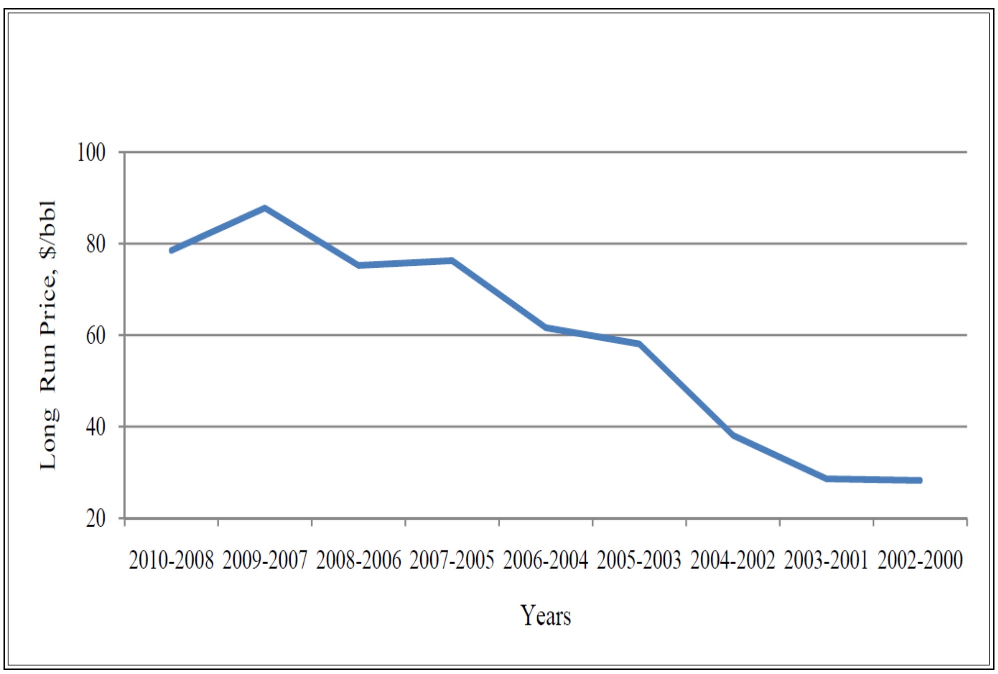


Figure 4-14: *WTI* Oil Price Mean Reversion Model Long Run Price from Daily Spot Price Data in Three Backward Rolling Years from 2010 to 2000

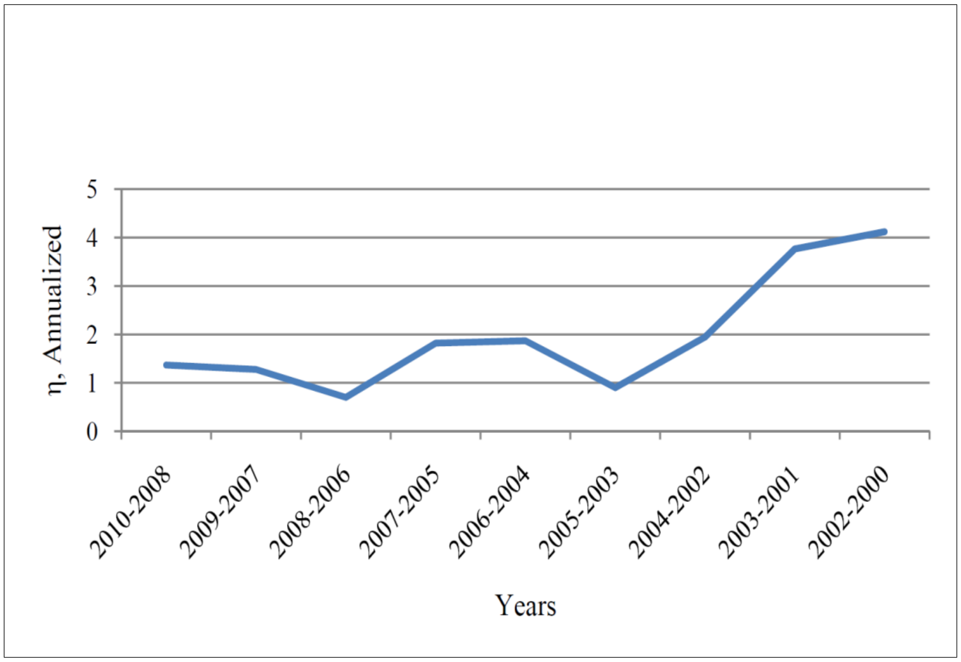


Figure 4-15: *WTI* Oil Price Mean Reversion Rate from Daily Spot Price Data in Three Backward Rolling Years from 2010 to 2000

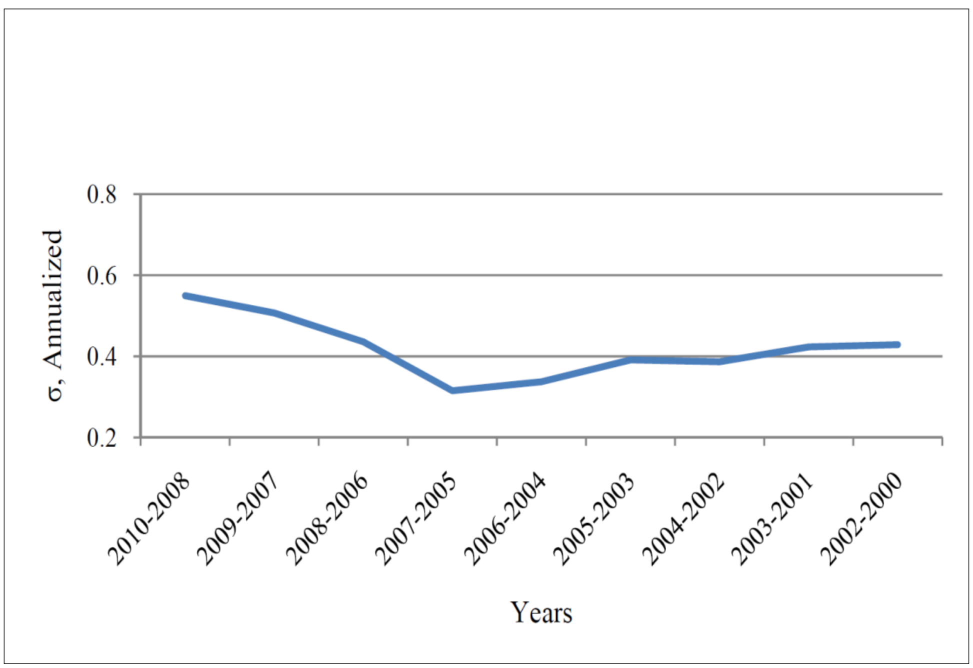


Figure 4-16: *WTI* Oil Price Mean Reversion Volatility from Daily Spot Price Data in Three Backward Rolling Years from 2010 to 2000

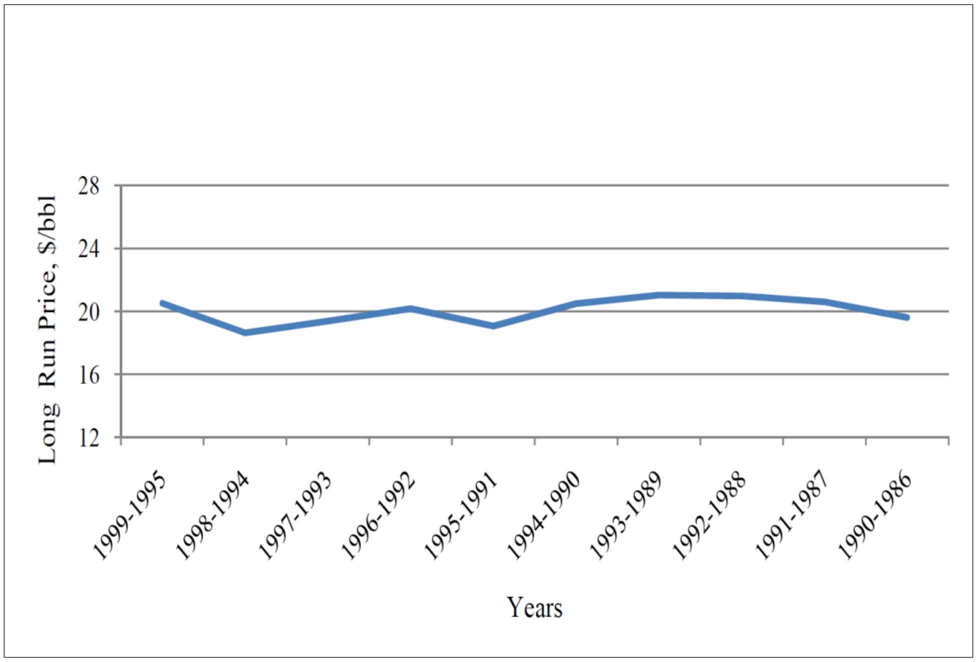


Figure 4-17: *WTI* Oil Price Mean Reversion Model Long Run Price from Daily Spot Price Data in Five Backward Rolling Years from 1999 to 1986

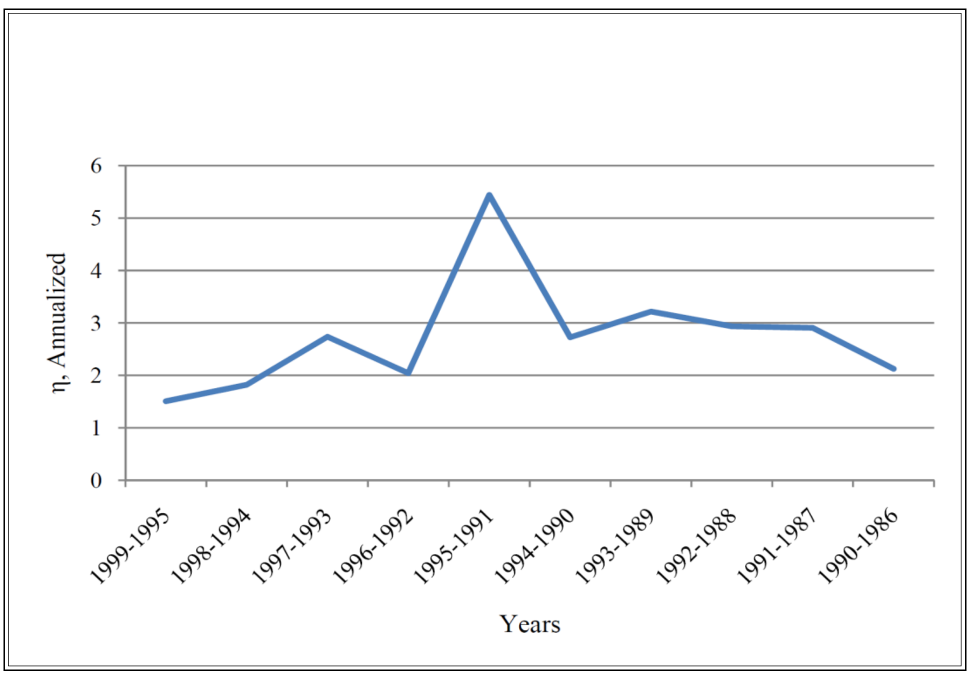


Figure 4-18: *WTI* Oil Price Mean Reversion Rate from Daily Spot Price Data in Five Backward Rolling Years from 1999 to 1986

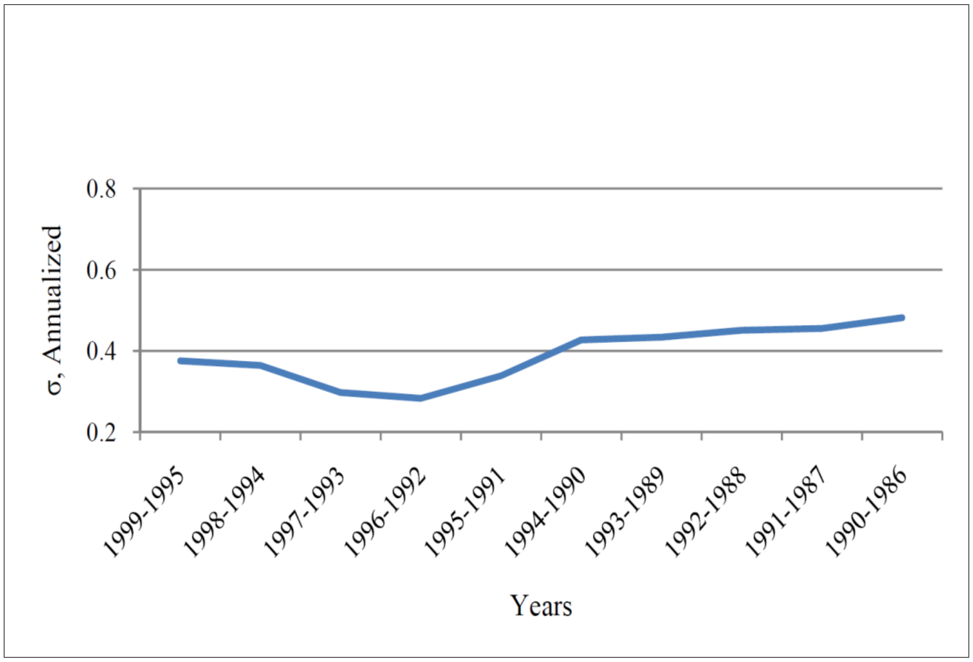


Figure 4-19: *WTI* Oil Price Mean Reversion Volatility from Daily Spot Price Data in Five Backward Rolling Years from 1999 to 1986

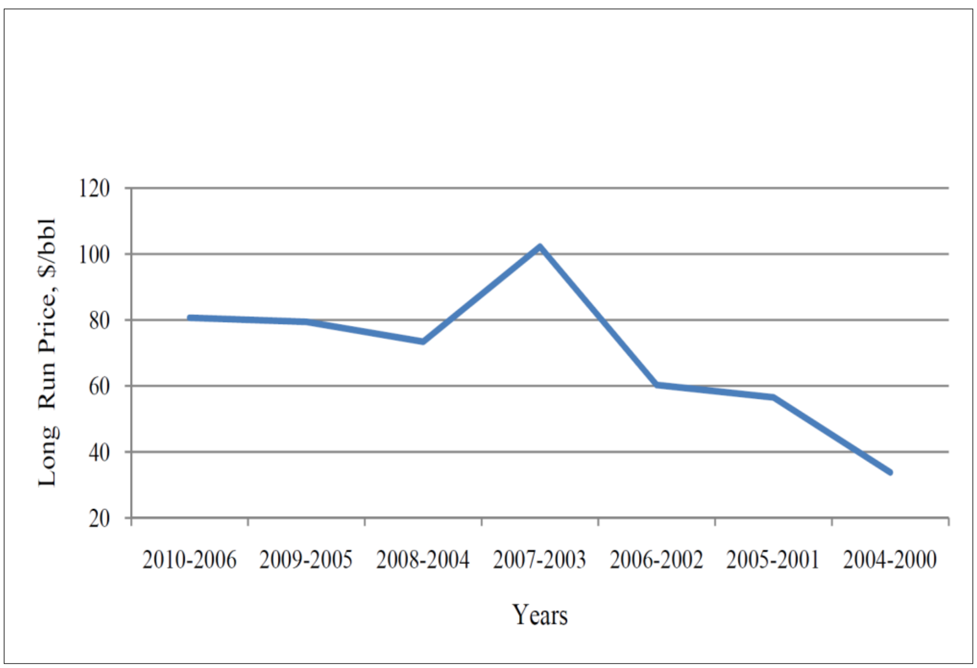


Figure 4-20: *WTI* Oil Price Mean Reversion Model Long Run Price from Daily Spot Price Data in Five Backward Rolling Years from 2010 to 2000

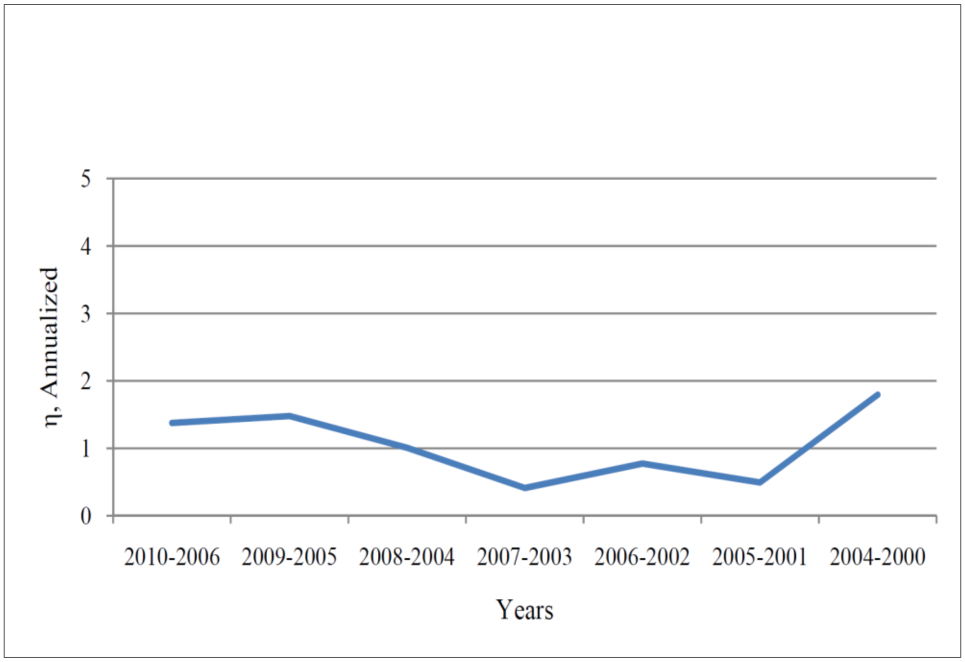


Figure 4-21: *WTI* Oil Price Mean Reversion Rate from Daily Spot Price Data in Five Backward Rolling Years from 2010 to 2000

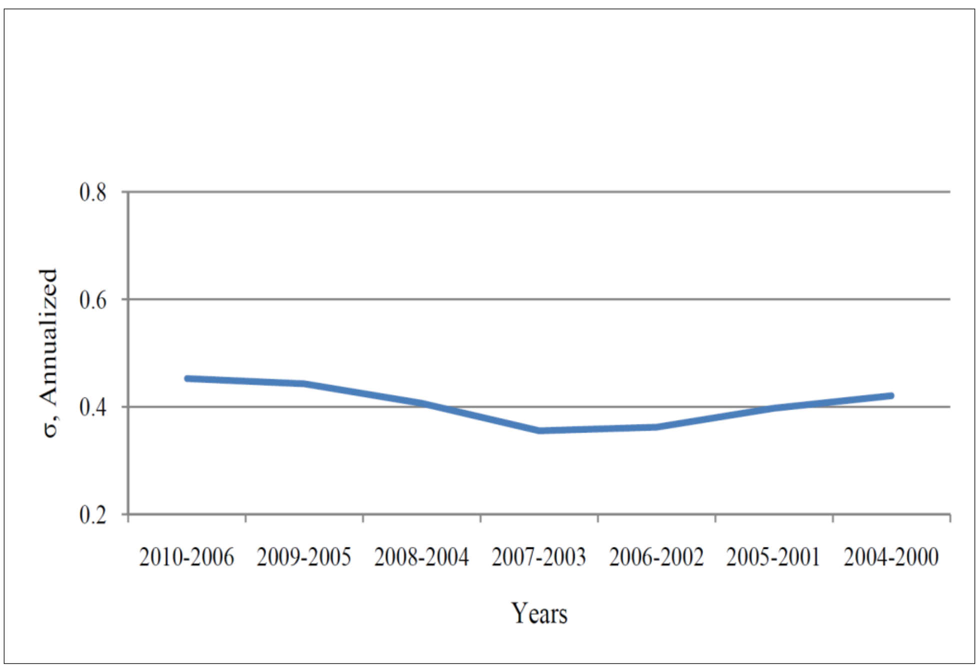


Figure 4-22: *WTI* Oil Price Mean Reversion Volatility from Daily Spot Price Data in Five Backward Rolling Years from 2010 to 2000

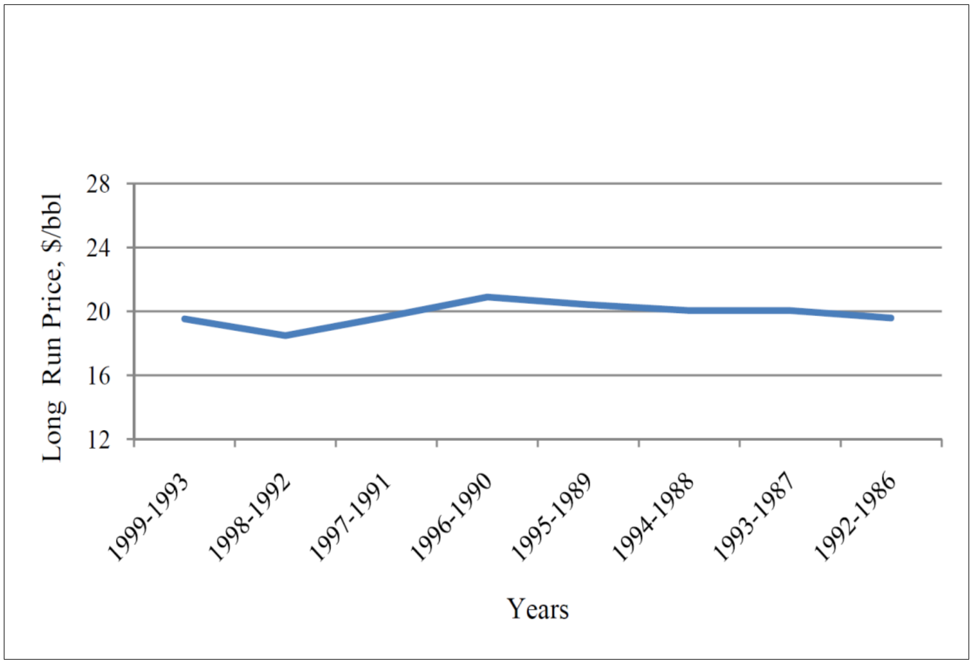


Figure 4-23: *WTI* Oil Price Mean Reversion Model Long Run Price from Daily Spot Price Data in Seven Backward Rolling Years from 1999 to 1986

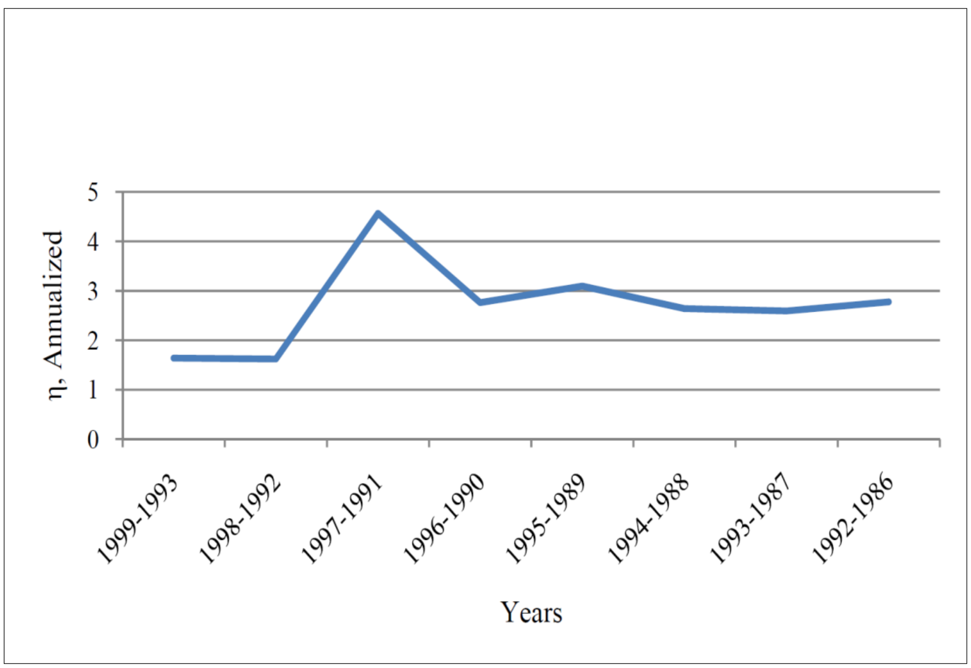


Figure 4-24: *WTI* Oil Price Mean Reversion Rate from Daily Spot Price Data in Seven Backward Rolling Years from 1999 to 1986

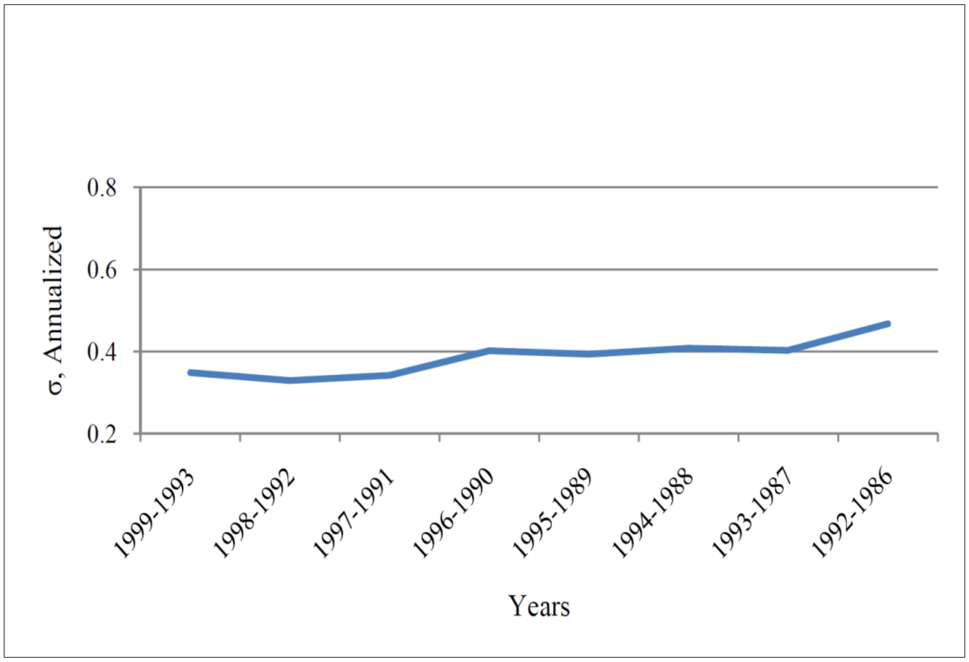


Figure 4-25: *WTI* Oil Price Mean Reversion Volatility from Daily Spot Price Data in Seven Backward Rolling Years from 1999 to 1986

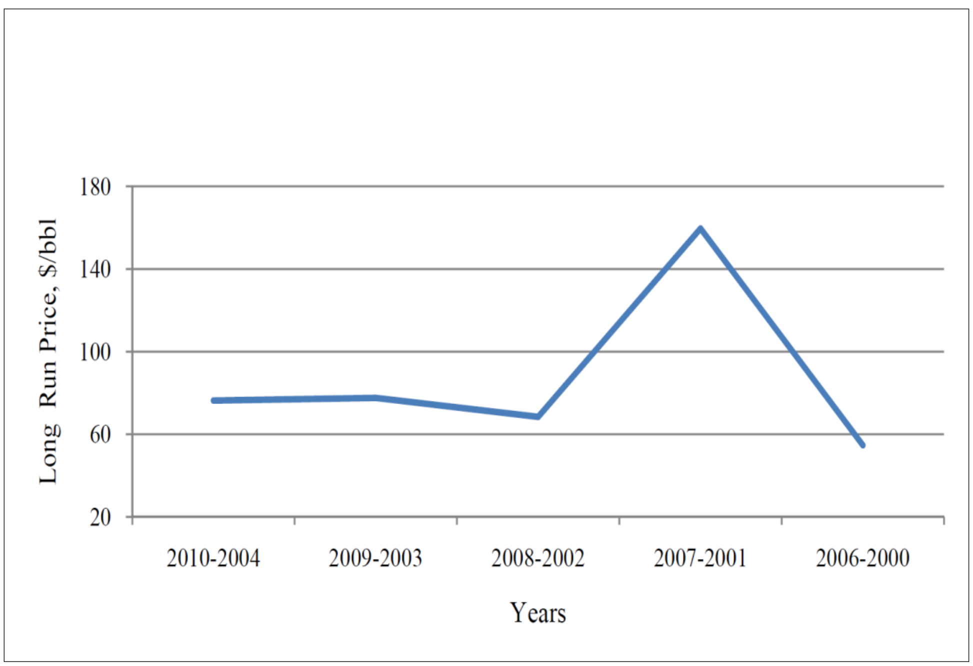


Figure 4-26: *WTI* Oil Price Mean Reversion Model Long Run Price from Daily Spot Price Data in Seven Backward Rolling Years from 2010 to 2000

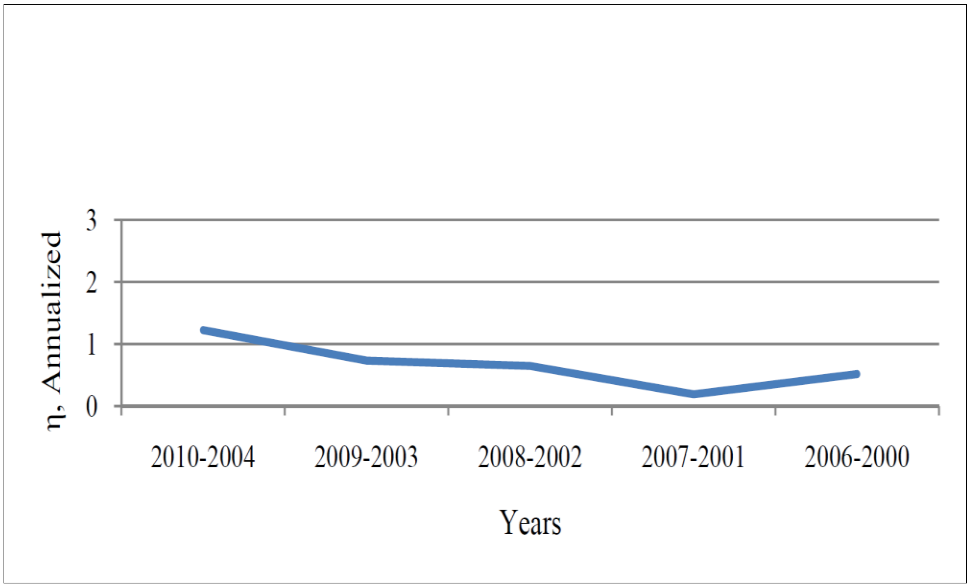


Figure 4-27: *WTI* Oil Price Mean Reversion Rate from Daily Spot Price Data in Seven Backward Rolling Years from 2010 to 2000

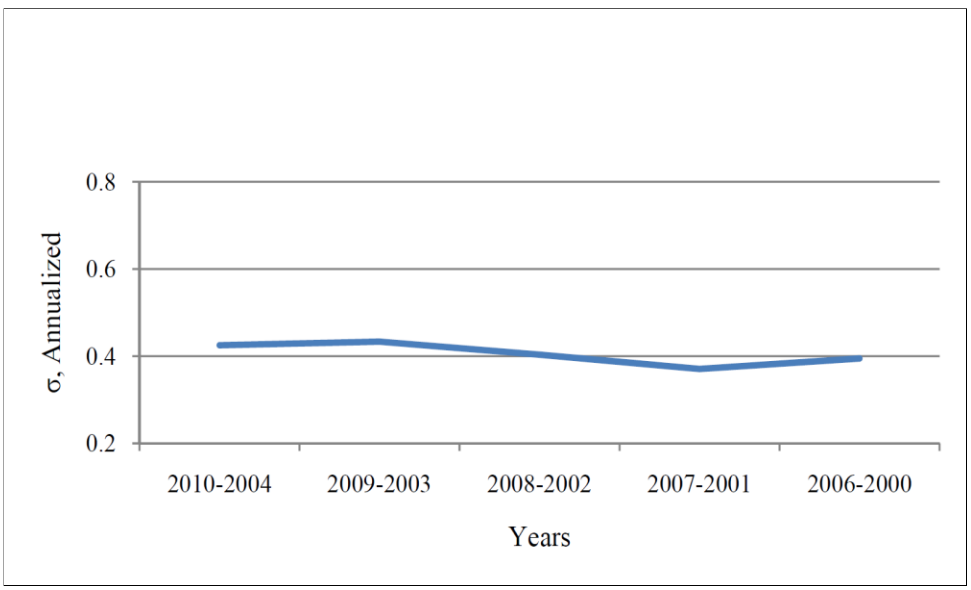


Figure 4-28: *WTI* Oil Price Mean Reversion Volatility from Daily Spot Price Data in Seven Backward Rolling Years from 2010 to 2000

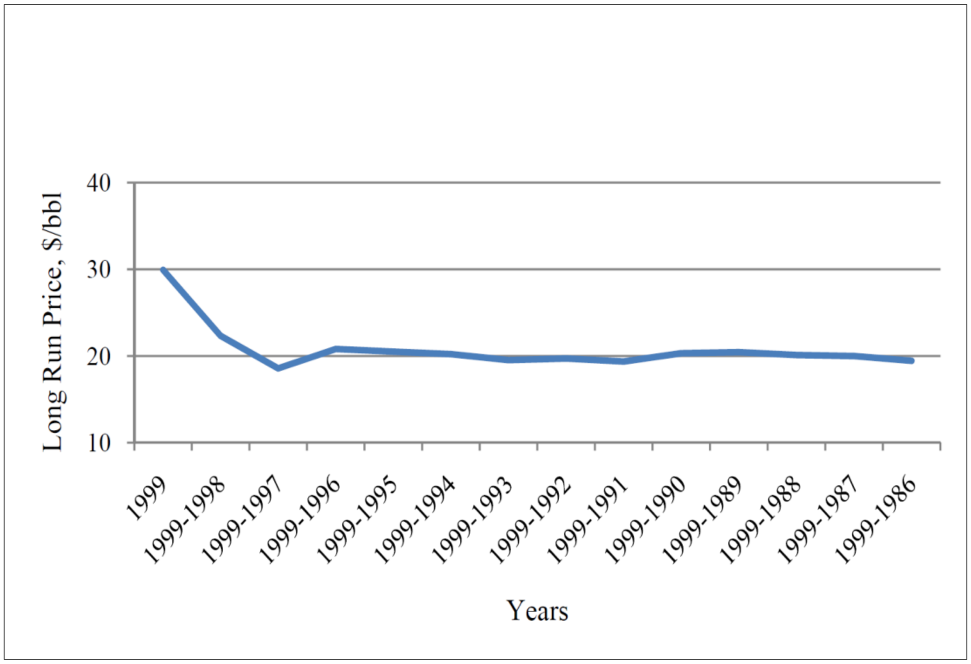


Figure 4-29: *WTI* Oil Price Mean Reversion Model Long Run Price from Daily Spot Price Data in Increasing Number of Backward Years from 1999 to 1986

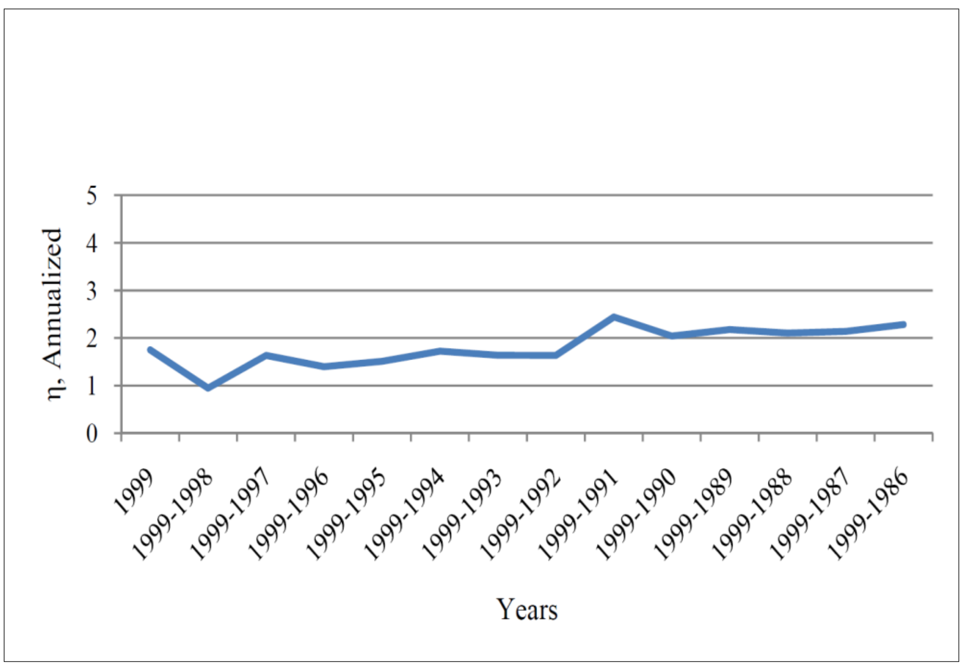


Figure 4-30: *WTI* Oil Price Mean Reversion Rate from Daily Spot Price Data in Increasing Number of Backward Years from 1999 to 1986

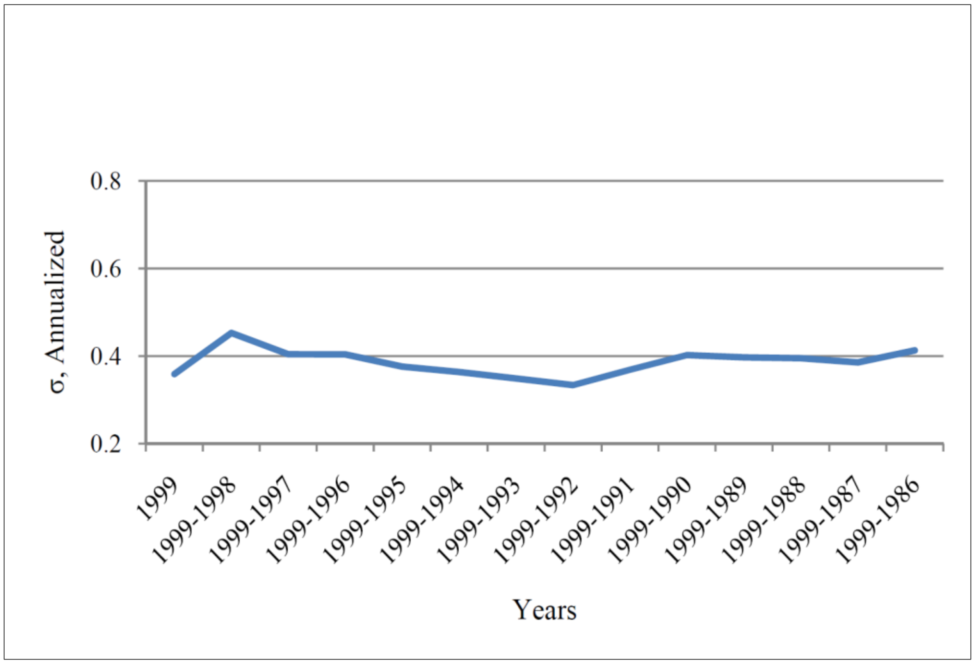


Figure 4-31: *WTI* Oil Price Mean Reversion Volatility from Daily Spot Price Data in Increasing Number of Backward Years from 1999 to 1986

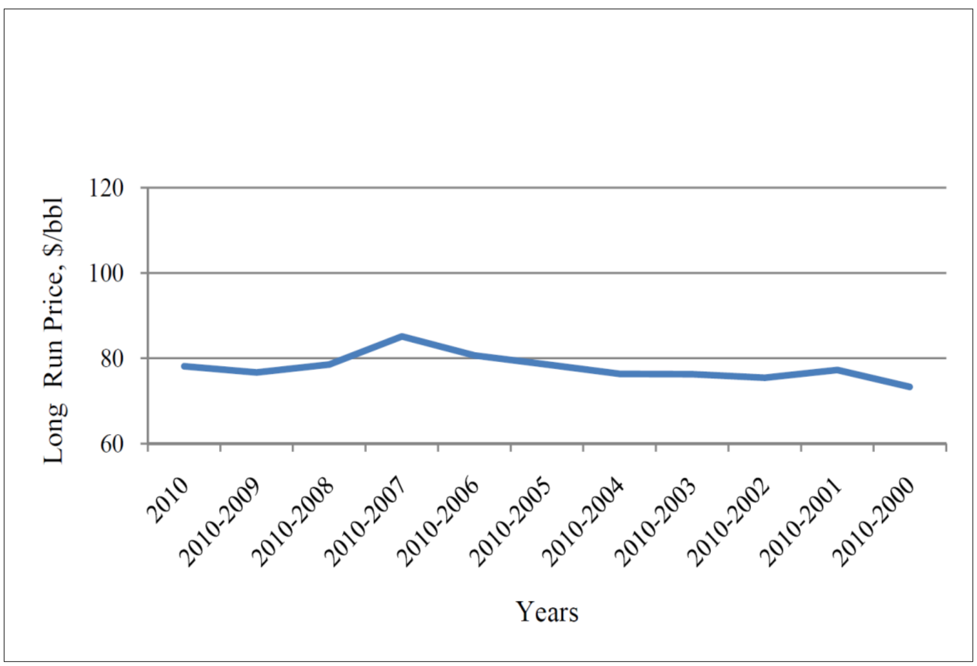


Figure 4-32: *WTI* Oil Price Mean Reversion Model Long Run Price from Daily Spot Price Data in Increasing Number of Backward Years from 2010 to 2000

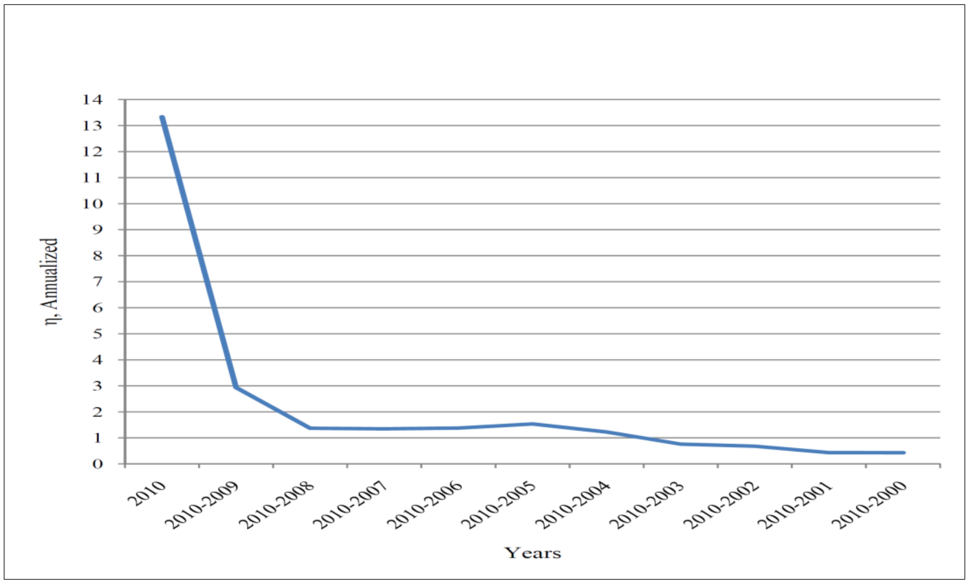


Figure 4-33: *WTI* Oil Price Mean Reversion Rate from Daily Spot Price Data in Increasing Number of Backward Years from 2010 to 2000

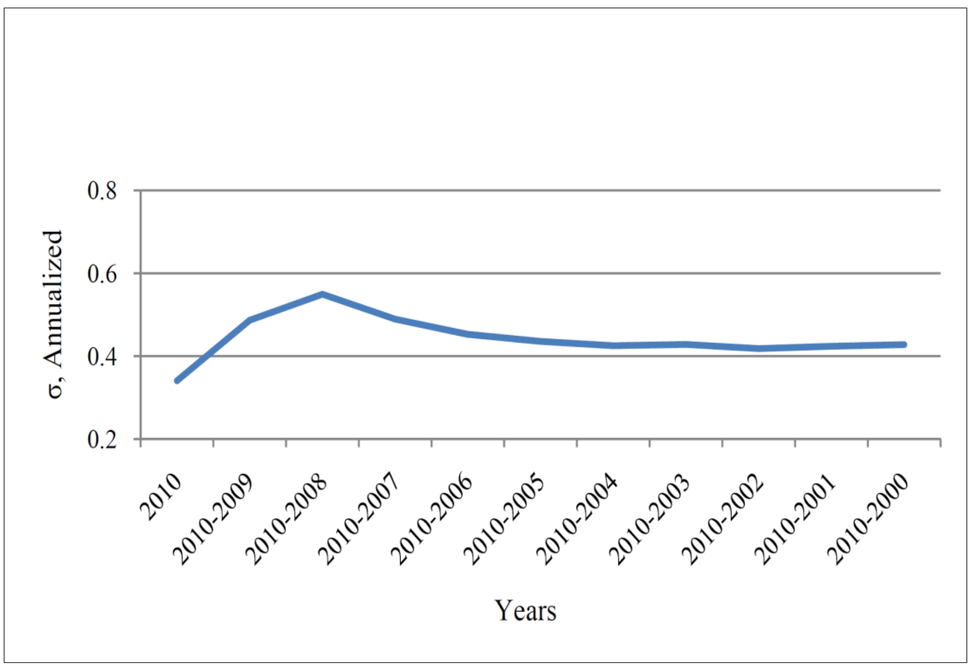


Figure 4-34: *WTI* Oil Price Mean Reversion Volatility from Daily Spot Price Data in Increasing Number of Backward Years from 2010 to 2000

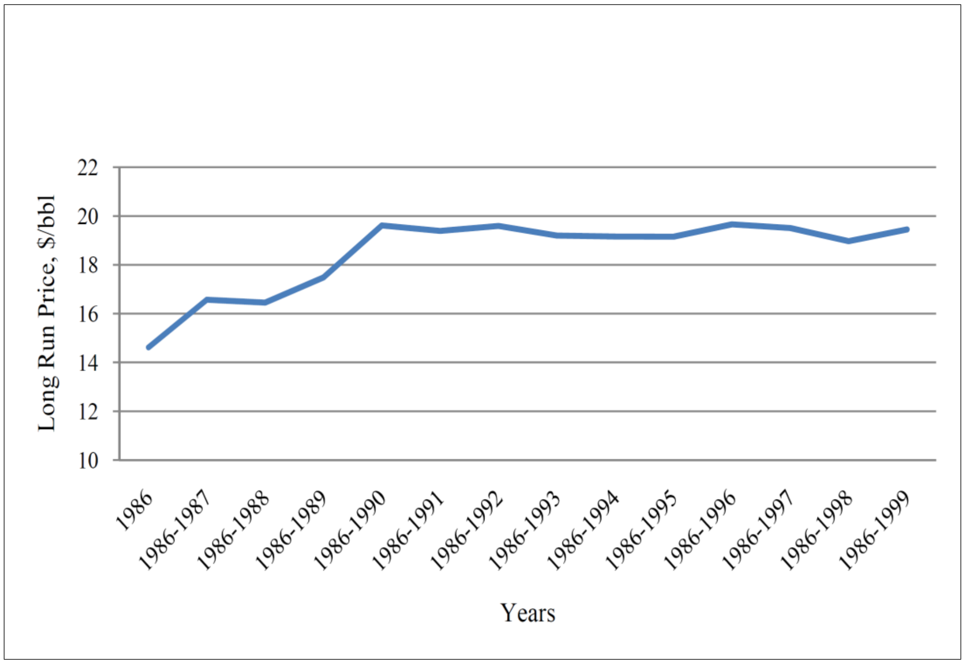


Figure 4-35: *WTI* Oil Price Mean Reversion Model Long Run Price from Daily Spot Price Data in Increasing Number of Forward Years in B4-2K Price Regime

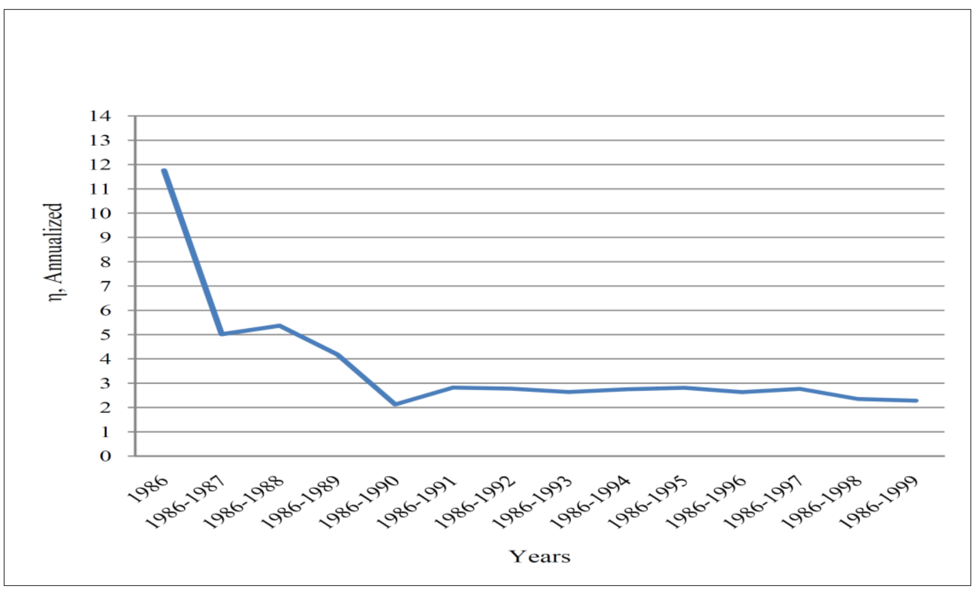


Figure 4-36: *WTI* Oil Price Mean Reversion Rate from Daily Spot Price Data in Increasing Number of Forward Years in B4-2K Price Regime

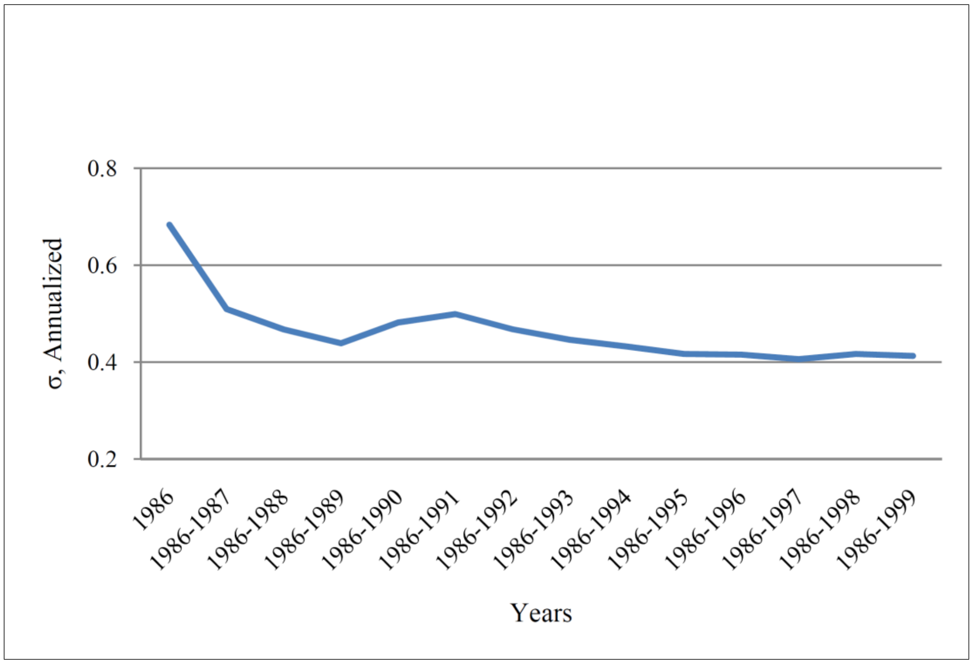


Figure 4-37: *WTI* Oil Price Mean Reversion Volatility from Daily Spot Price Data in Increasing Number of Forward Years in B4-2K Price Regime

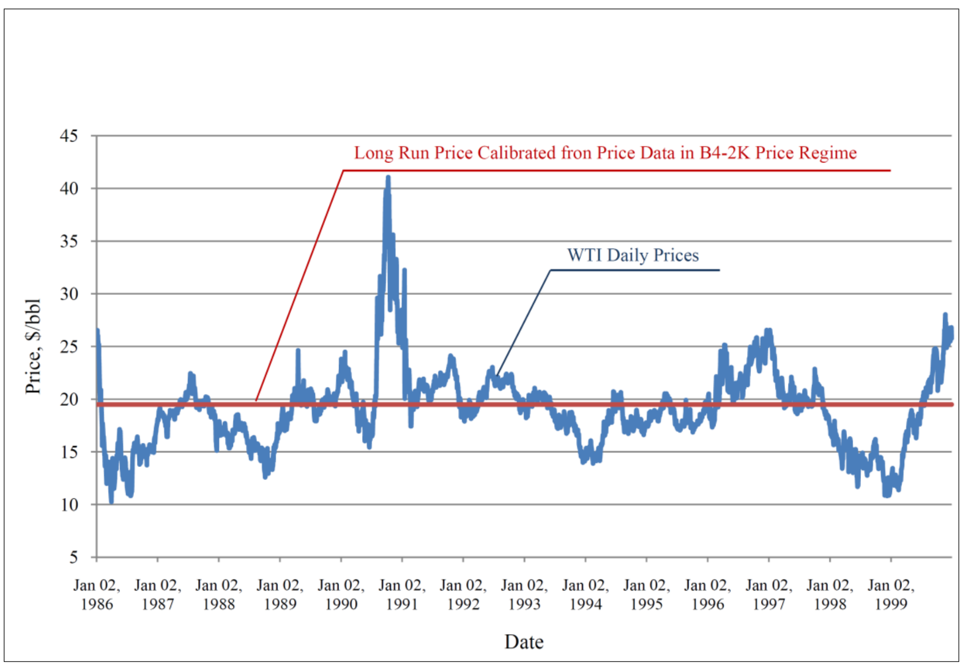


Figure 4-38: *WTI* Daily Spot Oil Prices in the B4-2K Price Regime and Mean Reversion Long Run Price Calibrated from Price Data in the B4-2K Price Regime

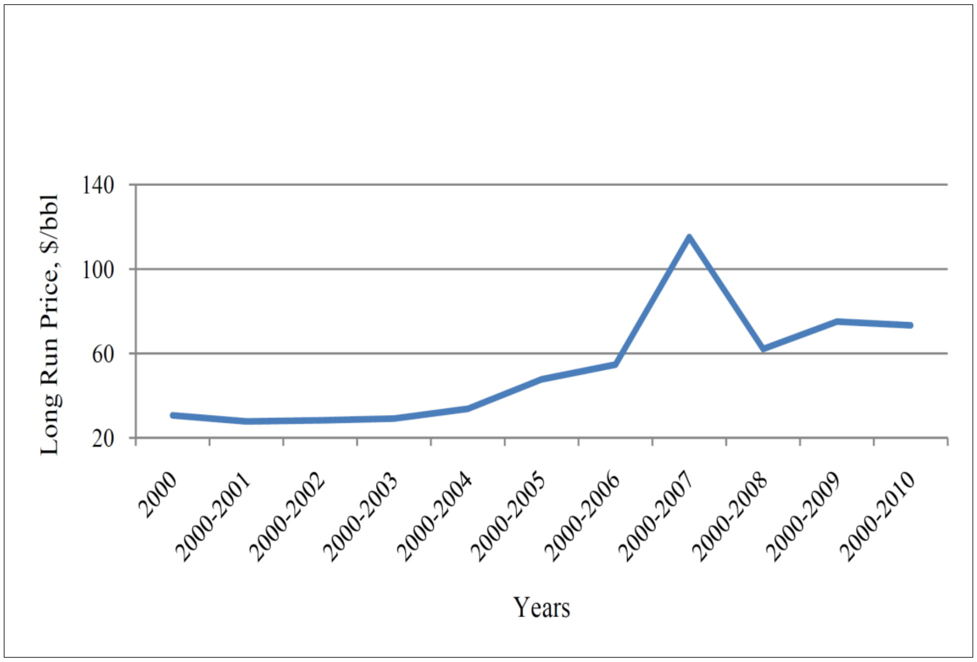


Figure 4-39: *WTI* Oil Price Mean Reversion Model Long Run Price from Daily Spot Price Data in Increasing Number of Forward Years from 2000 to 2010

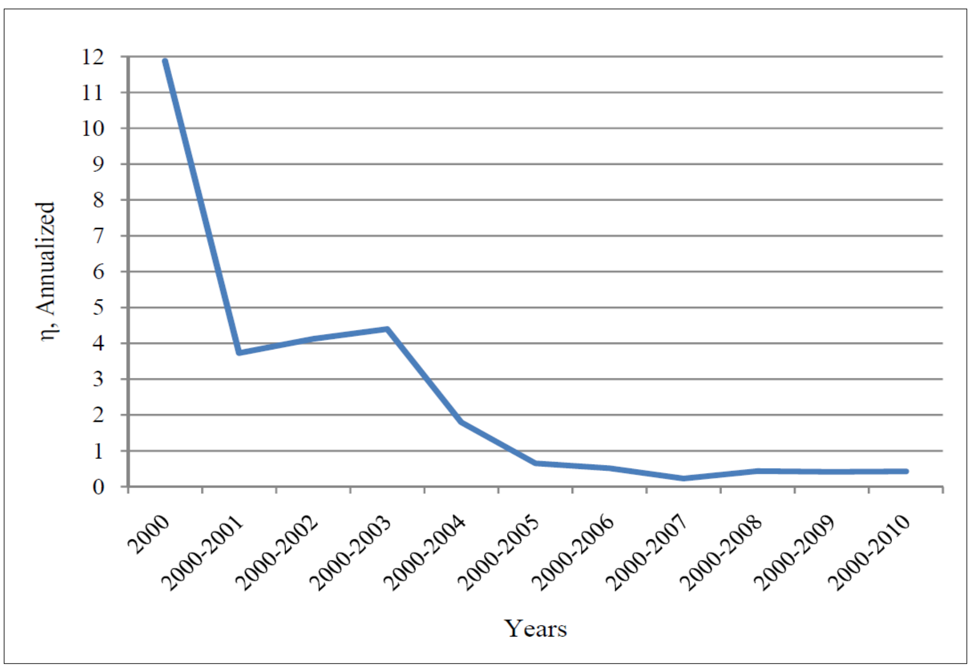


Figure 4-40: *WTI* Oil Price Mean Reversion Rate from Daily Spot Price Data in Increasing Number of Forward Years from 2000 to 2010

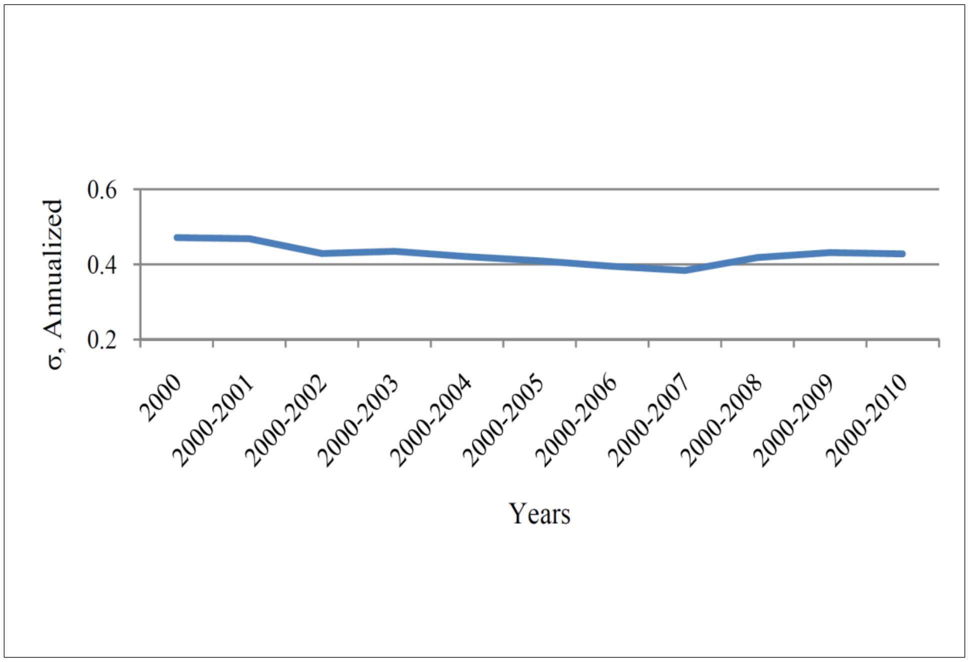


Figure 4-41: *WTI* Oil Price Mean Reversion Volatility from Daily Spot Price Data in Increasing Number of Forward Years from 2000 to 2010

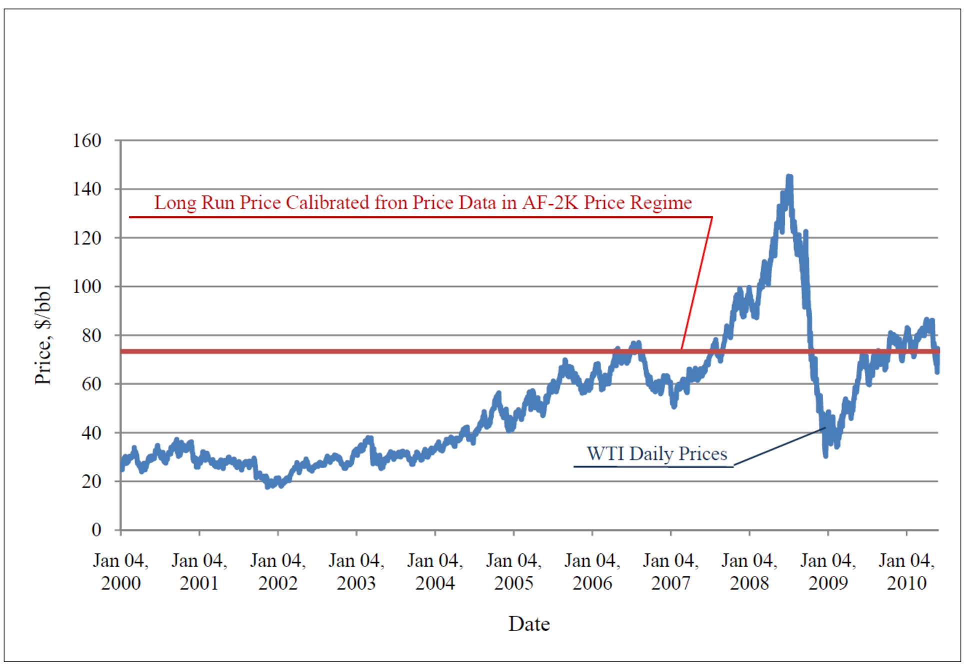


Figure 4-42: *WTI* Daily Spot Oil Prices in AF-2K Price Regime and Mean Reversion Model Long Run Price Calibrated from Price Data in AF-2K Price Regime

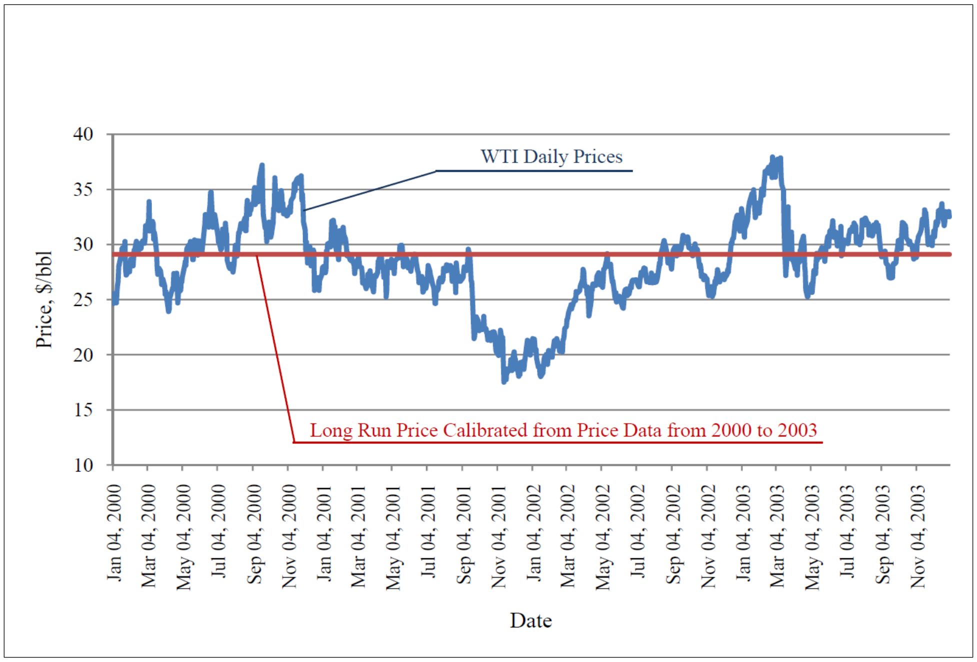


Figure 4-43: *WTI* Daily Spot Oil Prices in 2000-2003 and Mean Reversion Model Long Run Price Calibrated with Price Data from 2000-2003

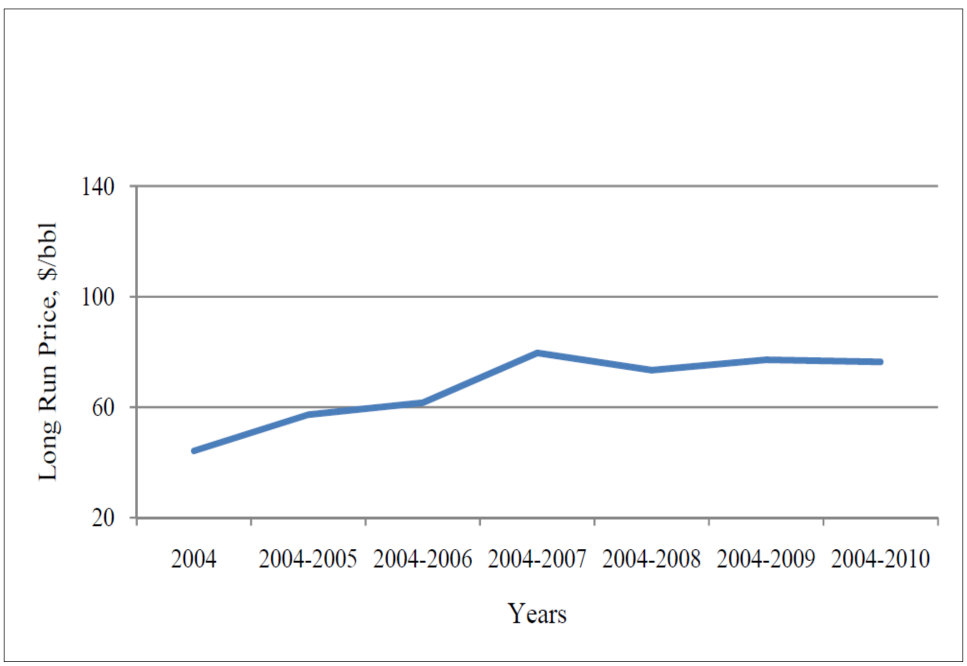


Figure 4-44: *WTI* Oil Price Mean Reversion Model Long Run Price from Daily Spot Price Data in Increasing Number of Forward Years from 2004 to 2010

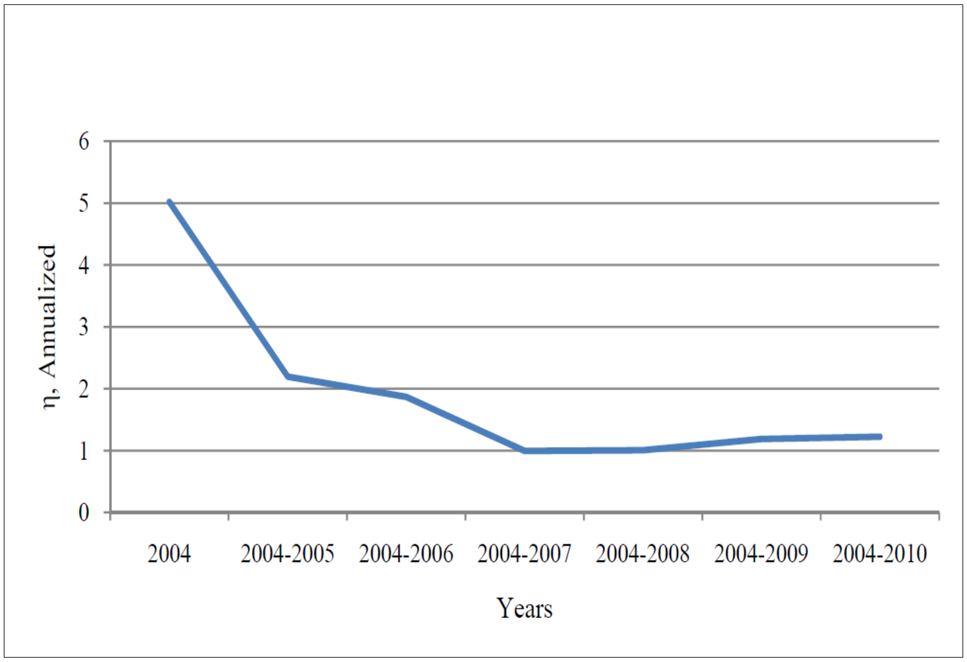


Figure 4-45: *WTI* Oil Price Mean Reversion Rate from Daily Spot Price Data in Increasing Number of Forward Years from 2004 to 2010

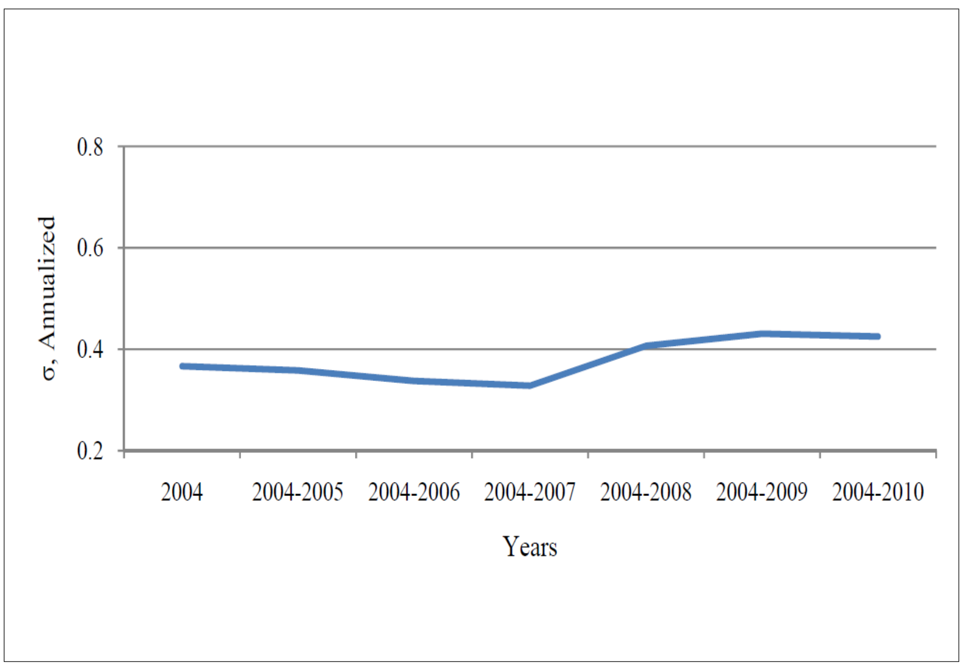


Figure 4-46: *WTI* Oil Price Mean Reversion Volatility from Daily Spot Price Data in Increasing Number of Forward Years from 2004 to 2010

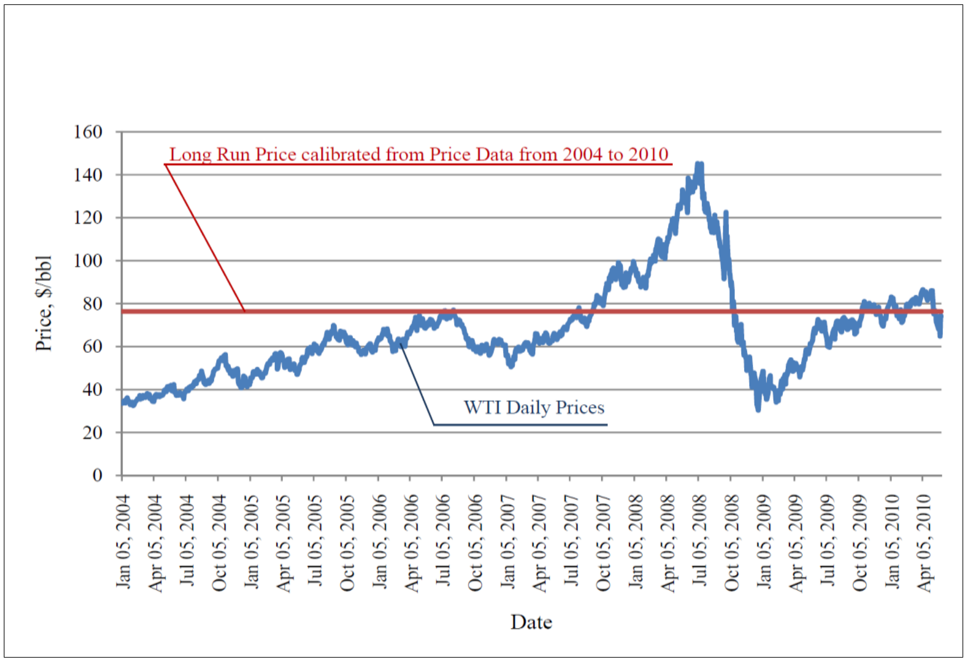


Figure 4-47: *WTI* Daily Spot Oil Prices in 2004-2010 and Mean Reversion Model Long Run Price Calibrated with Price Data from 2004-2010

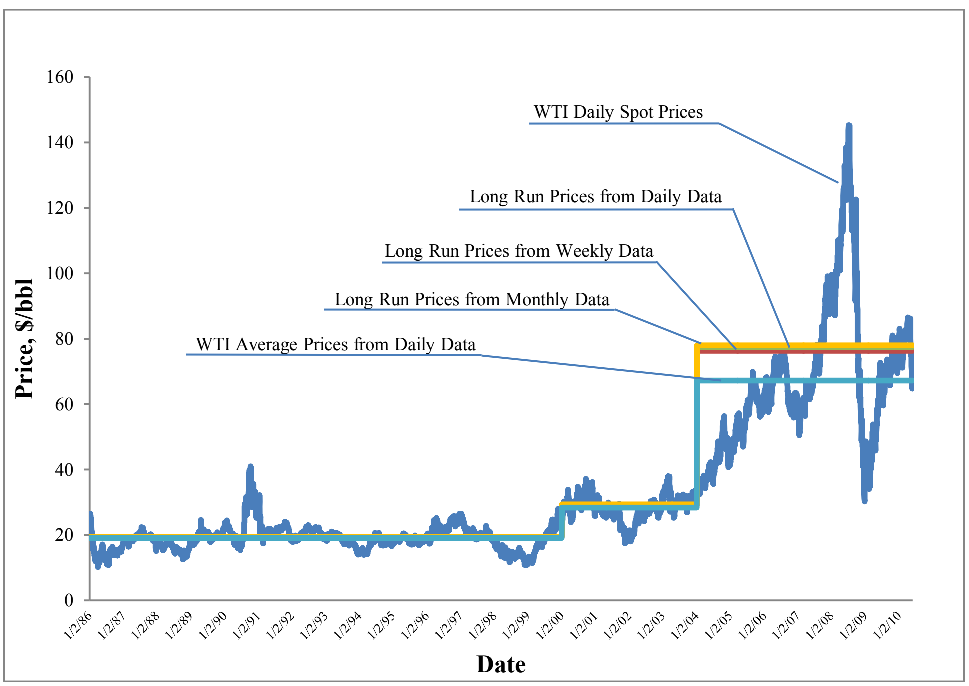


Figure 4-48: Long Run Price and Average Price for *WTI* Daily, Weekly, and Monthly Oil Price Data in Different Price Regimes from 1986 to 2010

Figure 4-49: Annualized Mean Reversion Rate for *WTI* Daily, Weekly, and Monthly Oil Price Data in Different Price Regimes from 1986 to 2010

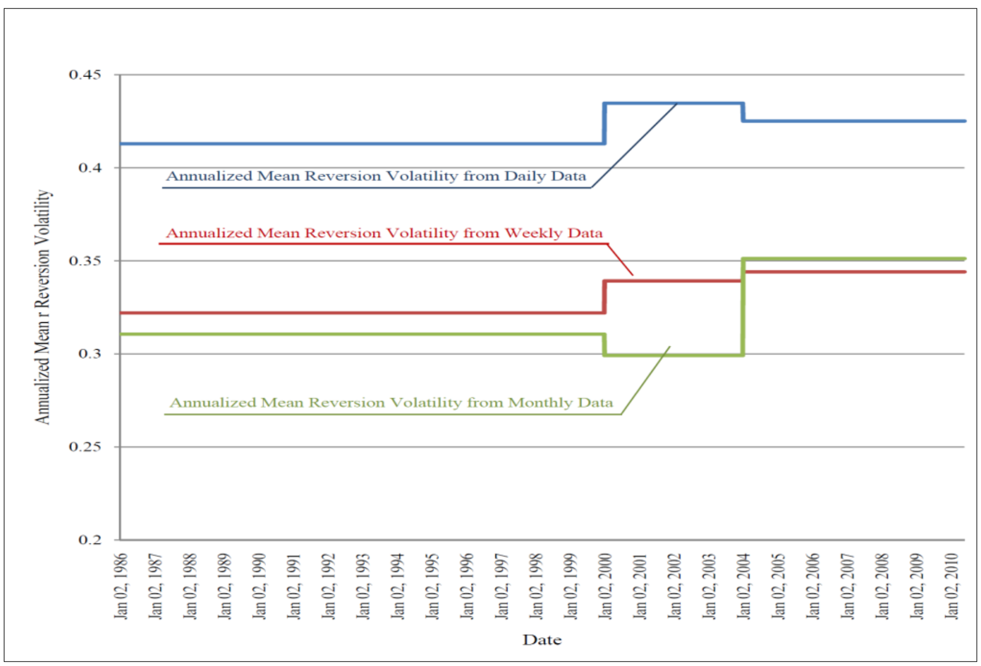


Figure 4-50: Annualized Mean Reversion Volatility for *WTI* Daily, Weekly, and Monthly Oil Price Data in Different Price Regimes from 1986 to 2010

CHAPTER 5: SIMULATIONS AND COMPARISONS OF THE GEOMETRIC BROWNIAN MOTION (GBM) AND ONE-FACTOR MEAN REVERSION PRICE MODELS FOR THE WTI OIL PRICES

In this chapter, Monte Carlo simulations are designed and performed in order to understand how the West Texas Intermediate (WTI) oil prices evolve according to the two price models, that is, the geometric Brownian motion (GBM) and the one-factor mean reversion price models. With the Monte Carlo simulation results, future WTI oil price distributions are obtained. The simulation results from the two price models are compared with each other and with the actual price movement of the WTI oil prices. Through the designed Monte Carlo simulations, which price model better fits the evolution of historical WTI oil prices is observed and analyzed.

5.1 MONTE CARLO SIMULATIONS FOR THE GEOMETRIC BROWNIAN MOTION PRICE MODEL FOR THE WTI OIL PRICES

5.1.1 Design Monte Carlo Simulations

From the continuous form of the GBM price model as shown in Eq. 3-5:

$$dLnP = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dW,$$

the price change in the time period of $[t,\ t + \Delta t]$ with the discrete form of the GBM price model is written as

$$\Delta LnP = \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma N(0,1)\sqrt{\Delta t}\,.$$

That is

$$LnP_{t+\Delta t} - LnP_t = \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma N(0,1)\sqrt{\Delta t},$$

$$LnP_{t+\Delta t} = LnP_t + \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma N(0,1)\sqrt{\Delta t}, \tag{5-1}$$

$$P_{t+\Delta t} = P_t Exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma N(0,1)\sqrt{\Delta t}\right]. \tag{5-2}$$

With Eq. (5-2), for any given price at time t, the distribution of oil prices at time $(t + \Delta t)$ is obtained. Considering the time duration $[\,0,\ T]$ and $T = N\Delta t$, in every time interval Δt, for each price at the beginning of the time interval, there is a price distribution at the end of the time interval. For example, suppose in the first time interval $[0,\ \Delta t\,]$, there is only one price at $t = 0$, that is, $P(t = 0) = P_0$. And assume that there are 1,000 random numbers generated from normal distribution of $N(0\,,1)$. Then at the end of Δt, there is 1,000 price realizations derived from P_0 according to Eq. (5-2). Each of these 1,000 price realizations then becomes the starting price for the next time interval $[\Delta t,$ $2\Delta t\,]$. By the end of $2\Delta t$, for each price reached at the end of the first time interval Δt, there is another 1,000 price realizations. The same procedure continues until reaching the last time interval. The amount of computation increases tremendously with the expansion of the time interval. @Risk with the Monte Carlo simulations is facilitated to perform the calculations described above.

The following Monte Carlo simulations are designed to compare the simulation results and the actual price movement for historical WTI oil prices. First, the two parameters, drift rate μ and volatility σ used in Eq. (5-2) are calibrated from the actual WTI weekly oil prices from January 4, 2000 to April 7, 2010 as described in Chapter 3, that is, $\mu = 0.17463,$ $\sigma = 0.3393$, and $\Delta t = 0.01923$. Second, the price at time $t = 0$ is set as the actual weekly price in the first week of year 2000, that is, $P_0 = \$24.95/bbl$. Third, the Monte Carlo simulations are run with @Risk from the second week of year 2000 to the week of May 28, 2010. Fourth, the simulation results are compared with the actual prices during the same period of time. This way, assume that it was at the beginning of the first week of year 2000, equipped with the knowledge of the actual oil prices in the future and the GBM parameters calibrated from these prices, simulations are run to forecast the future oil price movement. The difference between the actual prices and the simulation results are then observed and compared.

5.1.2 @Risk Simulation Results

Figures 5-1 through 5-3 are the simulation results with the GBM price model for the WTI weekly oil prices from January 4, 2000 to May 28, 2010. Due to the cell limitations in chart making using Excel 2007, the simulation results in Figures 5-1 through 5-3 can only have 100 iterations. Figures 5-1 and 5-2 show very similar results in two different simulation cases. The range of the simulated prices increases with the increase of simulation time.

Figure 5-2 includes the actual price movement and the expectation $E(P_t)$ for the forecasting prices according to Eq. (3-7). The actual prices are inside the simulation price ranges. However, the ranges of the simulation prices are far beyond the actual prices. Figure

5-3 shows that one of the simulation prices is as high as $\$1{,}000/bbl$ while the actual price is only $\$70/bbl$ the week of May 28, 2010.

Figure 5-4 shows the statistics of simulation results with 10K iterations, including the maximum, mean, minimum prices, and 5th percentile, 10th percentile, 90th percentile, and 95th percentile price ranges. The maximum simulated oil price can be as high as $\$7{,}000/bbl$. From the second week of year 2000 to May 28, 2010, more than half of the time, the simulated maximum prices are higher than $\$1{,}000/bbl$.

Since the curve of the maximum simulated prices is much higher than the rest of the curves, it is removed from the above chart so that the rest of price curves can be clearly compared, as shown in Figure 5-5. The actual price movement is between the 10th and 90th percentile curves but with a much narrower range.

Figure 5-6 shows the relationship among the actual prices and the simulation mean, minimum, 5th percentile, and 10th percentile prices. During the simulated time, the actual prices never reached the minimum prices. They are actually above the 10th percentile curve. Since the end of 2008, the difference between the simulation mean prices and the actual prices is much bigger than before, being in a range of around $\$50/bbl$ to $\$80/bbl$.

Figure 5-7 illustrates the relationship among the actual prices and the simulation mean, maximum, and 95th percentile prices. The difference between the curve of the simulated maximum prices and the other three curves becomes bigger with the increase of the simulation time, reaching the highest of $6,000/bbl around the middle of year 2009.

Figure 5-8 provides a closer look at the price curves for the 10th and 90th percentiles from simulations and the actual price movement. Nearly all of the actual prices lie between the 10th and 90th percentile price ranges. With the increase of time, the range between the 10th and 90th percentile curves becomes much wider than the actual price fluctuations could reach, especially for the upside prices. The 90th percentile curve is much higher than the actual price curve.

Figure 5-9 compares the actual price evolution with the expected forecast prices $E(P_t)$ of the GBM price model using Eq. (3-7). The expected forecast prices can be much different from the actual prices as showed from the two curves in Figure 5-9. Figure 5-10 shows the relationship between the expected forecast prices and the three drift rates, that is, $\mu(H) = 0.20920$, $\mu(M) = 0.17463$, and $\mu(L) = 0.05262$, calibrated from different historical WTI oil prices as described in Chapter 3. The expected forecast prices can be much different when different values of drift rates are applied for simulations with Eq. (5-2).

Figure 5-11 shows the impact of the number of iterations on the simulated mean prices and compares the simulated mean prices with the expected forecast prices $E(P_t)$ from Eq. (3-7). When the number of iterations is as high as 10K, the mean prices from the simulation are almost the same as the calculated expected future prices. There is a obvious difference in the simulated mean prices between 100 iterations and 10K iterations.

Figure 5-12 shows the relationship among the number of iterations, the variance of simulated prices, and the calculated variance for the forecasted future price distribution using Eq. (3-8). When the number of iterations is as high as 15K, the variance of the simulated prices is almost the same as the calculated variance. With the increase of time, the difference between the variance of the simulated prices with 100 iterations and 15K iterations becomes bigger.

Figures 5-13 and 5-14 are the skewness and kurtosis of the simulated prices with 10K iterations. Both figures show that the price distribution becomes less normal with the increase of simulation time.

Figure 5-15 is one realization of the simulated price evolution. The simulated prices are in an increasing trend most of the time. The highest price is higher than $\$400/bbl$. Figure 5-16 includes two simulation realizations, the expected forecast prices, and the actual prices. The simulated prices and the expected forecast prices can be lower or higher than the actual prices.

Table 5-1 contains examples of the simulated price distributions in eight weeks starting from the second week of year 2000 with 70 weeks in between. With the increase of simulation time, the simulated price distributions become more lognormal.

5.2 Monte Carlo Simulations for the One-factor Mean Reversion Oil Price Model

With the one-factor mean reversion price model, that is

$$\frac{dP}{P} = \eta(Ln\bar{p} - LnP)dt + \sigma d\mathrm{W},$$

the future prices can be generated with the Monte Carlo simulations. For any given time interval $[t,\ t+\Delta t]$, the prices at the end of the time interval can be derived according to the following equation:

$$P_{t+\Delta t} = Exp\left[e^{-\eta\Delta t}(LnP_t) + (1-e^{-\eta\Delta t})\left(Ln\bar{p} - \frac{\sigma^2}{2\eta}\right) + N(0,1)\sigma\sqrt{\frac{(1-e^{-2\eta\Delta t})}{2\eta}}\right]. \tag{5-3}$$

With @Risk, the values of random variable $N(0,1)$ can be generated with a defined number of iterations. For the purpose of comparison with simulation results from the GBM oil price model, the parameters used for the mean reversion price model are calibrated from the historical WTI weekly prices from January 4, 2000 to May 28, 2010. The parameters used in Eq. (5-3) are as follows:

$\eta = 0.2822$,

$\sigma = 0.3402$,

$\Delta t = 0.01923\ year$,

$\bar{p} = \$79.74/bbl.$

The starting price for simulation is

$P_0 = \$24.95/bbl.$

The simulation time is the same as that in the GBM model, from January 4, 2000 to May 28, 2010. Most simulation results including statistics, such as simulation mean prices, percentiles, and distribution normality measures, are obtained from simulation results with 1K iterations.

Figure 5-17 is one example of the simulation results. The highest simulated price is around $\$200/bbl$. The price ranges become wider with the increase of simulation time. However, the prices are condensed in a range between $\$20/bbl$ and $\$90/bbl$.

Figure 5-18 shows the results of another simulation case. The simulation results, including the 5th percentile, mean, and 95th percentile, are compared with the actual price movement. Most of the simulated prices are within the 5th percentile and 95th percentile ranges. And the ranges between the 5th and 95th percentiles cover the majority of the actual prices. The simulated mean prices are in the center of the price range of actual prices. Figures 5-17 and 5-18 also show that the simulation results from these two simulation cases are similar to each other.

Figure 5-19 further shows the statistics for the simulation results, including the maximum, minimum, and mean prices, and the 5th percentile, 10th percentile, 90th percentile, and 95th percentile ranges. After 2004, the trend for the maximum simulation prices is very similar to the actual price movement, but at much higher price levels. Actual oil prices never reach the maximum or minimum simulation prices during the simulation time.

Figure 5-20 shows that the simulation variance increases almost linearly with the simulation time. The highest value of variance is close to 1,200. Figure 5-21 shows that the skewness of the simulated prices ranges from zero to around 2.0 - a slightly increasing trend but in a cyclic pattern. The positive distribution skewness indicates that the simulated price distribution is not symmetric. It has longer right tails than that of the normal distribution. The distribution is centered on the left side. Figure 5-22 shows that the kurtosis of the simulated prices is higher than 3.0 but less than 11. With the increase of simulation time, the values of the kurtosis move up and down. The distribution kurtosis higher than 3.0 indicates that the simulated prices have a peak near the mean relative to a normal distribution.

Figure 5-23 includes three simulation realizations and compares them with the actual price movement. The simulated prices can be four times higher or eight times lower than the actual oil prices.

Table 5-2 is the summary of the simulation results for the selected eight weeks starting from the second week of year 2000 with 70 weeks in between. The distribution graphs match the simulation statistics.

5.3 Comparisons between the Geometric Brownian Motion and One-factor Mean Reversion Oil Price Models

Comparing the simulation results from both the GBM and the one-factor mean reversion oil price models can be used as guidance in selecting a fitted model for specific purposes. For the economic evaluations of petroleum projects, the uncertainty in future oil prices should be captured through a price model which takes the future price fluctuations into consideration. However, the estimated prices and ranges from the price model are desired to be as close to reality as possible so that the true values of projects can be obtained and right decisions can arrive.

Figures 5-3, 5-4, 5-18, and 5-19 demonstrate that the maximum simulation prices from the GBM price model are much higher than those from the one-factor mean reversion price model. In half of the simulation time from January 4, 2000 to April 7, 2010, the simulated maximum prices from the GBM price model are higher than $\$1{,}000/bbl$, reaching up to $\$7{,}000/bbl$; while from the mean reversion price model, the simulated maximum prices are less than $\$350/bbl$.

In Figure 5-24, the simulation results are shown in two charts with the same scale. From the two charts, it is clearly observed that the price range simulated from the GBM price model becomes wider with the increase of the simulation time, indicating that the GBM process is a non-stationary stochastic process. On the other hand, the price range simulated from the one-factor mean reversion model becomes very stable after a certain simulation time. In addition, many of the simulated prices from the GBM price model are much higher than the actual prices.

Figure 5-25 further shows that the simulated mean prices with the GBM price model, which are very close to the expected forecast future price distribution when the iteration number of simulations is large enough, are at the upside of the actual prices; and the simulated mean prices from the mean reversion price model are along the center of the actual prices.

From Figures 5-26 and 5-27, it is observed that: 1) The 90th and 95th percentiles from the GBM price model are much higher than the actual prices can reach. 2) The ranges of the 5th and 95th percentiles from the mean reversion price model are very close to the fluctuation ranges of actual prices. 3) The simulated price ranges from the GBM price model, which increase with the simulation time and can be a $\$500/bbl$ difference between the 5th and 95th percentiles, are much wider than those from the mean reversion price model. Referring to Figure 5-8, it is further understood that the disparities from the actual prices in the simulated up-bound prices is much more significant than those in the simulated low-bound prices.

Figures 5-12 and 5-20 show that the variance of the simulated prices from the GBM price model is much higher than that from the mean reversion price model. Volatility is one of the most important factors in option pricing. When the variance from the simulation prices is higher than that from the actual prices, so is the annualized standard deviation, *i.e*., volatility, then the options will be overpriced.

In summary, from the simulation results of the GBM and the mean reversion price models, a conclusion is reached that the one-factor mean reversion price model is a better model to fit the WTI historical oil prices. The GBM price model may be used for the stock prices so that Black–Scholes formula can be applied to price stock options. However, the mean reversion price model fits oil price evolutions, such as the historical WTI oil prices, better than the GBM price model.

Table 5-1: Summary of the Simulation Results from the GBM Price Model for the Selected Eight Weeks for the *WTI* Weekly Oil Prices from January 11, 2000 to May 28, 2010

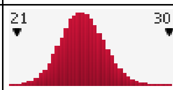
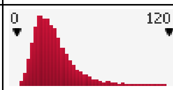
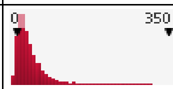
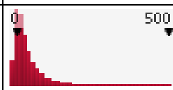
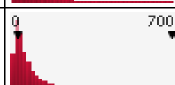
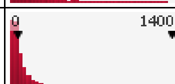
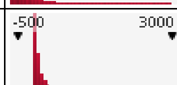
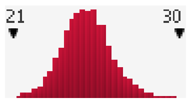
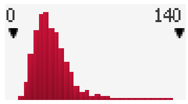
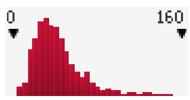
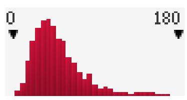
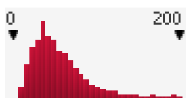
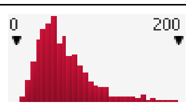
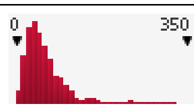
Price (Week)	Price Distribution	Min \$/bbl	Mean \$/bbl	Max \$/bbl	Standard Deviation \$/bbl	Skewness	Kurtosis	5th Percentile	95th Percentile
P(1)		21.01	25.03	29.94	1.18	0.14	3.02	23.15	27.01
P(71)		7.21	31.69	115.01	13.15	1.26	5.78	15.12	56.23
P(141)		3.16	40.06	306.35	24.54	2.04	10.68	13.53	86.39
P(211)		2.57	50.65	497.30	38.60	2.40	12.80	13.12	125.33
P(281)		2.06	63.70	661.04	57.17	2.75	14.76	13.09	172.14
P(351)		1.40	80.14	1355.46	83.66	3.83	30.39	12.76	229.11
P(421)		1.98	100.56	2572.37	117.98	4.72	50.50	13.37	302.84
P(491)		1.56	127.78	3230.50	170.76	5.72	66.33	13.69	404.81

Table 5-2: Summary of the Simulation Results from the One-factor Mean Reversion Price Model for the Selected Eight Weeks for the *WTI* Weekly Oil Prices from January 11, 2000 to May 28, 2010

Price (Week)	Price Distribution	Min \$/bbl	Mean \$/bbl	Max \$/bbl	Standard Deviation \$/bbl	Skewness	Kurtosis	5th Percentile	95th Percentile
P(1)		21.53	25.09	29.58	1.18	0.14	3.06	23.16	27.12
P(71)		10.18	35.65	130.34	12.06	1.20	7.05	19.31	56.49
P(141)		10.86	45.03	148.28	19.08	1.35	5.92	21.59	80.08
P(211)		9.95	51.88	166.61	22.52	1.30	5.66	23.82	92.95
P(281)		15.06	57.45	197.83	25.83	1.31	5.85	25.55	106.41
P(351)		13.08	61.32	192.62	27.40	1.24	5.06	26.88	112.40
P(421)		13.86	64.93	318.87	31.75	1.75	9.27	27.62	122.07

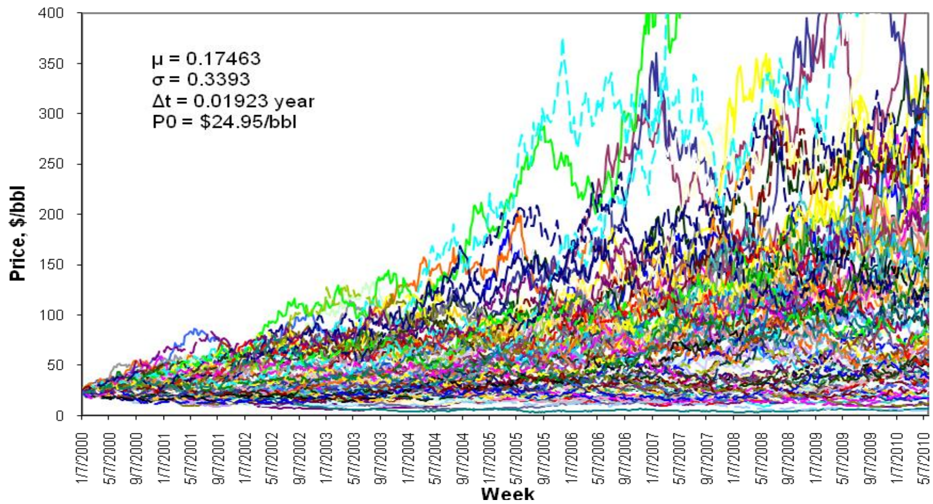


Figure 5-1: Simulation Results of the GBM Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010 (100 Iterations)

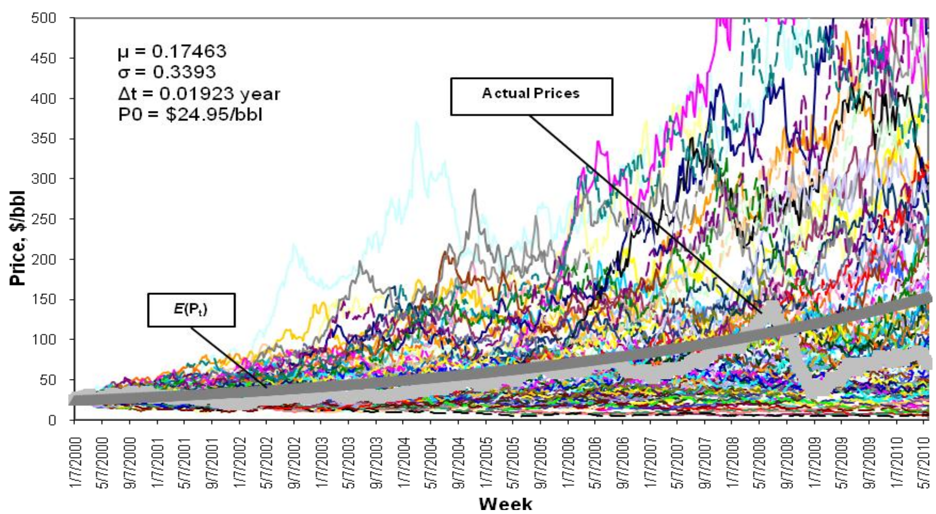


Figure 5-2: Comparison of Simulation Results of the GBM Price Model to the Actual *WTI* Weekly Oil Price Movement from January 4, 2000 to May 28, 2010 (100 Iterations)

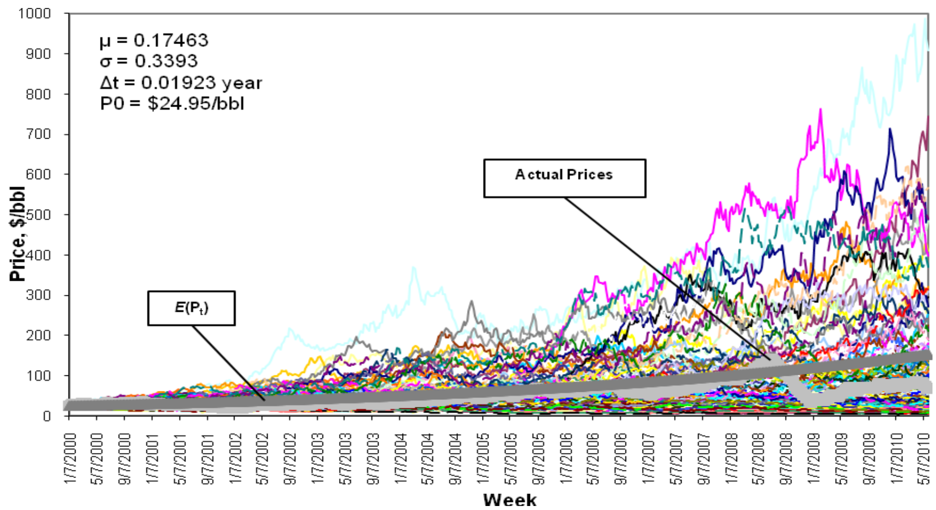


Figure 5-3: Simulation Results of GBM Price Model Including the Up-Bound Oil Price Realization for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010 (100 Iterations)

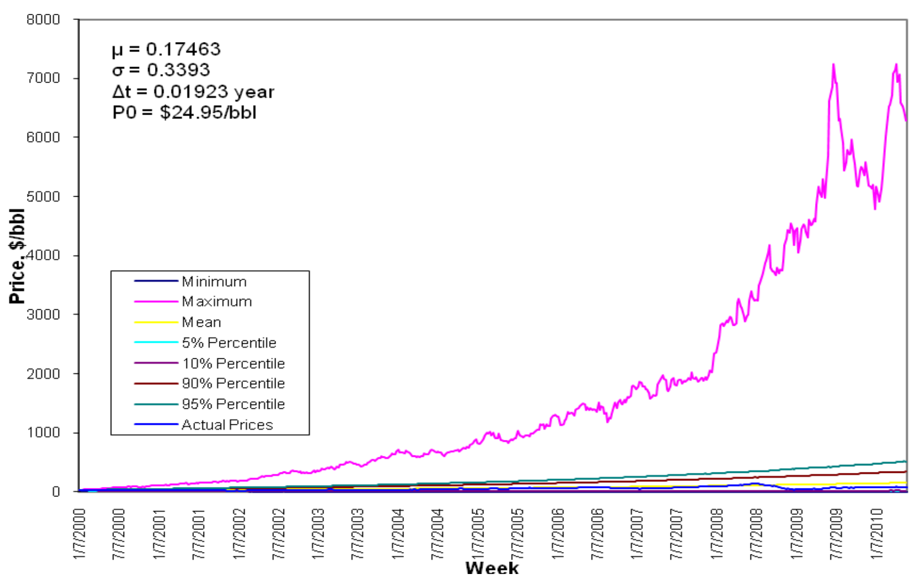


Figure 5-4: Simulation Price Ranges of GBM Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010 (10K Iterations)

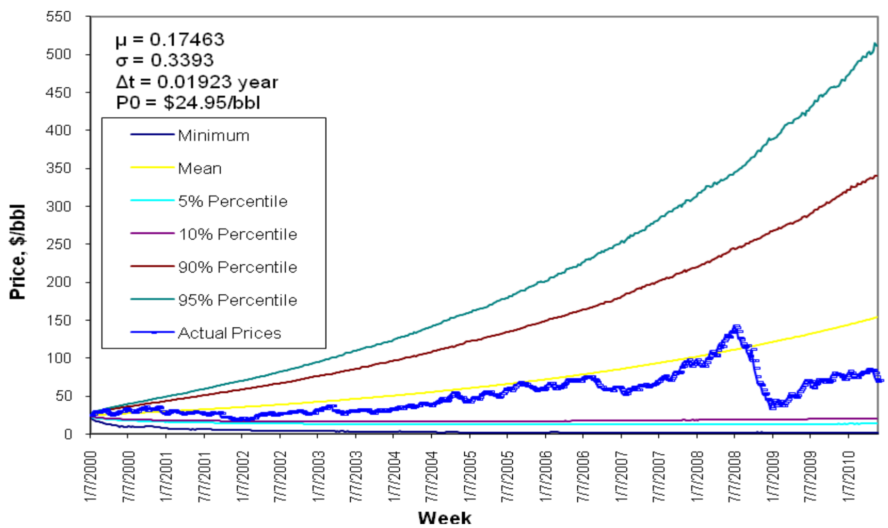


Figure 5-5: Simulation Price Ranges of GBM Price Model Excluding Maximum Prices for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010 (10K Iterations)

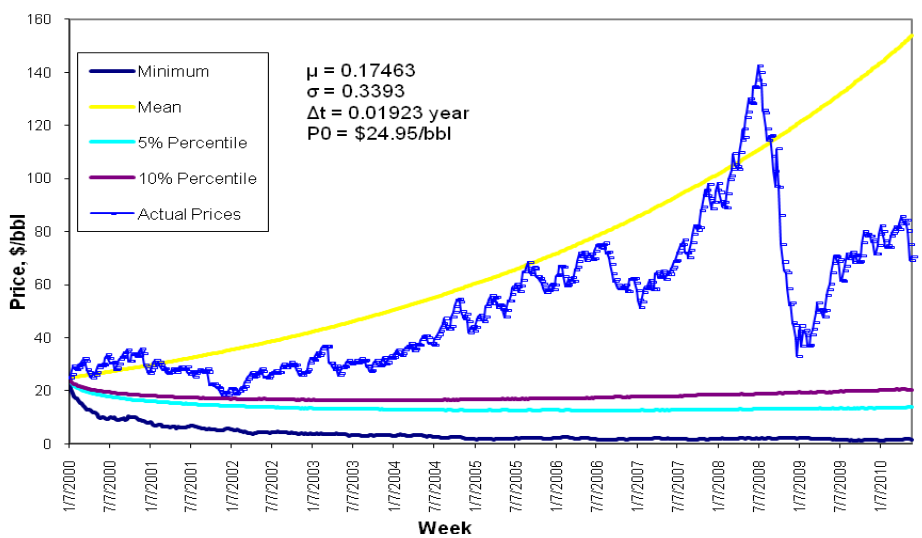


Figure 5-6: Simulation Low Bound Price Ranges of GBM Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010 (10K Iterations)

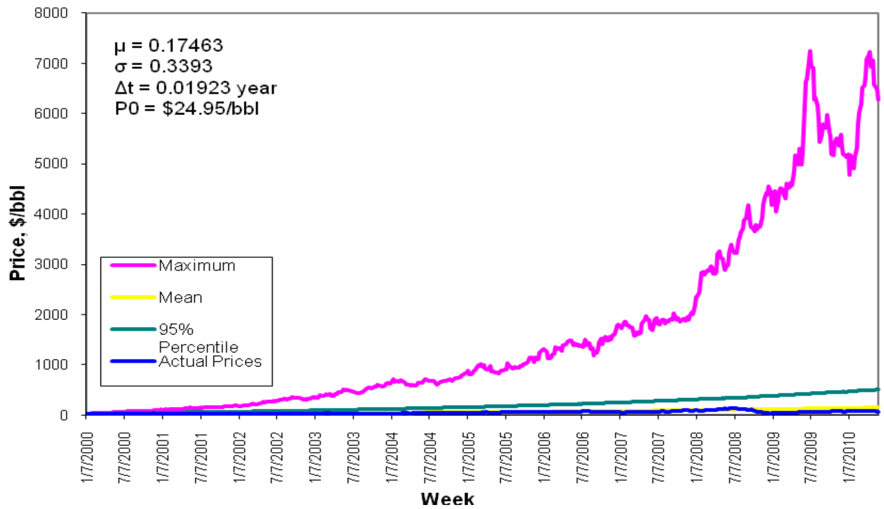


Figure 5-7: Simulation Up Bound Price Ranges of GBM Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010 (10K Iterations)

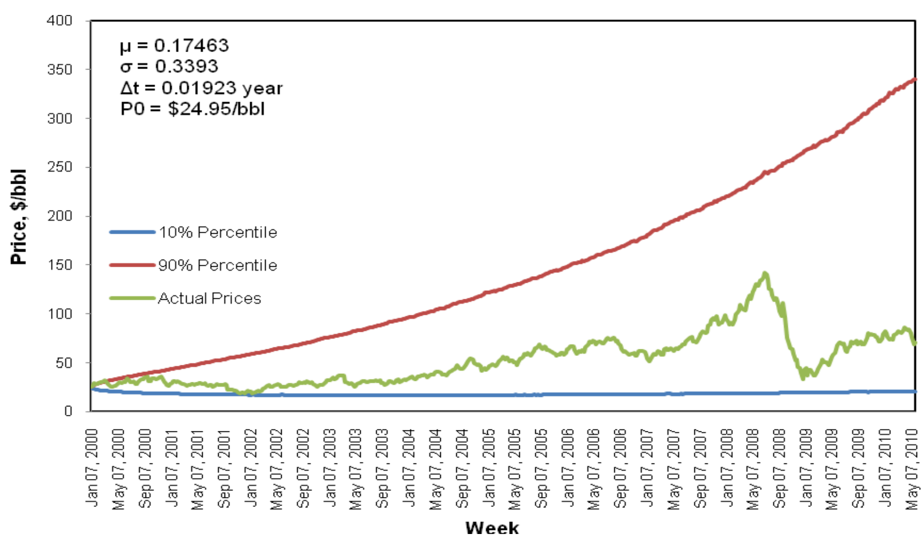


Figure 5-8: Simulation 80% Price Ranges of GBM Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010 (10K Iterations)

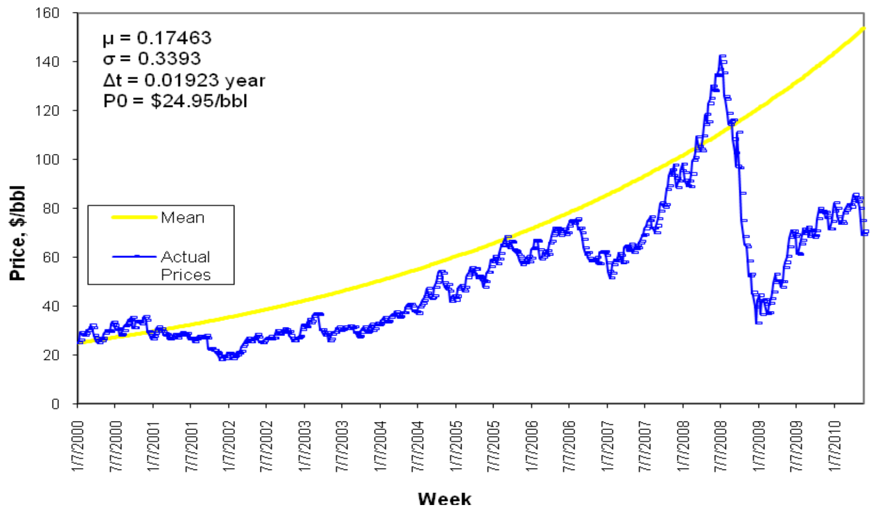


Figure 5-9: Comparison between the Expected Forecasting Prices $E(P_t)$ for the GBM Price Model and the Actual Prices for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010

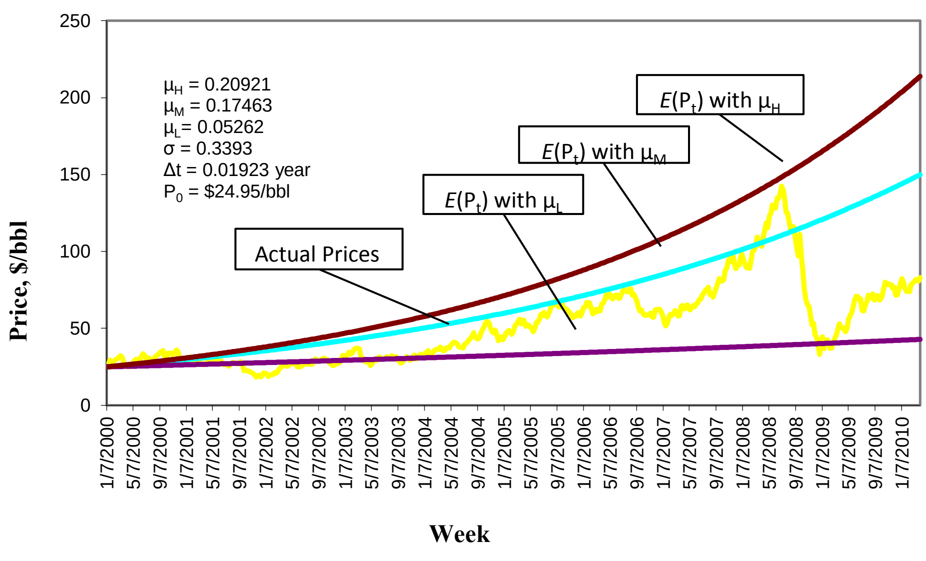


Figure 5-10: Comparison of the Expected Forecast Prices $E(P_t)$ from the GBM Price Model under Three Different Drift Rates and Actual Prices for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010

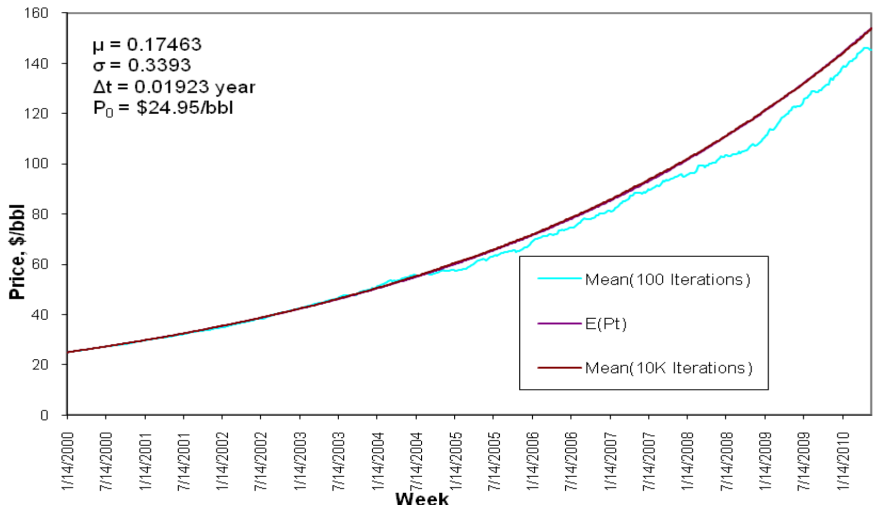


Figure 5-11: Comparison between the Simulated Mean Prices of the GBM Price Model and the Number of Iterations for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010

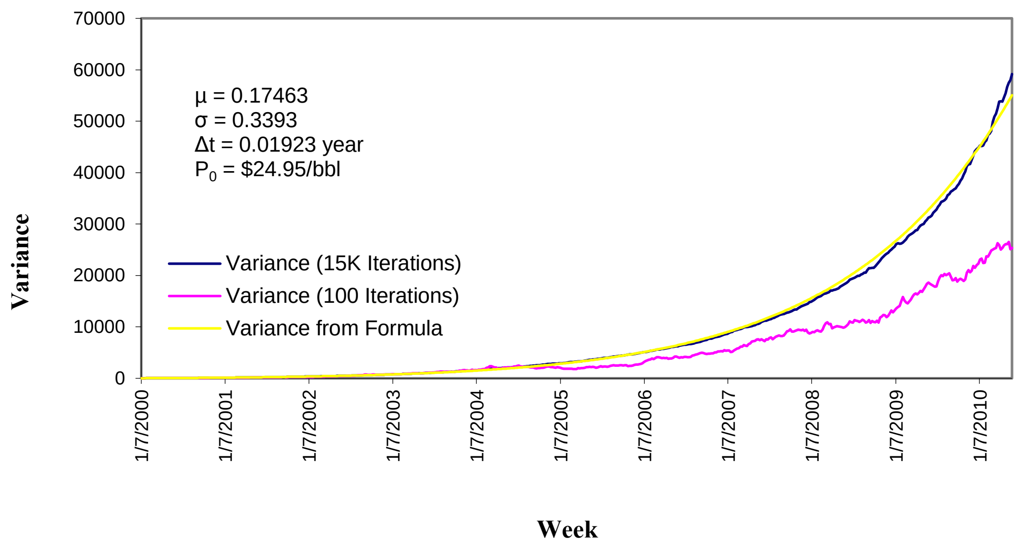


Figure 5-12: Comparison between the Variance of the Simulated Prices with Different Numbers of Iterations and the Calculated Variance with GBM Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010

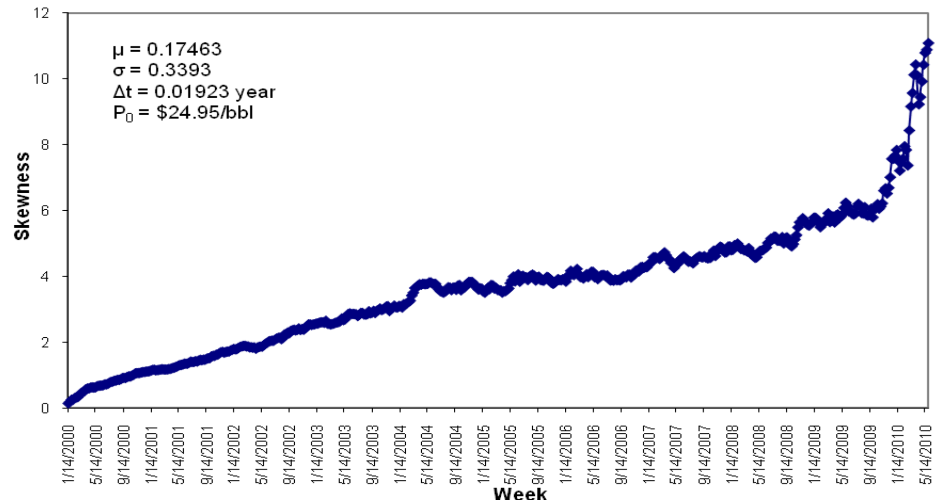


Figure 5-13: Simulation Skewness with the GBM Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010 (10K Iterations)

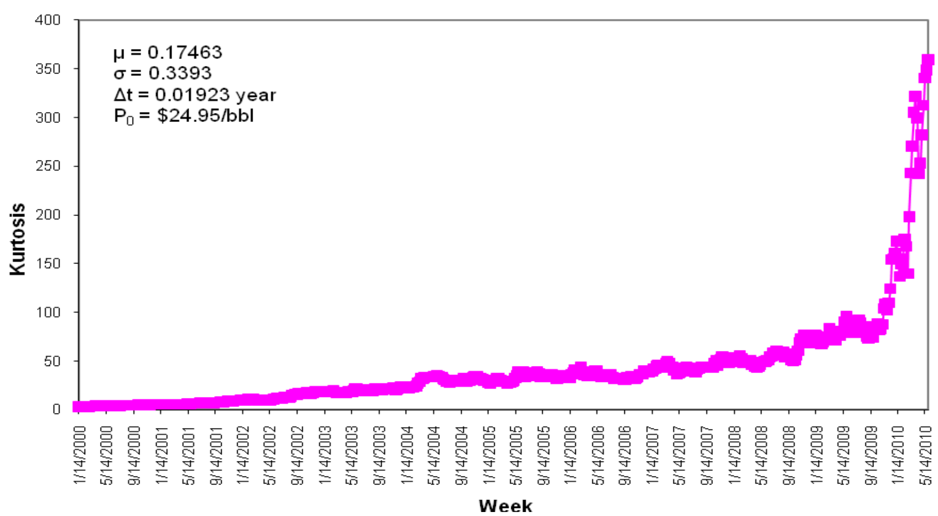


Figure 5-14: Simulation Kurtosis with the GBM Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010 (10K Iterations)

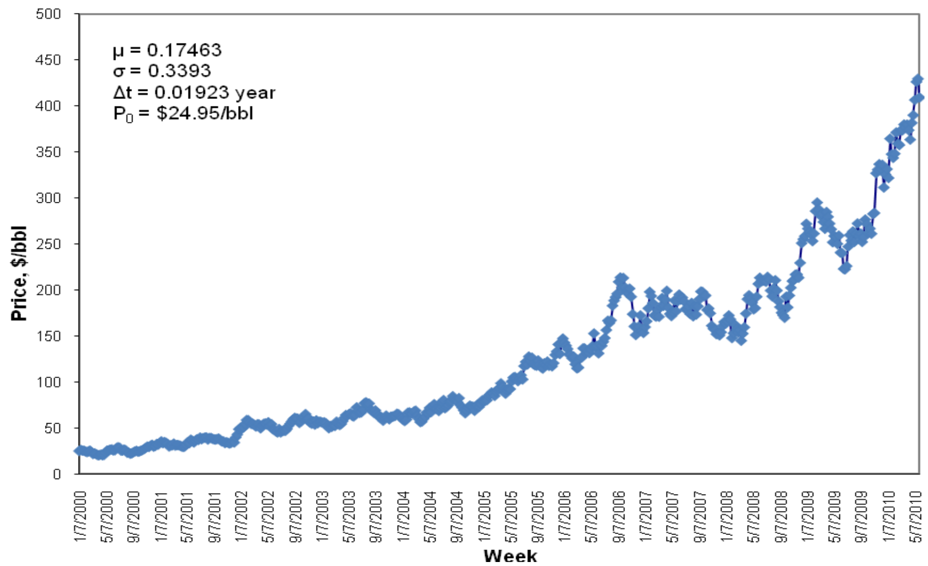


Figure 5-15: One Simulation Realization with the GBM Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010

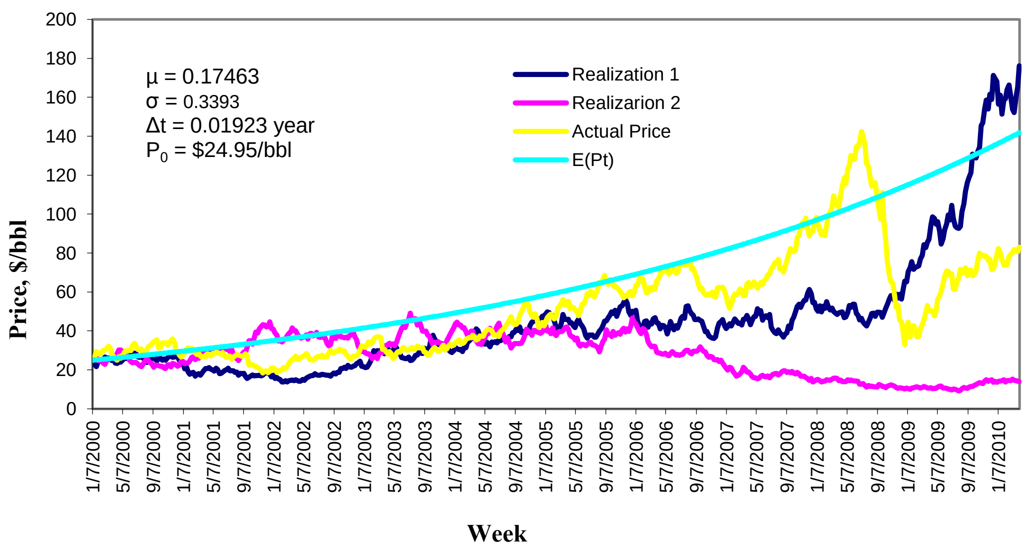


Figure 5-16: Comparison of Two Simulation Realizations and the Expected Forecast Prices with the GBM Price Model and the Actual Prices for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010

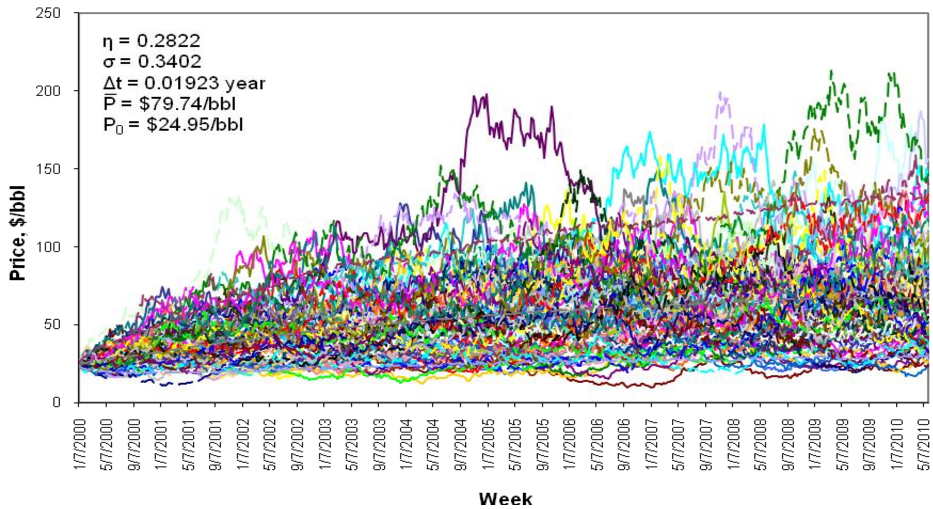


Figure 5-17: One Example of Simulation Results of the One-factor Mean Reversion Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010 (100 Iterations)

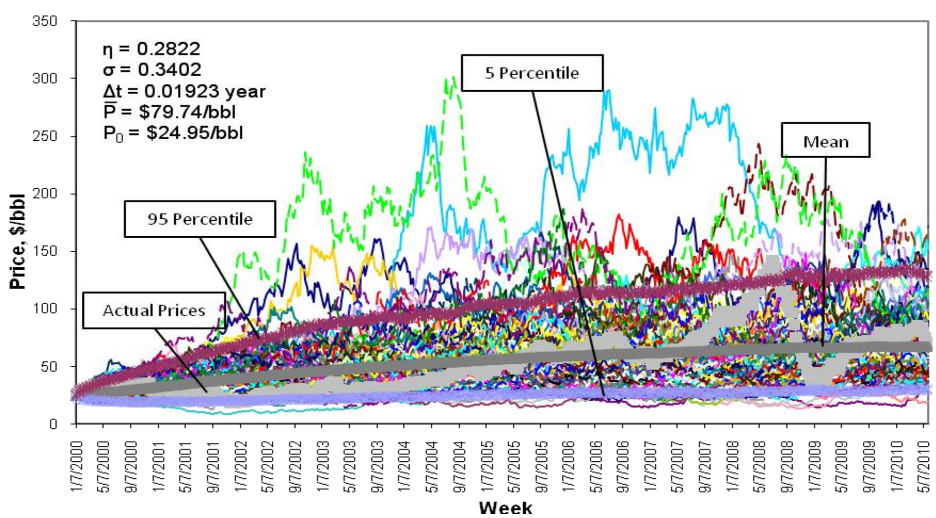


Figure 5-18: Simulation Results of One-factor Mean Reversion Price Model Compared with the Actual Price Evolution for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010 (100 Iterations)

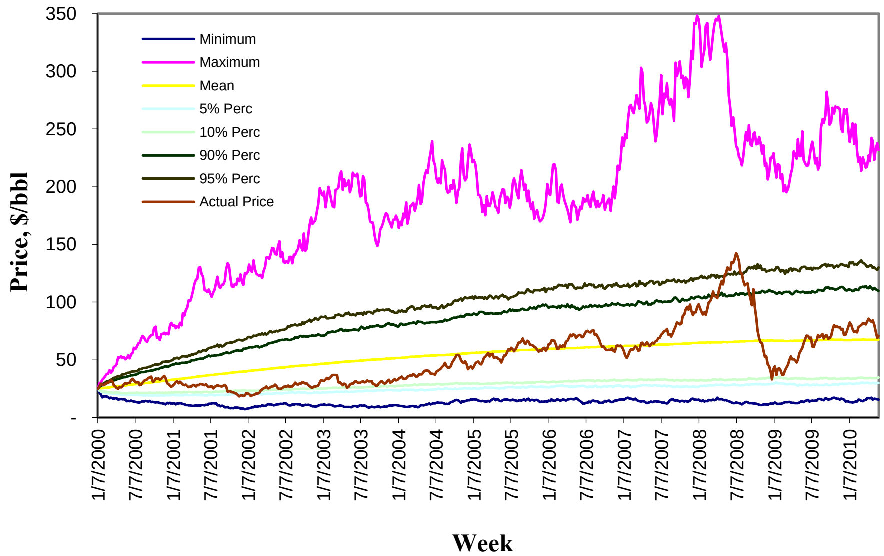


Figure 5-19: Simulation Statistics of One-factor Mean Reversion Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010

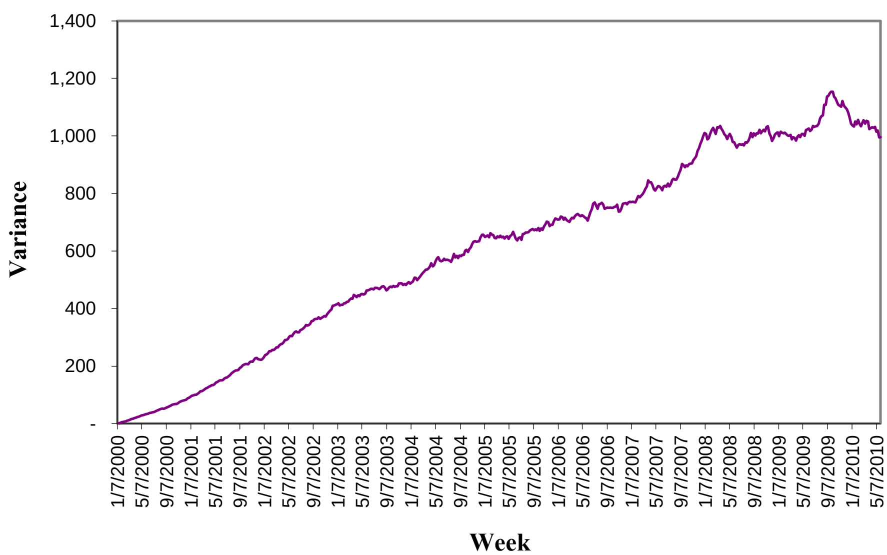


Figure 5-20: Simulation Variance of One-factor Mean Reversion Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010

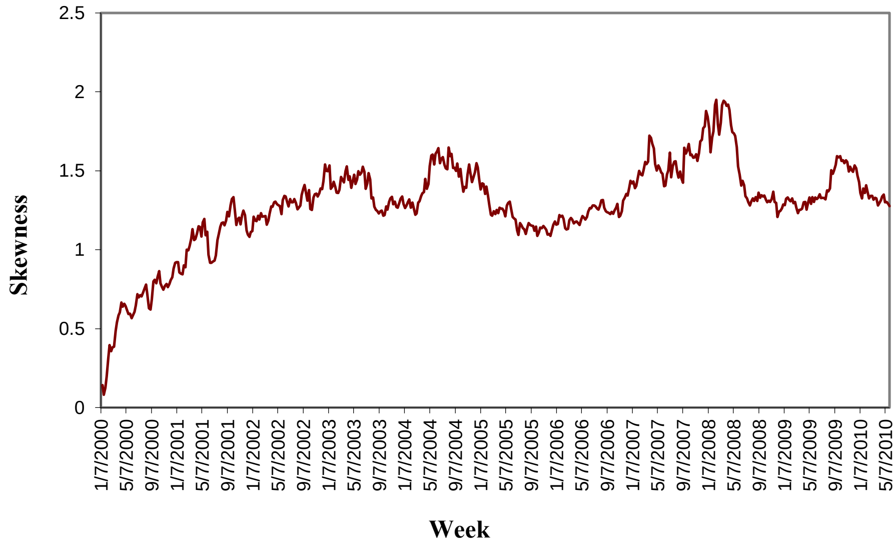


Figure 5-21: Simulation Skewness of One-factor Mean Reversion Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010

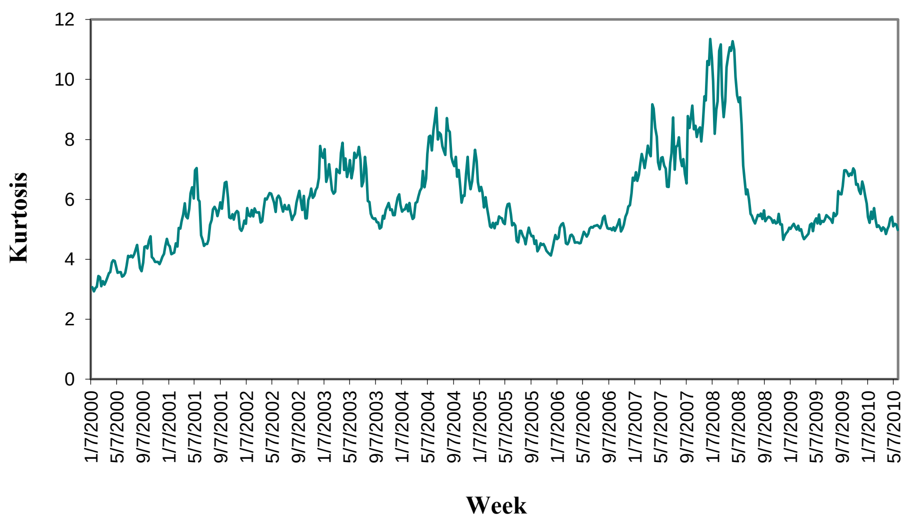


Figure 5-22: Simulation Kurtosis of One-factor Mean Reversion Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010

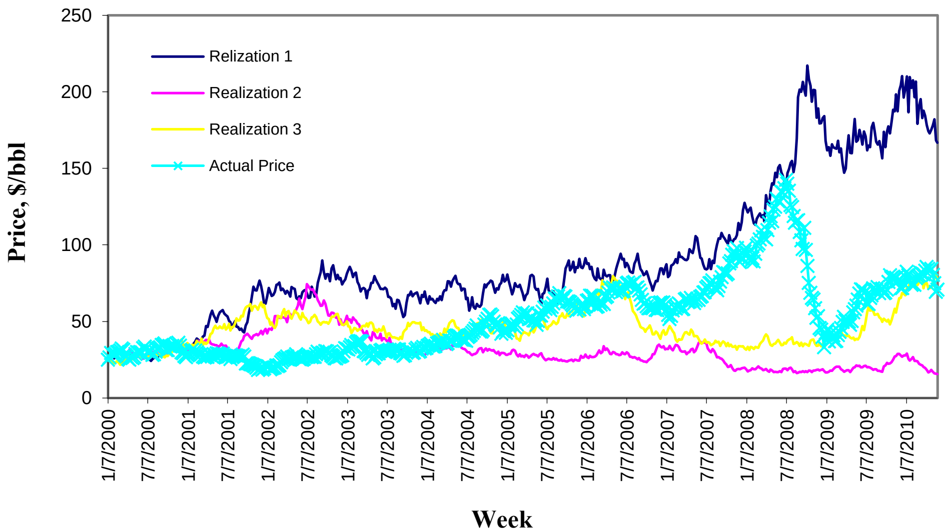


Figure 5-23: Three Simulation Realizations of One-factor Mean Reversion Price Model for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010

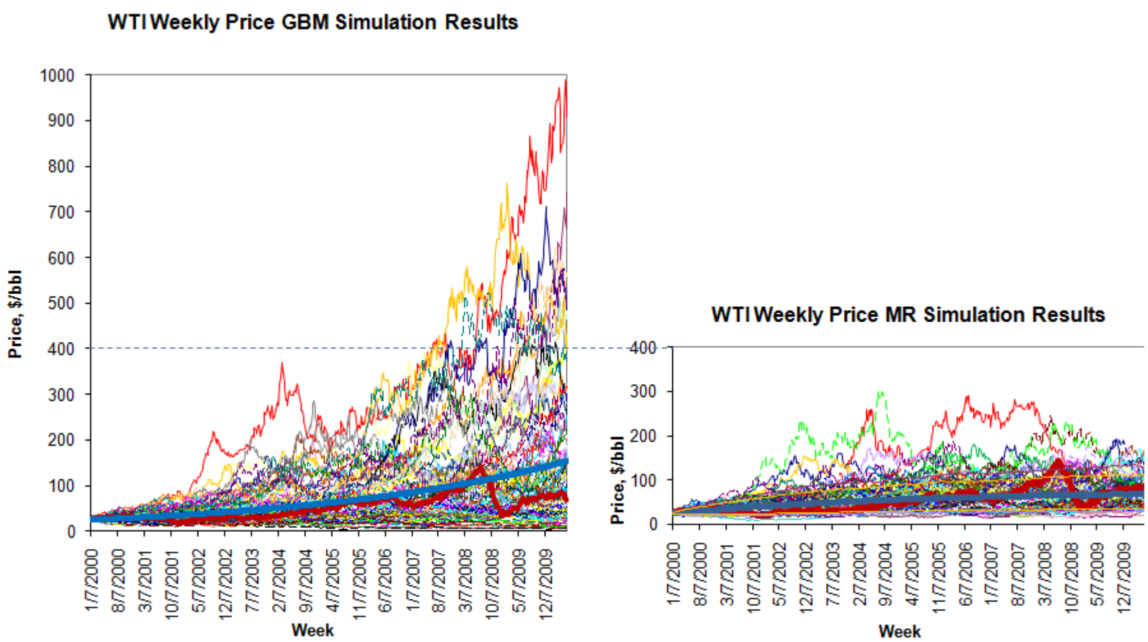


Figure 5-24: Comparison of the Simulation Results of the GBM and One-factor Mean Reversion Price Models for the *WTI* Weekly Oil Prices from January 4, 2000 to May 28, 2010

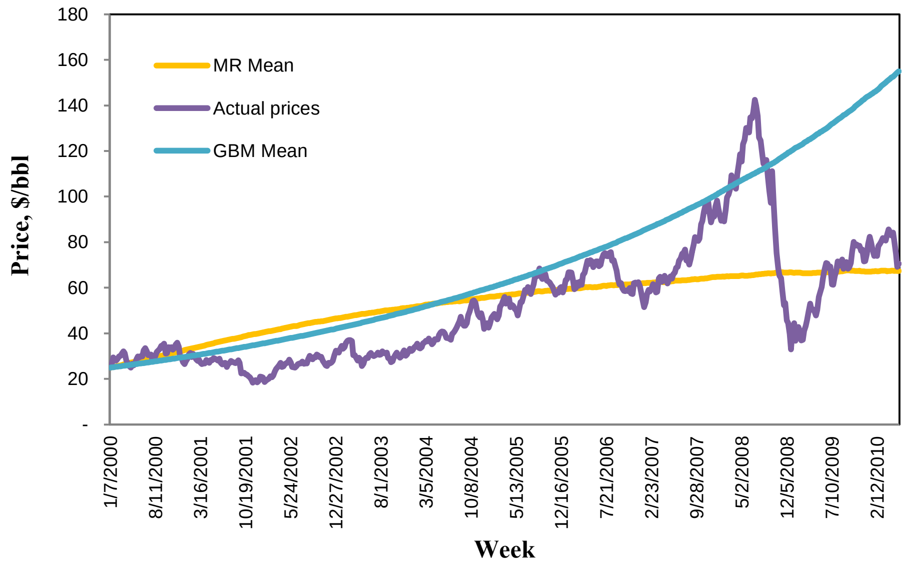


Figure 5-25: Comparison of the Simulation Means of the GBM and One-factor Mean Reversion Price Models for the *WTI* Weekly Prices from January 4, 2000 to May 28, 2010

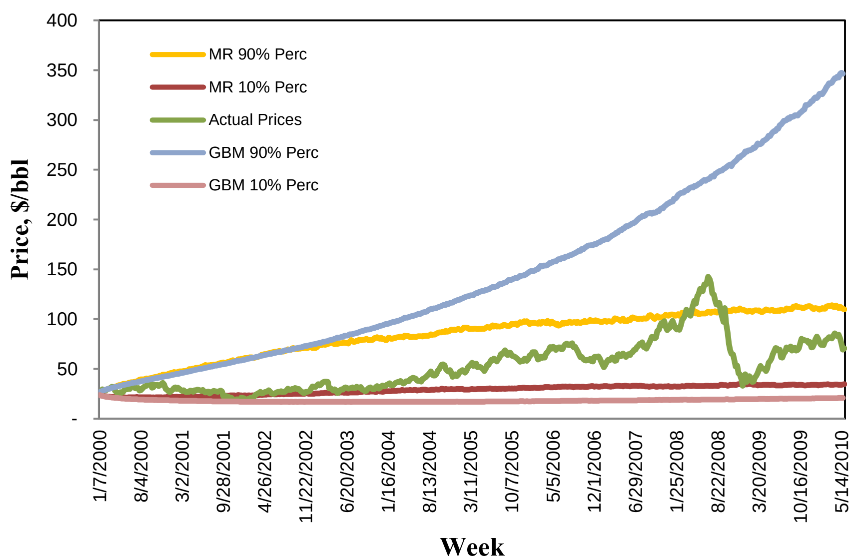


Figure 5-26: Comparison of the Simulation 10th and 90th Percentiles of the GBM and One-factor Mean Reversion Price Models for the *WTI* Weekly Prices from January 4, 2000 to May 28, 2010

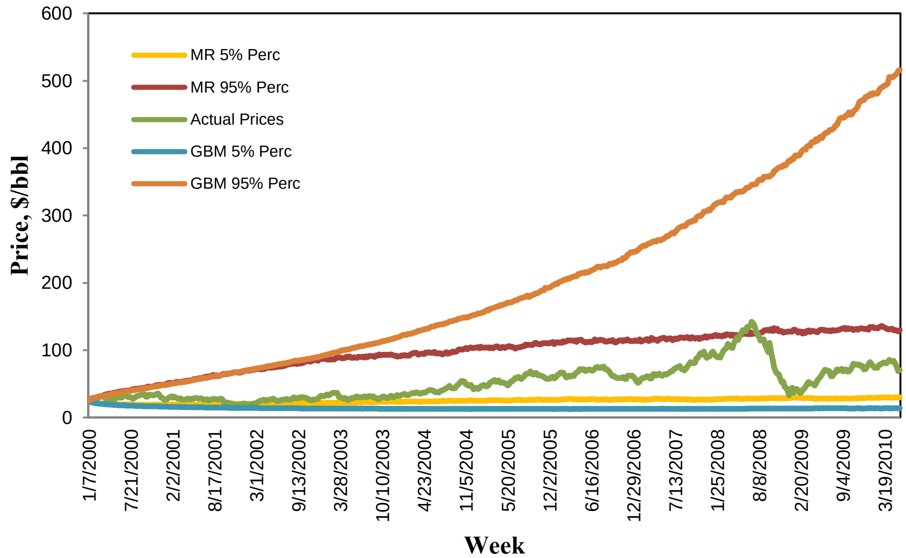


Figure 5-27: Comparison of the Simulation 5th and 95th Percentiles of the GBM and One-factor Mean Reversion Price Models for the *WTI* Weekly Prices from January 4, 2000 to May 28, 2010

CHAPTER 6: RESERVOIR SIMULATIONS AND OIL PRODUCTION PROFILE

Specific reservoir simulation cases are designed to conduct real options evaluation on the switching time from primary to water flooding oil production. The reservoir simulations are conducted through the University of Texas Chemical Flooding Simulator (UTCHEM) (Center for Petroleum and Geosystems Engineering, 2000) on a synthetic onshore oil reservoir. To capture the underground reservoir uncertainty, reservoir heterogeneity is applied in reservoir simulations. The oil production profile is generated for 29 oil production cases with different water flooding switch times.

6.1 UTCHEM SIMULATOR

The oil production simulations are conducted by UTCHEM simulator. UTCHEM is a three-dimensional chemical flooding simulator. Two input files need to be created in order to run the non-restart simulations: HEAD and INPUT.

The information regarding the name of the case, number of grid blocks, and number of wells in simulations should be included in the “HEAD” input file. All information regarding simulations is given in the “INPUT” input file. The “INPUT” file includes sections of reservoir descriptions, reservoir properties, physical property data, and well data. Under the reservoir description section, the number and size of the simulation grid blocks are defined according to the actual reservoir size and simulation goal. Under the reservoir properties section, rock compressibility, reservoir permeability in different directions, porosity, initial water saturation, initial reservoir pressure, and the maximum simulation time are required as input. Under the physical property data section, density and viscosity of oil, compressibility of oil, residual saturation for both oil and water need to be specified. Under the well data section, the number and locations of the producers and injectors, bottom hole pressure for pressure constraint wells, injection rate, the switching time from primary production to water flooding are defined.

After running reservoir simulations with UTCHEM, the commercial software Kraken is used to post-process the simulation results in order to visualize the oil production rate, reservoir pressure, cumulative oil recovery, and other related data.

In this study, a synthetic onshore oil reservoir is created according to the study on the Tertiary Oil Recovery Information System (TORIS) database to include the underground uncertainty about reservoir permeability, porosity, and initial oil saturation. The following section includes the research results on TORIS data.

6.2 Related Research Results on TORIS Data

TORIS database is an analytical system maintained by the Department of Energy's (DOE) Bartlesville Project Office. The most updated TORIS database (updated in 1995) covers over 2,540 oil reservoirs, accounting for over 64% of the original oil in place ($OOIP$) estimated to exist in discovered oil reservoirs in the U. S. TORIS uses reservoir-level data to evaluate the technical and economic recovery potential of specific crude oil reservoirs (DOE's Bartlesville Project Office, 1995).

The research on TORIS data includes correction for the original data, BestFit for the distributions of reservoir data, and correlation and dependencies analysis among reservoir properties. While comprehensive studies have been undertaken on the TORIS data, only the related results are described and used here as guidance to create a synthetic oil reservoir for this study, including reservoir porosity, permeability, and initial oil saturation.

There are more than 1,000 sets of paired data including porosity, permeability, reservoir vertical depth, geological age code, geological play indicator, depositional system, trap type, net pay, initial formation pressure, formation temperature, oil API gravity, initial oil saturation, and initial formation volume factor. Errors in the original data are found when a dependency study is performed for porosity and permeability. For example, as shown in Figure 6-1, two groups of reservoir permeability data can be observed from the correlation results between Ln(Permeability) and Ln(Porosity) for the uncorrected TORIS permeability data. Table 6-1 shows that after the permeability data in the small group are taken out, the correlations between Ln(Permeability) and porosity, and between Ln(Permeability) and Ln(Porosity), both improve. Inconsistency in the units of oil saturation is also found in the original data. Professional judgments are applied to make the correction so that the data used all have uniform units.

@Risk software is used to obtain the BestFit results for reservoir data including porosity, permeability, initial oil saturation, and initial oil formation volume factor (FVF). BestFit function in @Risk identifies the distributions that most likely produced the input data. For input sample data, distribution parameters are estimated using maximum-likelihood estimators. BestFit does not produce only one absolute distribution; instead, fitted distributions are ranked using one or more fit statistics, such as Chi-square. Thus professional judgments are applied to select one distribution among the first three top ranked distributions from BestFit for each type of the reservoir data of interests.

The TORIS reservoir porosity data are from core studies or from electric log data. 1,337 data were used to find the BestFit distribution for porosity. The minimum porosity is 1.5%. The maximum porosity is 58%. Normal distribution is the third BestFit function for TORIS porosity data. As shown in Figure 6-2, TORIS porosity data resemble the normal distribution with a mean of 20.29% and a standard distribution of 8.76%.

The TORIS reservoir permeability data are from whole core studies, pressure buildup test, or sidewall core analysis. The unit of permeability is millidarcies (mD). 1275 data are used to find the BestFit distribution for permeability and Ln(Permeability). The minimum permeability is 0.1 mD. The maximum permeability is 26,817.0 mD. The average permeability is 469.6 mD. Standard deviation of permeability is 1,132.4 mD. The minimum of Ln(Permeability) is -2.3. The maximum of Ln(Permeability) is 10.20. Normal distribution is the third BestFit function for TORIS Ln(Permeability) data. Figure 6-3 shows that the logarithm of permeability is close to a normal distribution with a mean of 4.55 and a standard distribution of 2.14. According to the relationship between the standard deviation of Ln(Permeability) (σ) and the Dykstra-Parsons coefficient (V_{DP}) (Jensen, Lake, Corbett, and Goggin, 2003), that is

$V_{DP} = 1 - Exp(-\sigma)$,

the value of V_{DP} is 0.882 when σ is 2.14, which is very high in comparison to the typical Dykstra-Parson's coefficient of reservoir permeability property in the U.S., which is between 0.7 and 0.8.

The TORIS initial oil saturation data are usually derived from electric log analysis. 1,338 initial oil saturation data are used to determine the BestFit distribution. The minimum value of the initial oil saturation is 26% and the maximum value is 96%. Normal distribution is the second BestFit function for the initial oil saturation. Figure 6-4 shows that the initial oil saturation is close to a normal distribution with a mean of 69.75% and a standard deviation of 9.54%.

There are 587 initial oil formation volume factor (FVF) data from the TORIS database. The minimum FVF is 1.0 RB/STB, and the maximum FVF is 2.13 RB/STB. The average FVF is 1.26 RB/STB and the standard deviation of FVF is 0.21. Lognormal distribution is the third BestFit function for the initial formation volume factor, as shown in Figure 6-5.

The dependency study on TORIS data shows that there are strong correlations between reservoir permeability (in the form of logarithm of permeability) and porosity, as shown in Figure 6-6, between initial reservoir pressure and reservoir vertical depth, as shown in Figure 6-7, with regression R-Square being 0.8948, and between reservoir temperature and reservoir vertical depth, as shown in Figure 6-8, with regression R-Square being 0.6342. There is also relationship between initial oil formation volume factor and reservoir vertical depth, as shown in Figure 6-9, and between initial oil formation volume factor and oil API gravity, as shown in Figure 6-10.

As showed in Figure 6-7, the approximate relationship between initial reservoir pressure and reservoir vertical depth can be described with the following equation:

$$Initial\ Reservoir\ Pressure\ (psi)$$
$$= 14.7 + 0.4399 \times Depth(ft) \qquad (6-1)$$

The slope is very close to the pressure gradient of water at $4\,^{\circ}C$: $0.4335\ psi/ft$.

As shown in Figure 6-8, reservoir temperature changes with the reservoir vertical depth as the following equation describes:

$$Reservoir\ Temperature\ (°\mathrm{F}) = 76.66 + 0.0122 \times Depth\ (ft) \qquad (6-2)$$

As shown in Table 6-2, the initial oil formation volume factor can be related to the oil API gravity and reservoir vertical depth with the following equation:

$$FVF\left({}^{RB}/_{STB}\right) = 0.813 + 0.00835 \times °API + (3.32E-5) \times Depth\ (ft) \qquad (6-3)$$

With the knowledge of reservoir vertical depth, reservoir temperature can be approximated by Eq. (6-2). With this reference temperature, the important properties of reservoir fluids (oil and water), such as density and viscosity can be obtained. According to the one-dimensional form of Darcy's law (Walsh and Lake, 2003), that is

$$q = -\frac{k}{\mu} \times \frac{\mathrm{dP}}{\mathrm{dL}}, \qquad (6\text{-}4)$$

where q is production rate, k is reservoir permeability, μ is fluid viscosity, and dP/dL is the pressure gradient in the fluid-flow direction, a good estimation of reservoir temperature is important in determining fluid viscosity, and thus, the simulated oil production rate.

6.3 Reservoir Construction

A three-dimensional synthetic oil reservoir is constructed as follows: the reservoir is at a vertical depth of 4,370 feet with a 60 feet net pay; the size of the reservoir is 6,969,600 square feet, which is four times a 40-acre reservoir (one 40 acre equals 1,742,400 square feet); and the size in both X and Y directions of the reservoir is 2,640 feet. For reservoir simulations, the grid size is 64.4 feet in both X and Y directions and 20 feet in Z direction. So the total simulation number of grid blocks is 5,043 $(41 \times 41 \times 3)$. The general reservoir field properties are summarized in Table 6-3.

The reservoir is a carbonate mixed-wet reservoir. The reservoir formation compressibility is 0.000004$(1\ /\ psi)$. The initial reservoir pressure is 4,000 psi. Since the reservoir is at a depth of 4,370 ft, the 4,000 psi, initial pressure is much higher than that from the correlated equation between initial reservoir pressure and the reservoir vertical depth described in Eq. (6-1). The reservoir is located in a high pressure formation zone.

With a formation vertical depth of 4,370 feet, the formation temperature is around 130 $°F$ (or 54.5 $°C$) according to Eq. (6-2). This temperature is the reference temperature for density and viscosity of reservoir fluids (oil and water).

The properties of the oil in the reservoir is very close to the West Texas Intermediate (WTI), in both API gravity and sulfur content; thus, the oil produced in this reservoir can be sold at the same prices as WTI when transportation costs from well-head to spot market and inventory cost are neglected. The API gravity of the oil in the reservoir is 39.6. The specific gravity of the oil is 0.827 g/cm^3, or 0.3585 $psi/ft,$ at 60 $°F$; and 0.805 g/cm^3, or 0.3490 $psi/ft,$ at 130 $°F$. The viscosity of the oil is 9.7cp at 60 $°F$ and 2.8174 cp at 130 $°F$. The properties of reservoir fluids are listed in Table 6-4.

6.4 RESERVOIR HETEROGENEITY

In order to take the underground reservoir uncertainty into consideration, three categories of reservoir heterogeneity, i.e., initial water saturation, permeability, and porosity, are included for reservoir simulations.

The distribution from TORIS data on the initial oil saturation is used to generate the initial water saturation field. The initial oil saturation is normally distributed with a mean of 69.75% and a standard deviation of 9.54%. 5,043 initial oil saturation data are generated accordingly and assigned to each grid block of the reservoir. Since the sum of oil and water saturation equals one, the initial water saturation is determined from the initial oil saturation. The initial water saturation heterogeneity data for the entire reservoir are included in the UTCHEM input file.

The Matrix Decomposition Method (MDM) (Yang, 1990) is used and the results from the TORIS permeability distribution are adjusted to generate a permeability heterogeneity field for the synthetic reservoir. As mentioned above, the Dykstra-Parson's coefficient of 0.882 from TORIS data, together with the permeability standard deviation of 1,132.4 mD and the highest permeability of 26,817.0 mD, indicates that permeability heterogeneity from TORIS data is higher than the typical permeability heterogeneity in the U.S. petroleum reservoir. Both the mean and standard deviation from TORIS data are reduced for this study. The Monte Carlo simulation is first conducted with a mean of 4.55 and a standard deviation (SD) of 2.14 to obtain a normal distribution of Ln(Permeability) as showed in Figure 6-3; then the obtained distribution is truncated at (mean−1SD) and (mean+1SD), which is at 2.40 and 6.69 in the values of Ln(Permeability) and at 11 mD and 801 mD in the values of the permeability; and then the permeability data series between 11 mD and 801 mD from the above Monte Carlo simulation results is used to calculate the adjusted mean permeability, which is 172 mD. With a mean permeability of 172 mD, a Dykstra-Parsons coefficient of 0.7, which is a typical value of the permeability heterogeneity in the U.S. petroleum reservoir, and correlation lengths of 128 feet in the X and Y directions, 12.8 feet in the Z direction, the heterogeneity permeability field in the horizontal direction for the entire reservoir of 5,043$(41 \times 41 \times 3)$grid blocks is generated using the MDM program. Appendix A is the input file for the MDM program. The generated permeability heterogeneity field has a mean of 174.7 mD, which is very close to the input mean permeability, a Dykstra-Parsons coefficient of 0.7016, which is very close to the input Dykstra-Parsons coefficient, and a standard deviation of 320.9 mD. The summary of the permeability distributions in different layers is shown in Table 6-5. The permeability heterogeneity in the Y direction is the same as that in the X direction. The permeability heterogeneity in the Z direction is 10% of that in the X direction. The permeability heterogeneity fields in the X, Y, and Z directions for the entire reservoir are included in the UTCHEM input file.

The reservoir porosity heterogeneity field is generated according to the relationship between permeability and porosity for a typical carbonate reservoir as shown in the following equation (Ghomian, 2010):

$$k = 7.38 \times 10^6 \times \emptyset^{6.72}, \qquad (6-5)$$

where k is reservoir permeability in mD, and $\emptyset$ is reservoir porosity in fraction. With the heterogeneity permeability field data, porosity in each of the 5043 grid blocks is calculated from Eq. 6-5. The resulting porosity field has a mean of 18.66%, which is slightly smaller than that of the TORIS data, and a standard deviation of 3.39%, which is about half of that from the TORIS data. The heterogeneity porosity data for the entire reservoir are included in the UTCHEM input file.

6.5 Reservoir Simulations, Results, and Oil Production Profile

The reservoir simulations are conducted with the UTCHEM simulator to forecast the oil production on the synthetic reservoir described above. The oil production includes two stages: primary oil recovery and then water flooding oil recovery, except for one case in which water flooding starts at the very beginning of oil production. Oil is produced with a 5-spot well configuration, that is, a production well (producer) is drilled in the middle of the reservoir and the four water injection wells (injectors) are drilled at four corners of the reservoir. Figure 6-11 shows how the five wells are located on the reservoir. The 3-

dimensional initial reservoir pressure, initial reservoir oil saturation, reservoir porosity, reservoir permeability in X direction, and reservoir permeability in Z direction are shown in Figures 6-12 through 6-16.

As shown in Table 6-6, the oil production well is controlled by bottom hole pressure (BHP) at 500 psi while the water injection wells are controlled by the water injection rate at 4,000 bbl/day for each well. An artificial lift is used to take the oil from the bottom hole of the reservoir to surface since the 500 psi BHP is not high enough to overcome the gravitational force and other resistant forces.

6.5.1 Design of Reservoir Simulations

In order to conduct the real options analysis and capture the value of the flexibility in water flooding switching time, 29 oil production cases are designed to run the reservoir simulations, as shown in Table 6-7. The difference among cases is the starting time of water flooding after primary oil production with an increment of 91 or 92 days. For example, in Case 1 (G440T0), water flooding starts at time zero; in Case 2 (G44002), water flooding starts at day 91; and in Case 3 (G44005), water flooding starts at day 182, etc.

In the primary oil production, oil in the reservoir is driven by the natural reservoir pressure, *i.e.*, the difference between reservoir pressure and the BHP. According to Eq. (6-4), when the reservoir pressure is close to the BHP, the oil production rate will become very small. Figure 6-17 shows that if there is only primary oil recovery, after about 2,550 days, the oil production rate is less than 10 bbl/day. So in this study, the longest simulation time for primary oil recovery is 2,555 days (7 years). The longest simulation time for both primary and water flooding oil production is 50 years (18,250 days) when the oil production rate is about 20 bbl/day in water flooding oil recovery, as shown in Figure 6-18.

6.5.2 Reservoir Simulation Results for Reservoir Oil Saturation and Reservoir Pressure

6.5.2.1 Calculation of Original Oil in Place (OOIP)

Reservoir oil saturation measures the potential amount of oil that can be produced in the future. Initial oil saturation is used to calculate the original oil in place ($OOIP$), which is the maximum amount of oil that may be produced from a reservoir, according to the following equation:

$$OOIP = \frac{7758 V_b \phi S_{io}}{B_{io}} STB \, ,$$

or

$$OOIP = \frac{7758 A H \phi S_{io}}{B_{io}} STB \, , \qquad (6\text{-}6)$$

where V_b is reservoir volume, A is reservoir size in acres, H is net pay of the reservoir in feet, ϕ is reservoir porosity (%), S_{io} is initial oil saturation (%), and B_{io} is the oil initial formation volume factor, a dimensionless factor converting the volume of oil from reservoir condition to the standard conditions at the surface (60 °F and 14.7 psi).

Because of the heterogeneity of reservoir, average reservoir porosity and average initial oil saturation are used in Eq. (6-6), that is

$$OOIP = \frac{7758 \times A \times H \times \overline{\phi} \times \overline{S_{io}}}{B_{io}} \; STB. \qquad (6-7)$$

Figures 6-9, 6-10, and Table 6-2 show that both oil API gravity and reservoir depth affect the formation volume factor. For crude oil with an API gravity of 40 at 4,370 feet depth, the formation volume factor can be estimated with Eq. (6-3), that is,

$$B_{io} = 0.813 + 0.00835 \times 40 + 3.32 \times 10^{-5} \times 4370 = 1.292 \left({}^{RB}/_{STB} \right).$$

So the OOIP for this reservoir is estimated as follows:

$$OOIP = \frac{7758 \times 160 \times 60 \times 18.22\% \times 70\%}{1.292} = 7{,}529{,}535\ (bbls).$$

The $OOIP$ will be used as proved reserve and as the base to calculate the amount of depletion according to the units-of-production depreciation method, which is discussed in Chapter 7.

6.5.2.2 Reservoir Oil Saturation Change with Oil Production Time

With the production of oil, the reservoir oil saturation decreases. Figures 6-19(a), 6-19(b), and 6-20 show the change in reservoir oil saturation with oil production time for the cases of G440T0 (water flooding starts at time zero) and G440T7 (water flooding starts at day 2,555 or year seven). The scale for the oil saturation is kept unchanged so that the change of oil saturation is observed clearly with the color scale.

Images (1) and (2) in Figure 6-19(a) show that, for the case of G440T7, during seven years of primary oil production, there is no significant change in reservoir oil saturation, both in distribution and values. Image (3) in Figure 6-19(a) shows the oil saturation distribution after 40 days of water flooding. The four corners of the reservoir have the lowest oil saturation. The low oil saturation zones expand quickly with the increase of water injection time. About three years after water injection, the oil saturation is kept unchanged only along the two center cross bands of the reservoir, as shown in Image (5) of Figure 6-19(b); for the rest of the reservoir, the oil saturation decreases dramatically. At day 4,995 of oil production, the oil saturation for the entire reservoir decreases very quick with less than 7 years of water injection, as shown in Image (6) of Figure 6-19(b).

Figures 6-19 and 6-20 show that after water injection, the oil saturation of the reservoir has three zones: the low oil saturation zone, the intermediate oil saturation zone, and the high oil saturation zone. With the increase of water injection time, the low oil saturation zone expands and the high oil saturation zone shrinks. In addition, the absolute values of oil saturation in both the high oil saturation zone and intermediate oil saturation zone become less and less. Finally, the reservoir oil saturation becomes two zones with the disappearance of the high oil saturation zone.

Figure 6-21 shows the comparison of average reservoir oil saturation change with oil production time for eight cases, $i.e.,$ G440T0, G440T1, G440T2, ..., and G440T7, of different water flooding switching times. The water flooding switching time for each of the eight cases is from year zero to year seven, respectively, as described in Table 6-7. In all the eight cases, the average reservoir oil saturation decreases slightly during the primary oil production. In the first several years of water injection, the average reservoir oil saturation decreases very quickly. Afterwards, reservoir oil saturation changes much slowly. After a certain number of years of water flooding, average reservoir oil saturation changes very little with the increasing of water injection time. Figure 6-22 gives a closer look of the average reservoir oil saturation change for two cases of G440T0 and G440T7. For the

G440T7 case, average reservoir oil saturation changes less than 0.05 in the first seven years of primary oil recovery, which indicates a limitation of oil recovery efficiency from primary oil recovery. Table 6-8 contains a detailed analysis of the average reservoir oil saturation change with oil production time for the case of G440T0. After water injection, the average reservoir oil saturation changes dramatically in the first 3.72 years (1,360 days). The average reservoir oil saturation decreases from 0.7 to 0.43 in a straight line speed, which means that oil is extracted from the reservoir very quickly in the first 3.72 years of water injection. Then the average reservoir oil saturation reaches to a slow-change period from year 3.72 to year 7.45 (2,720 days). During this time, the average reservoir oil saturation changes from 0.43 to 0.35. After 2,720 days, the average reservoir oil saturation changes very slowly.

The study of the change in reservoir oil saturation with the time of primary and water flooding oil recovery shows that there is a limit in oil recovery efficiency in each stage after a certain period of oil production time. It also indicates that there is an opportunity to make changes in the oil production option when there is very little oil saturation decrease in the reservoir.

6.5.2.3 Reservoir Pressure Change with Oil Production Time

Figures 6-23(a), 6-23(b), and 6-23(c) show the reservoir pressure change with oil production time for the case of G440T7 (water flooding starts at year seven). Figures 6-23(a) and 6-23(b) use the same color scale while Figure 6-23(c) changes to another scale. Figure 6-23(a) shows the reservoir pressure change in the primary oil recovery stage. The reservoir pressure decreases very quickly from the initial reservoir pressure of 4,000 psi for the first three years of primary oil production, as shown from Images (1) to (8) in Figure 6-23(a). However, from day 1,040 to day 2,555, in about four years, the reservoir pressure changes very little, as shown in Images (8) and (9) in Figure 6-23(a). The two images look very similar, suggesting that the reservoir pressure is close to the bottom hole pressure (BHP), indicating a very small driving force for oil production after day 1040 in the primary oil production.

Figure 6-23(b) shows the first stage of the reservoir pressure change after water flooding. The reservoir pressure increases quickly with the increase of water injection time. Figure 6-23(c) shows the second stage of the reservoir pressure change after water flooding. Image (18) in Figure 6-23(b) and Image (19) in Figure 6-23(c) are the same reservoir pressure with different color scales. Reservoir pressure reaches the highest at day 4,595, as shown in Image (24) in Figure 6-23(c), and decreases afterwards with the increase of oil production time.

Figure 6-24 shows the pattern of reservoir pressure change with oil production time for the case of G440T0 (water flooding starts at time zero). The reservoir pressure first increases with the injection of water, reaching the highest level at 2,040 days as shown in Image (5) in Figure 6-24. It then decreases after the pressure peak.

All of the images in Figures 6-23 and 6-24 show that the center of the reservoir, where the producer is located, has the lowest pressure, and the four corners, where the injectors are located, have the highest pressure. The difference in pressure between the producer and injectors is the driving force for oil production as described through Darcy's law in Eq. (6-4).

Figure 6-25 shows the average reservoir pressure change with oil production time for the selected eight cases, which are G440T0, G440T1, G440T2, ..., and G440T7, of different water flooding switching times. For each of the eight cases, water flooding starts from year zero to year seven, respectively. During the primary oil production, average reservoir pressure decreases towards the BHP of 500 psi. During the water flooding oil production, the average reservoir pressure first increases to a maximum reservoir pressure, and then decreases until it reaches a pressure of about 4,000 psi. The point of water breakthrough for each oil production case is shown in the average reservoir pressure curve. After water breakthrough, reservoir pressure continues to increase towards the maximum reservoir pressure. Unlike primary oil production, reservoir pressure is not the controlling factor for water flooding oil production. As discussed in later section, water cut is the controlling factor for water flooding oil production.

6.5.3 Production Profile from Reservoir Simulations

Figure 6-26 shows the oil production rate change with the oil production time for the 29 designed cases of different water flooding switching times, as shown in Table 6-7. For the primary oil production, oil production rate decreases with the production time. At the beginning of water flooding, the oil production rate first increases until reaching the highest level, and then decreases rapidly. Afterwards, the oil production rate continues to decrease but more gradually. Figure 6-27 provides a closer look at the oil production rate change for the three cases of different water flooding switching times.

Figure 6-28 shows the cumulative oil production (in $\% \, OOIP$) change with oil production time for the eight cases, which are G440T0, G440T1, G440T2, ..., and G440T7, of different water flooding switching times. For each of the eight cases, water flooding starts from year zero to year seven, respectively, as shown in Table 6-7. Among the eight cases, the maximum cumulative oil recovery from primary oil recovery is about 10% of $OOIP$; and the maximum cumulative oil recovery from both primary and water flooding oil production is less than 60% of the $OOIP$.

Figure 6-29 shows water cut (volume percentage of water in total produced fluids) change with oil production time for the eight cases mentioned above. For each case, before water breakthrough, water cut remains zero since it takes time for water to travel from injectors to the producer after water flooding starts. Once water breakthrough happens, water cut increases very quickly with the injection of water. Water cut increases slowly after about 90% water cut is reached.

Table 6-9 summarizes the oil production time, total cumulative oil production (in $\% \, OOIP$), and average reservoir pressure at the time when 99% water cut first reaches for the 15 cases of different water flooding switching times from time zero to year seven, respectively, with a half-year increment in between. Table 6-9 shows that when 99% water cut first reaches, the total production time is from 36.60 years for the case when water flooding starts at time zero to 42.62 years for the case in which water flooding starts from year seven; the cumulative oil recovery ranges from 56.21% to 56.26% of $OOIP$; and the average reservoir pressure is about 4,280 $psi,$ which is much higher than the BHP.

Figure 6-30 shows the relationship among changes in oil production rate, cumulative oil recovery, and average reservoir pressure for the case of G440T7 for 50 years of oil production, in which there are seven years of primary oil recovery and 43 years of water flooding oil recovery. Figure 6-30 shows that before water flooding starts, both oil production rate and average reservoir pressure decrease with oil production time. Once water flooding starts, oil production rate, cumulative oil recovery, and average reservoir pressure all increase rapidly until water breakthrough happens. After water breakthrough, oil production rate starts to decrease dramatically; cumulative oil recovery continues to increase; and average reservoir pressure continues to increase until reaches the highest level. After the average reservoir reaches the highest value, it starts to decrease with the oil production time. At the same time, the changing rates of both the cumulative oil recovery and oil production rate become much smaller.

Figures 6-31 demonstrates the relationship among changes in oil production rate, water cut, and average reservoir oil saturation for the case of G440T7. After water flooding and before water breakthrough, oil production rate increases very quickly with the oil production time. Thus, average reservoir oil saturation decreases very quickly with oil production time because of the rapid oil production. Since it takes time for the injected water to reach the oil producer, there is no injected water in the produced fluids before water breakthrough. After about four years of water flooding, water breakthrough happens. Since then, the injected water occurs in the produced fluids and thus the oil production rate decreases dramatically; water cut increases dramatically; and average reservoir oil saturation decreases much slowly because oil is extracted much slowly. Once the injected water reaches all the oil producers from all directions and in all layers of reservoir after water breakthrough, the entire reservoir is immersed by the injected water. Since then, the average reservoir pressure starts to decrease; and the changes with the increase of oil production time, in water cut, oil and water production rates, and average reservoir oil saturation, all become smaller and smaller, as shown in Figures 6-31 and 6-32. Water cut is the controlling factor for the water flooding oil production.

Figure 6-32 illustrates the relationship among oil production rate, water production rate, and average reservoir pressure for the case of G440T7. When oil production reaches its limits, that is, pressure limit for primary oil recovery and water cut limit for water flooding oil recovery, continuing with same production option will results in low oil production efficiency, as seen in changes of the reservoir oil saturation and the cumulative oil recovery with the oil production time. Challenges arise when trying to find out the best time to switch from one production option to another, both in the stages of primary and secondary oil production, so that the maximum value of the oil reservoir can be obtained with all possible states of future oil prices being considered. The following chapters are about the switching options from primary oil production to water flooding oil recovery.

Table 6-1: Correlations between Permeability and Porosity for TORIS Data before and after Separating the Small Group Permeability Data

	Correlation Coefficient before Separating the Small Group Permeability Data	Correlation Coefficient after Separating the Small Group Permeability Data
Perm vs Porosity	0.3315	0.3587
Ln(Permeability) vs Porosity	0.6608	0.7157
Ln(Permeability) vs Ln(Porosity)	0.3918	0.6863

Table 6-2: Summary Output for the Regression of Initial Oil Formation Volume Factor (RB/STB) on Oil API Gravity and Reservoir Vertical Depth

Regression Statistics	
Multiple R	0.705381065
R Square	0.497562447
Adjusted R Square	0.49584177
Standard Error	0.146021295
Observations	587

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	12.33135337	6.165677	289.1668	5.20E-88
Residual	584	12.45217572	0.021322		
Total	586	24.78352909			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.812686463	0.023519404	34.55387	2.60E-143
Oil API Gravity	0.008349973	0.000767994	10.87244	3.36E-25
Reservoir Vertical Depth, ft	3.32E-05	2.25E-06	14.75242	4.28E-42

$$FVF\ \left({}^{RB}/_{STB}\right) = 0.813 + 0.00835 \times {}^{\circ}API + (3.32E-5) \times Depth\ (ft)$$

Table 6-3: General Reservoir Field Properties

Reservoir Size	2640 ft × 2640 ft × 60 ft
Simulation Grid Block Size	64.4 ft × 64.4 ft × 20 ft
Number of Simulation Grid Blocks	41 × 4 1 × 3
Reservoir Vertical Depth	4370 ft
Reservoir Initial Pressure	4000 psi
Reservoir Rock Type	Carbonate
Initial Oil Saturation	Normally distributed: Mean=70%, Standard deviation=10%
Reservoir Permeability	Log normally distributed: Mean =172mD, $V_{DP} = 0.7$
Reservoir Porosity	Random variable
Reservoir Formation Compressibility	0.000004 psi^{-1}
Oil Compressibility	0.00002 psi^{-1}
Water Compressibility	0.000001 psi^{-1}

Table 6-4: Properties for Reservoir Fluids under Different Temperatures

Fluid Properties	Temperature, °F(℃)	
	60 (15.6)	130 (54.4)
Crude Oil API Gravity	40	40
Crude Oil Specific Gravity (Reference Water Density at 60 °F)	0.825	0.805
Water Density, g/cm^3 (psi/ft)	0.999 (0.4331)	0.986 (0.4275)
Crude Oil Density, g/cm^3 (psi/ft)	0.825 (0.3577)	0.805 (0.3490)
Water Kinematic Viscosity, CentiStokes (cSt)	1.13	0.55
Water Dynamic Viscosity, CentiPoise (cp)	1.13	0.54
Crude Oil Kinematic Viscosity, CentiStokes (cSt)	9.7	3.5
Crude Oil Dynamic Viscosity, CentiPoise (cp)	8.00	2.82

Sources:
http://www.simetric.co.uk/si_liquids.htm
http://www.engineeringtoolbox.com/specific-gravity-liquid-fluids-d_294.html
http://en.wikipedia.org/wiki/Density
http://www.csgnetwork.com/sgvisc.html
http://www.engineeringtoolbox.com/water-dynamic-kinematic-viscosity-d_596.html
http://www.engineeringtoolbox.com/viscosity-converter-d_413.html

Table 6-5: Reservoir Permeability Heterogeneity Field Generated from the MDM Program (Input: Mean=172mD, V_{DP}=0.7)

SAMPLE MEAN = 174.7mD, STANDARD DEVIATION = 320.9mD
MEAN (lnk) = 4.423 , SD(lnk) = 1.209
CV = 1.837, V_{DP} = 0.7016

Layer = 1
MEAN = 183.7mD, SD = 368.0 mD
MEAN (lnk) = 4.473, SD(lnk) = 1.193
CV = 2.003, V_{DP} = 0.6967

Layer = 2
MEAN = 171.2mD, SD = 286.3 mD
MEAN (lnk) = 4.414, SD(lnk) = 1.208
CV = 1.672, V_{DP} = 0.7011

Layer = 3
MEAN = 169.1mD, SD = 302.5 mD
MEAN (lnk) = 4.381, SD(lnk) = 1.226
CV = 1.789, V_{DP} = 0.7064

Table 6-6: Well Constraints for Oil Production Well and Water Injection Wells

Well Type	Constraint Type	Constraint Value
Oil Production Well	Pressure Constraint	Bottom Hole Pressure: 500 psi
Water Injection Well	Rate Constraint	Water Injection Rate: 4000 bbl/day

Table 6-7: Water Flooding Switching Schedule for the 29 Oil Production Cases

Case Number	UTCHEM Run Case Name	Switching Time, Year	Switching Time, Days	Time Length between Two Switching Time, Days
1	G440T0	0.00	0	
2	G44002	0.25	91	91
3	G44005	0.50	182	91
4	G44007	0.75	274	92
5	G440T1	1.00	365	91
6	G44012	1.25	456	91
7	G44015	1.50	547	91
8	G44017	1.75	639	92
9	G440T2	2.00	730	91
10	G44022	2.25	821	91
11	G44025	2.50	912	91
12	G44027	2.75	1004	92
13	G440T3	3.00	1095	91
14	G44032	3.25	1186	91
15	G44035	3.50	1277	91
16	G44037	3.75	1369	92
17	G440T4	4.00	1460	91
18	G44042	4.25	1551	91
19	G44045	4.50	1642	91
20	G44047	4.75	1734	92
21	G440T5	5.00	1825	91
22	G44052	5.25	1916	91
23	G44055	5.50	2007	91
24	G44057	5.75	2099	92
25	G440T6	6.00	2190	91
26	G44062	6.25	2281	91
27	G44065	6.50	2372	91
28	G44067	6.75	2464	92
29	G440T7	7.00	2555	91

Table 6-8: Average Reservoir Oil Saturation Change with Oil Production Time for the G440T0 Case (Water Flooding Starts at Time Zero)

	Water Injection Time Duration, day		
	0~1360	1360~2720	2720~6960
Time Length of Water Injection, year	3.726	3.726	11.626
Average Reservoir Oil Saturation, (ft^3/ft^3)	0.7~0.432	0.4232~0.354	0.354~0.321
Changes of Average Reservoir Oil Saturation, %	0.268	0.078	0.033
Rate of Average Reservoir Oil Saturation Change, %/year	0.0713	0.0209	0.0028

Table 6-9: Selected UTCHEM Simulation Results at 99% Water Cut

UTCHEM Run Case Name	Simulation Time, Days	Simulation Time, Years	Cumulative Oil Recovery $(\%OOIP)$	Average Reservoir Pressure (psi)	Water Cut (ft^3/ft^3)
G440T0	13360.0	36.60	56.2	4283.33	0.99
G44005	13422.2	36.77	56.2	4284	0.99
G440T1	13565.1	37.16	56.2	4281.91	0.99
G44015	13667.1	37.44	56.2	4283.4	0.99
G440T2	13810.1	37.84	56.2	4283.53	0.99
G44025	13992.0	38.33	56.3	4282.05	0.99
G440T3	14135.1	38.73	56.3	4283.22	0.99
G44035	14317.0	39.22	56.3	4282.4	0.99
G440T4	14500.1	39.73	56.3	4282.26	0.99
G44045	14642.1	40.12	56.3	4283.86	0.99
G440T5	14825.1	40.62	56.3	4283.54	0.99
G44055	15007.0	41.12	56.3	4283.32	0.99
G440T6	15190.1	41.62	56.3	4283.13	0.99
G44065	15372.1	42.12	56.3	4282.98	0.99
G440T7	15555.0	42.62	56.3	4282.86	0.99

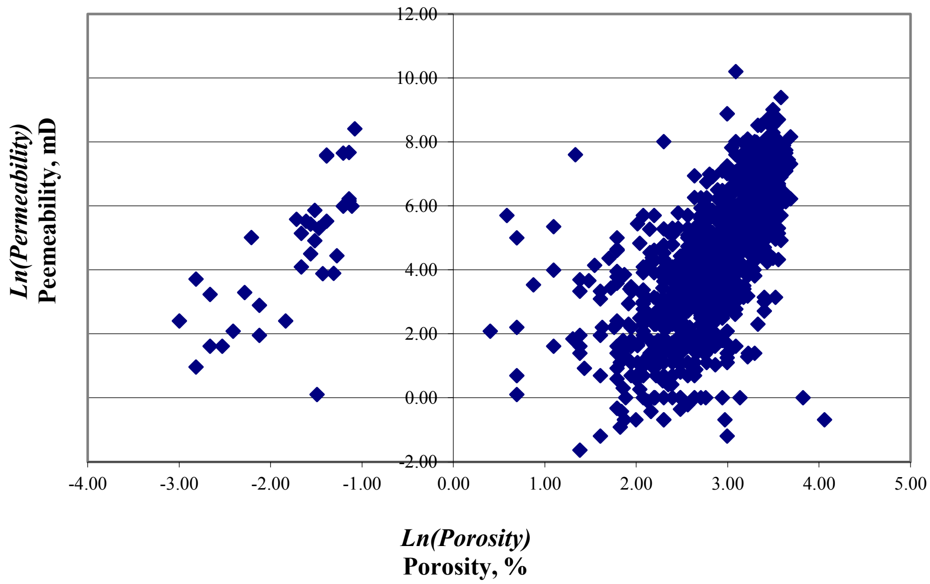


Figure 6-1: Correlation between *Ln(Permeability)* and *Ln(Porosity)* for the Uncorrected TORIS Data

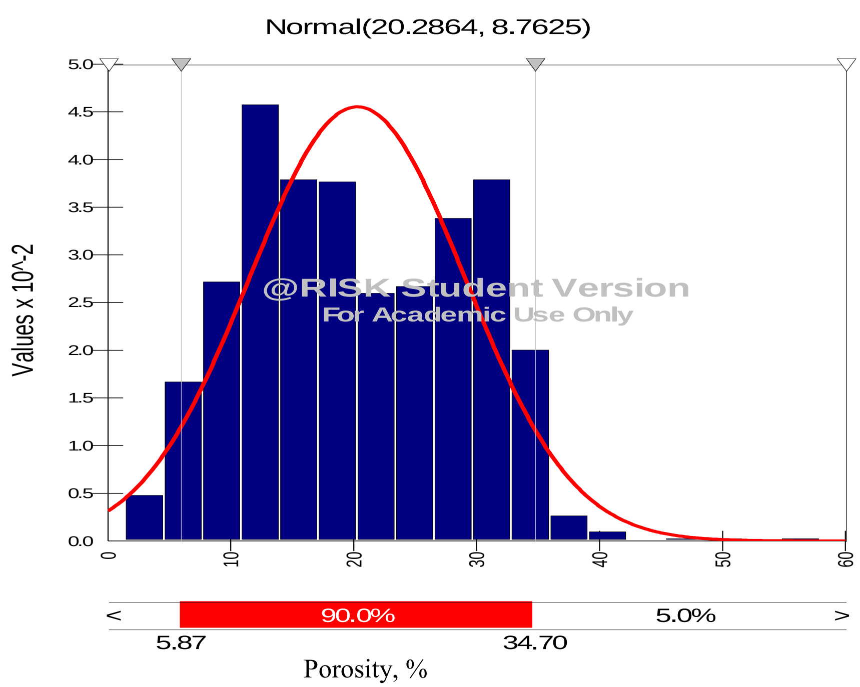


Figure 6-2: Porosity Distribution for TORIS Data

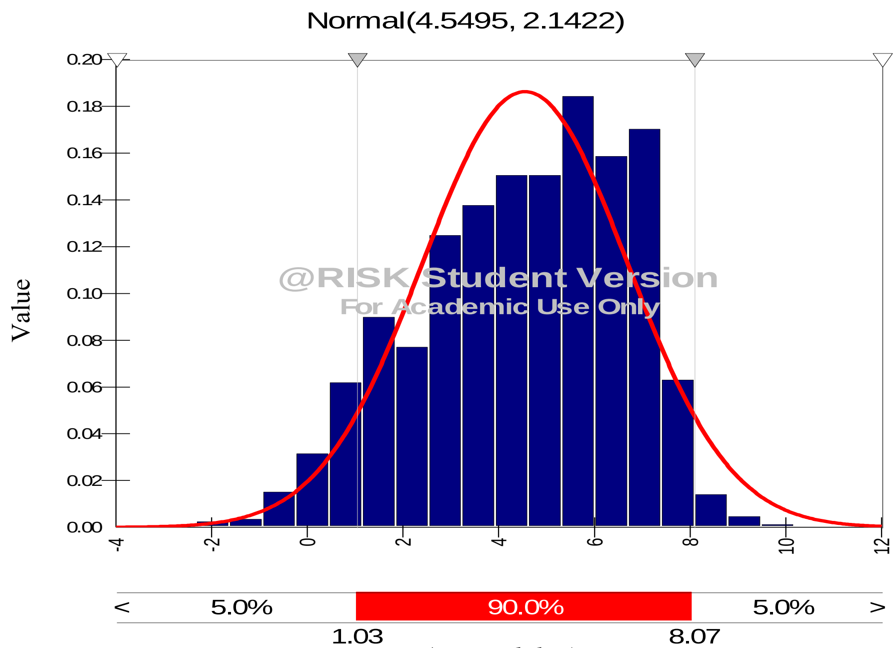


Figure 6-3: *Ln(Permeability)* Distribution for TORIS Data

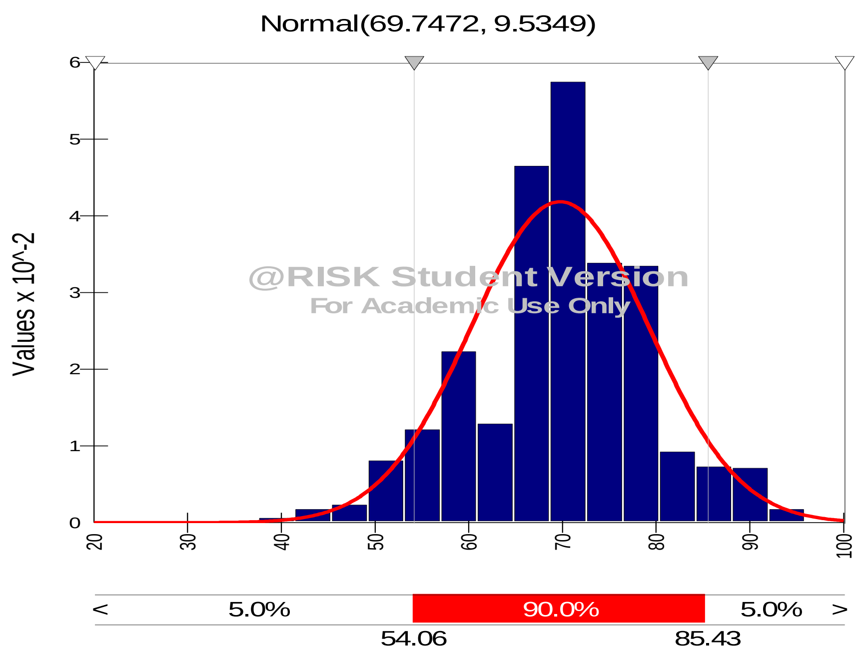


Figure 6-4: Distribution of Initial Oil Saturation for TORIS Data

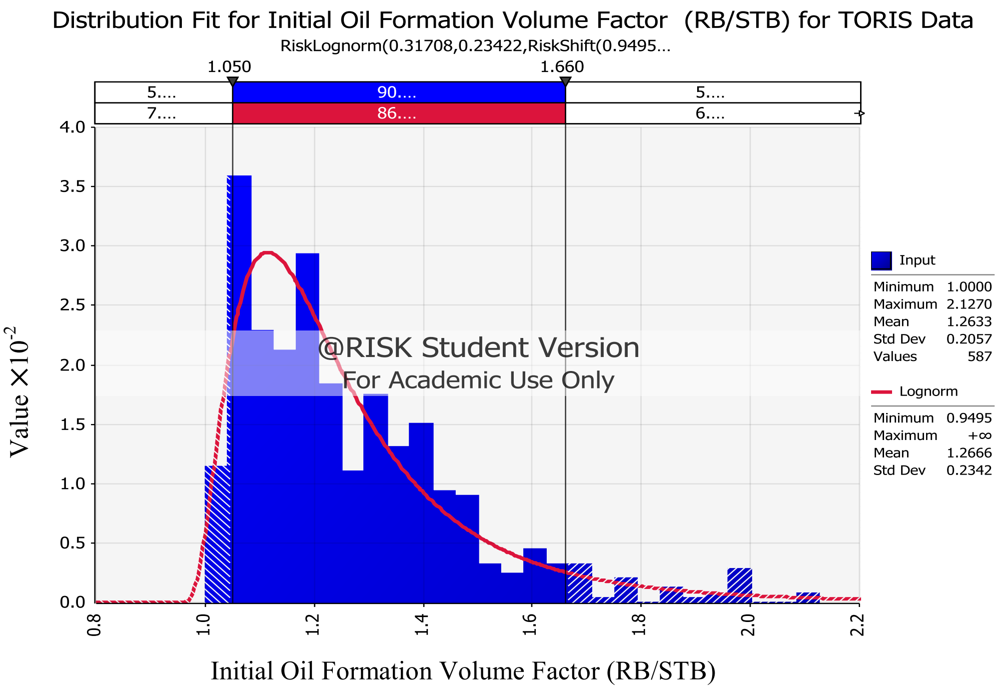


Figure 6-5: Distribution of Initial Oil Formation Volume Factor for TORIS Data

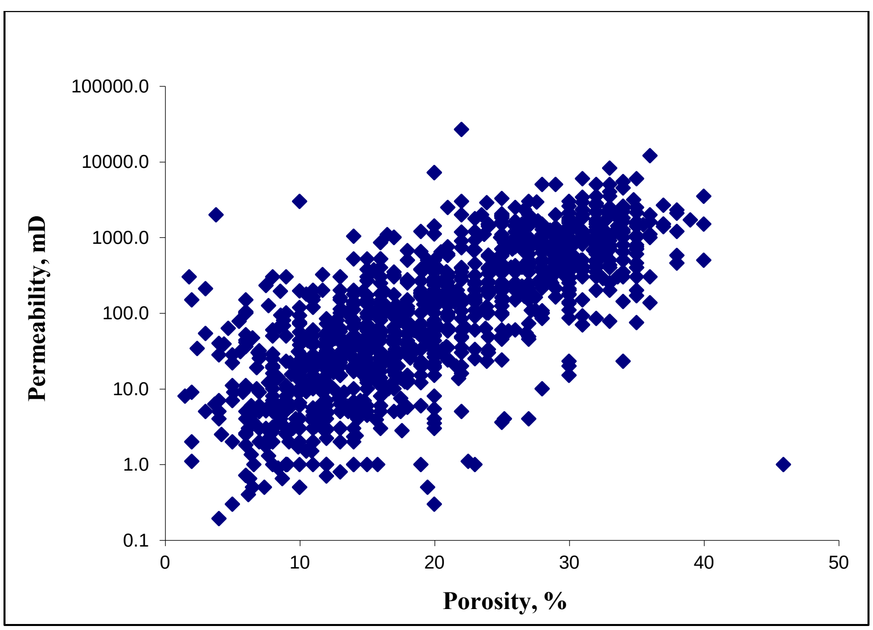


Figure 6-6: Correlation between Permeability and Porosity for the Corrected TORIS Data

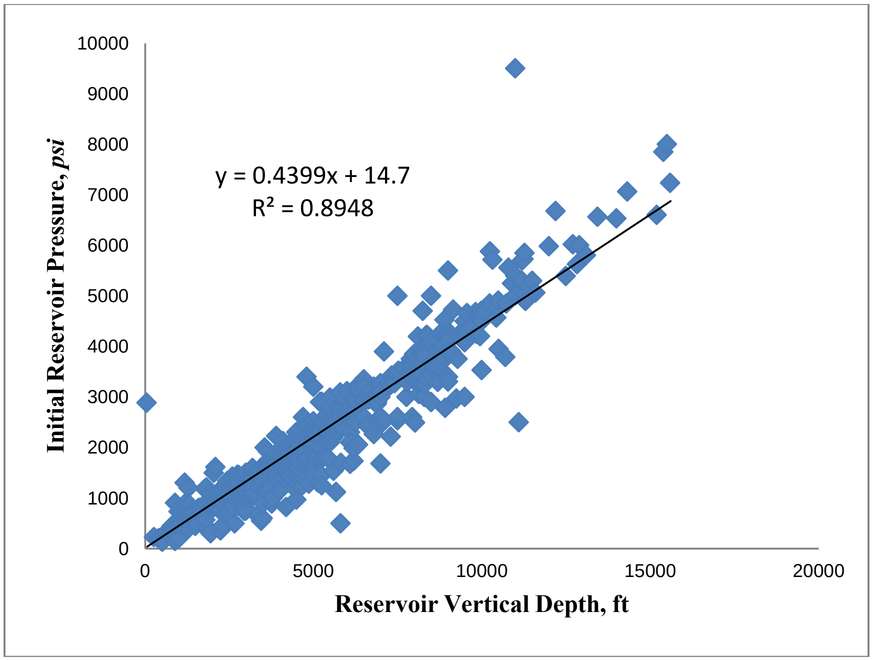


Figure 6-7: Correlation between Initial Reservoir Pressure and Reservoir Vertical Depth for TORIS Data

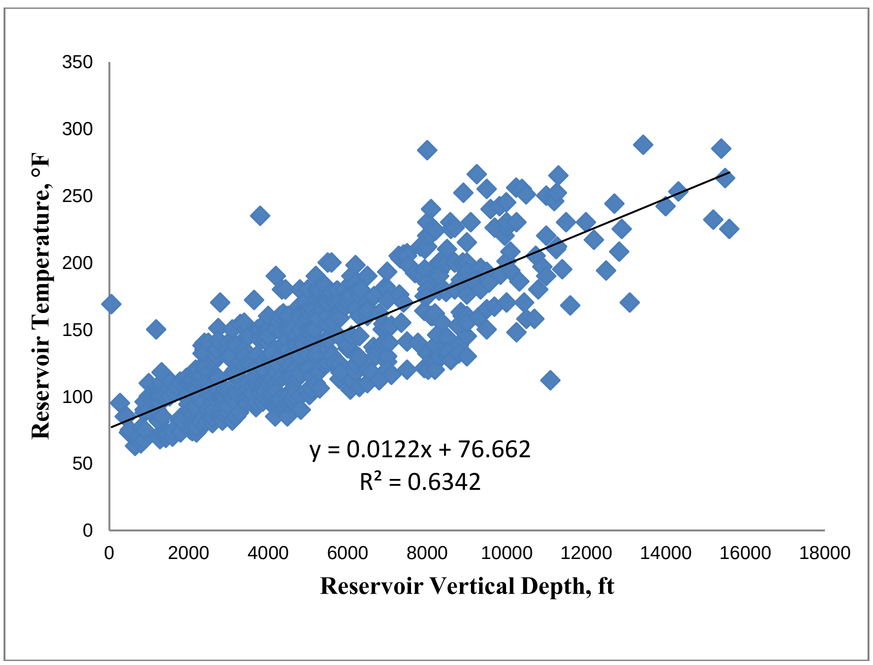


Figure 6-8: Correlation between Reservoir Temperature and Reservoir Vertical Depth for TORIS Data

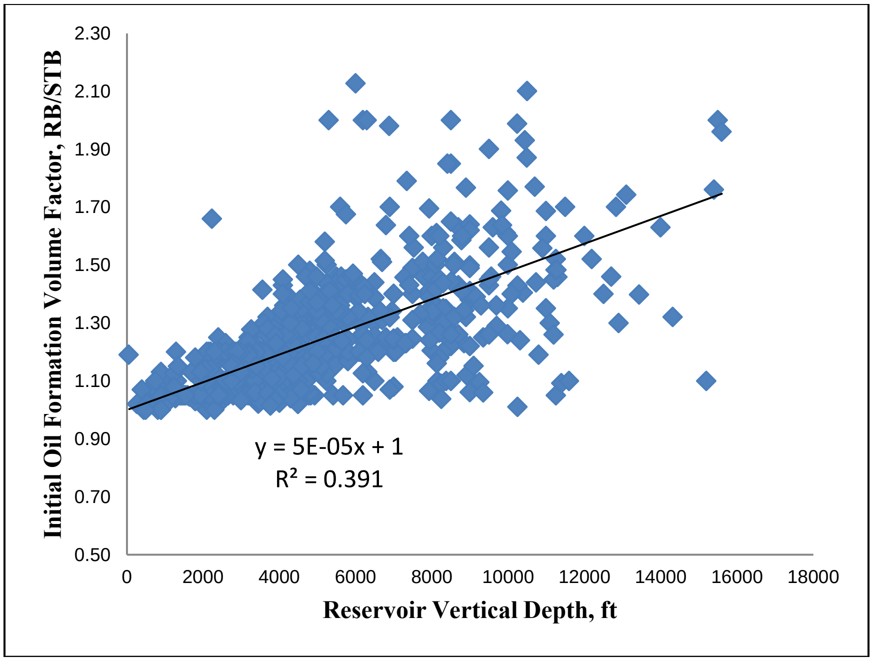


Figure 6-9: Correlation between Initial Oil Formation Volume Factor and Reservoir Vertical Depth for TORIS Data

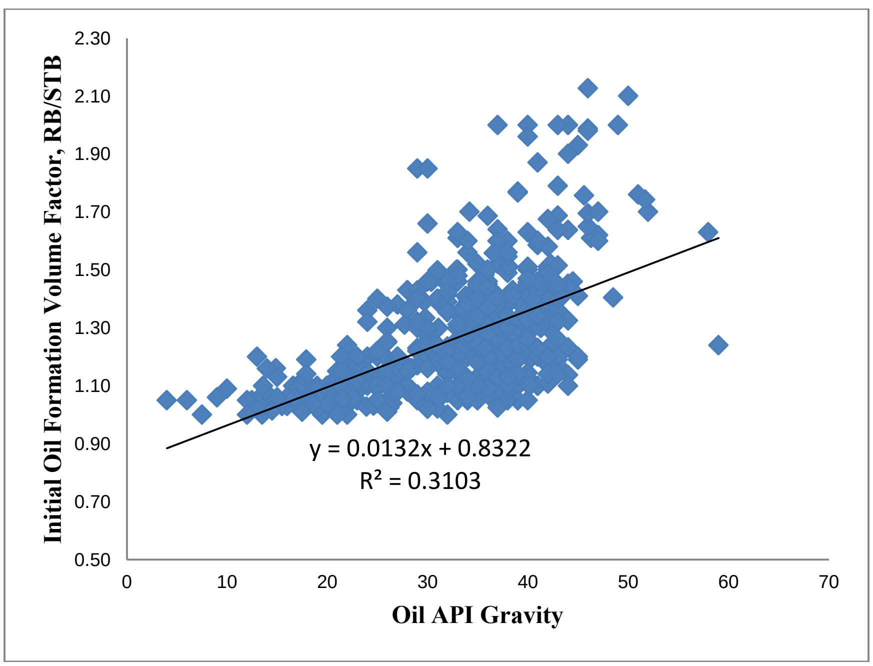


Figure 6-10: Correlation between Initial Oil Formation Volume Factor and Oil API Gravity for TORIS Data

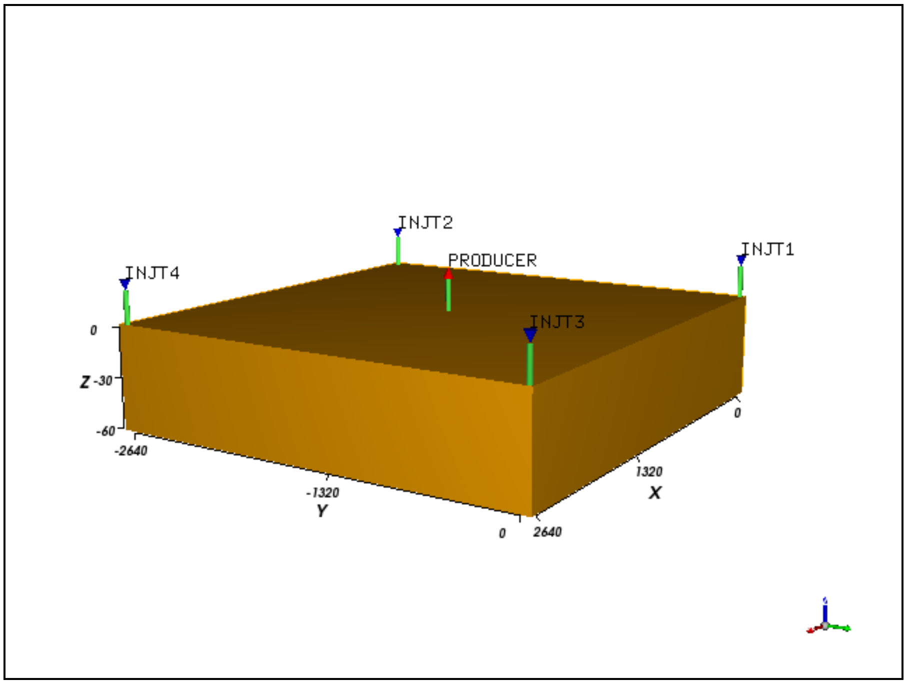


Figure 6-11: 5-Spot Well Configuration for Oil Production

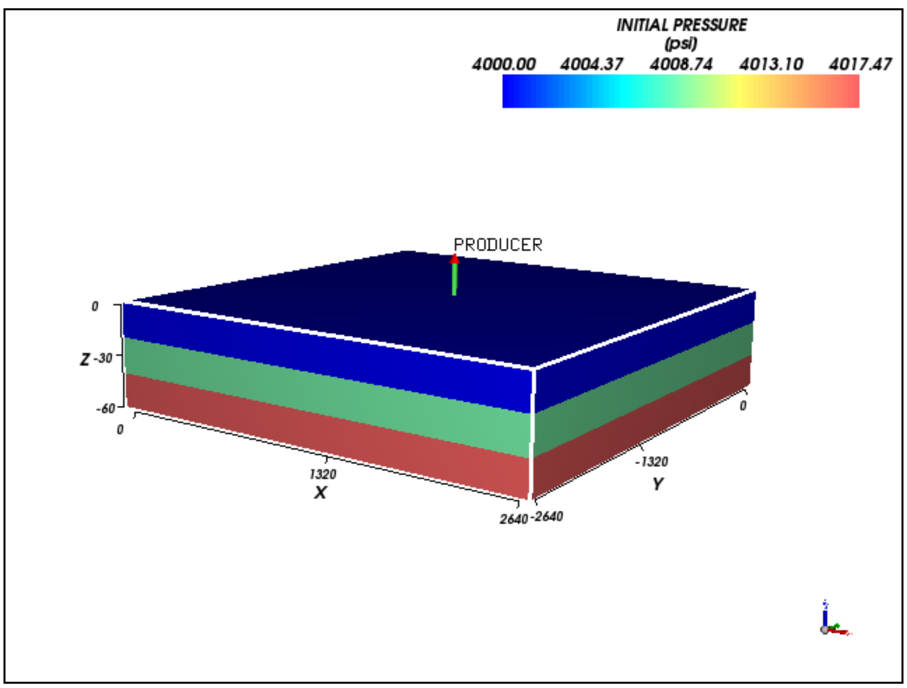


Figure 6-12: Initial Reservoir Pressure

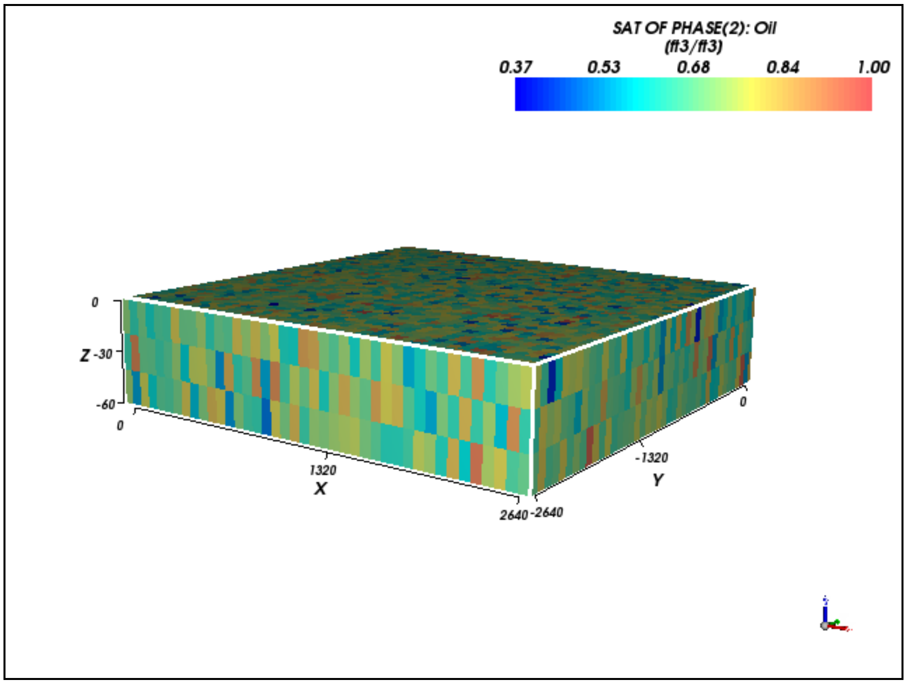


Figure 6-13: Initial Reservoir Oil Saturation

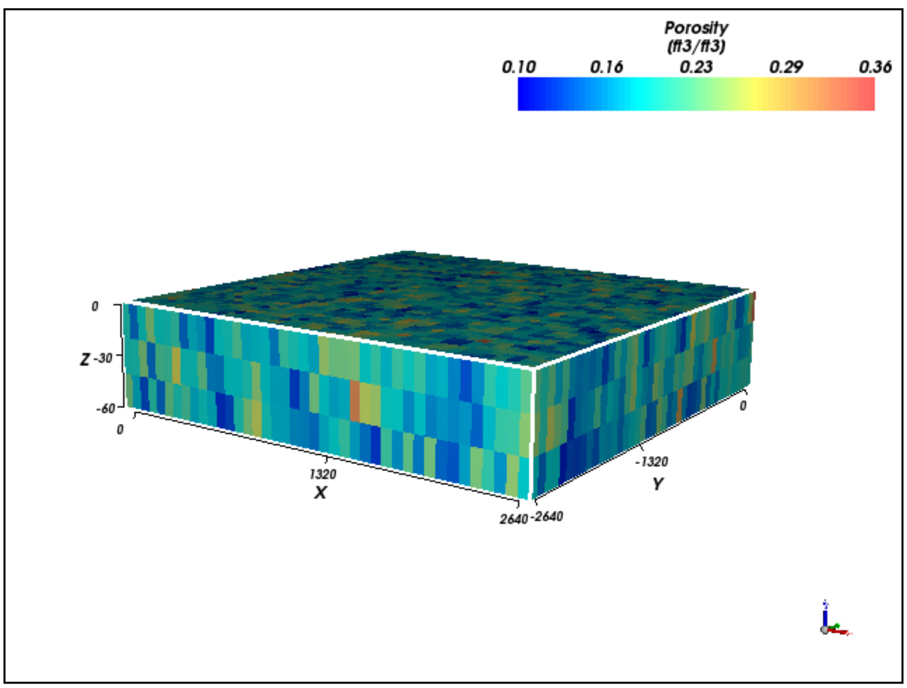


Figure 6-14: Reservoir Porosity

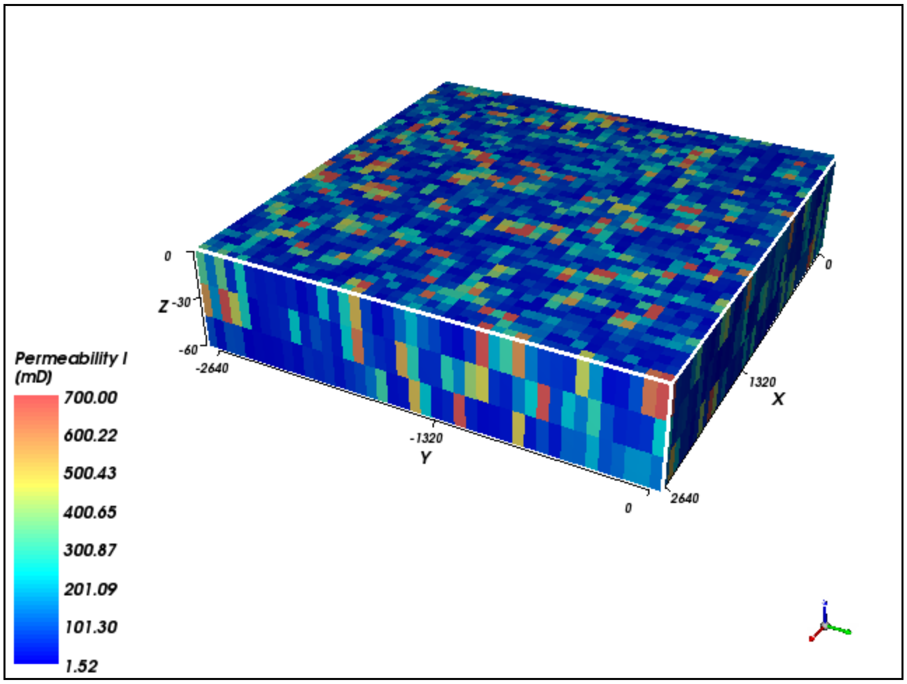


Figure 6-15: Reservoir Permeability in X Direction

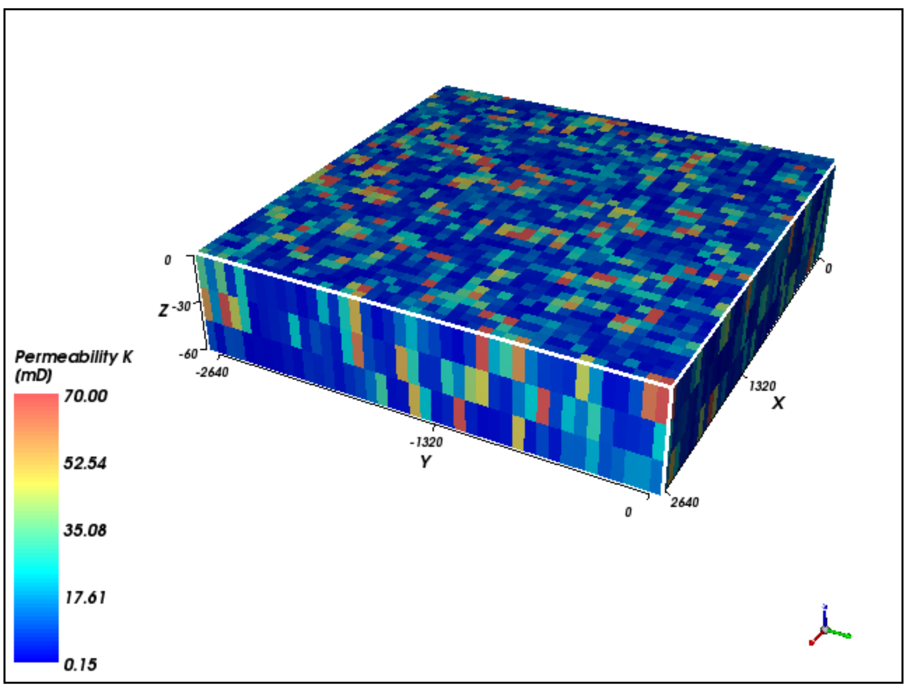


Figure 6-16: Reservoir Permeability in Z Direction

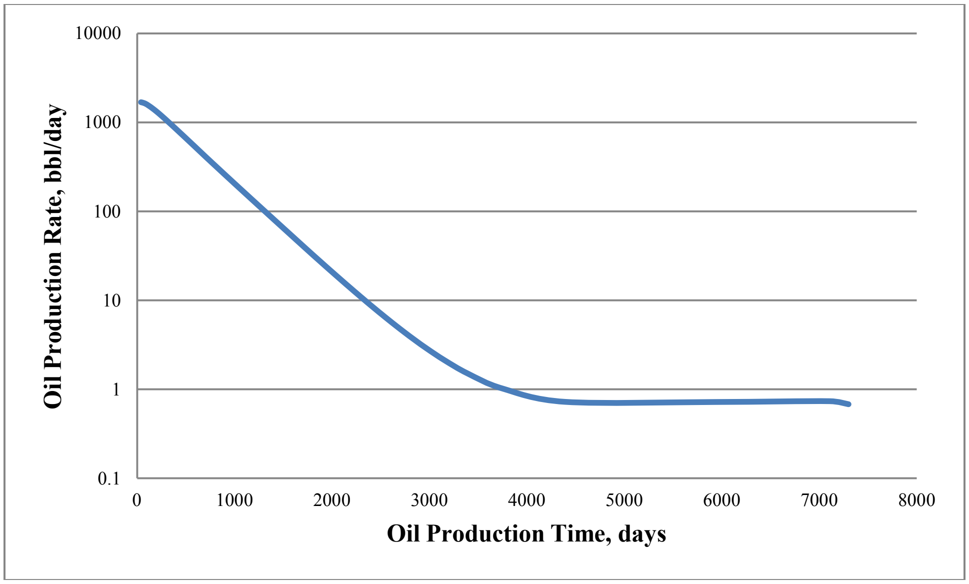


Figure 6-17: Oil Production Rate Change with Production Time in Primary Oil Recovery

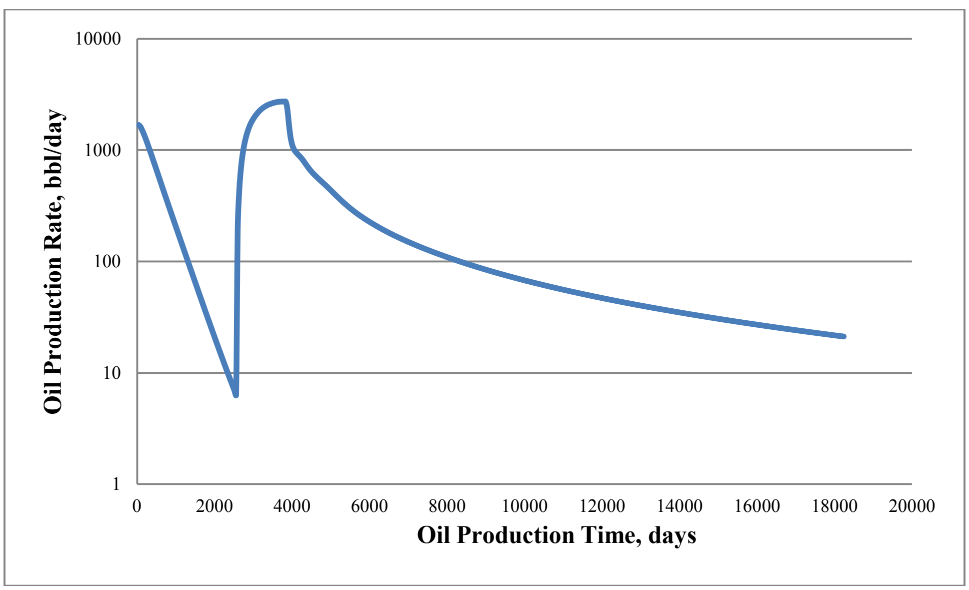
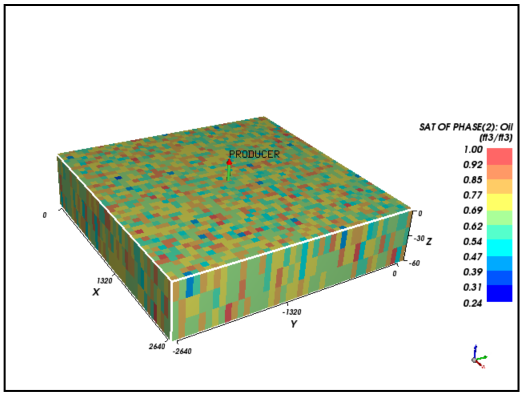
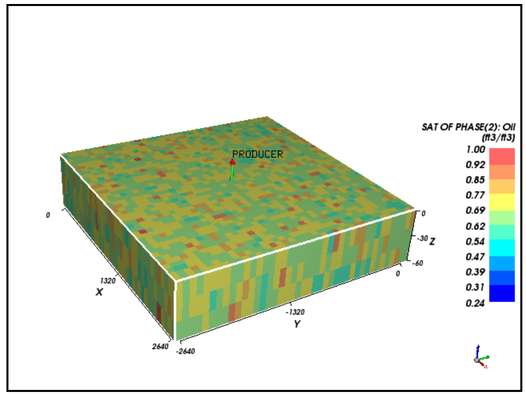


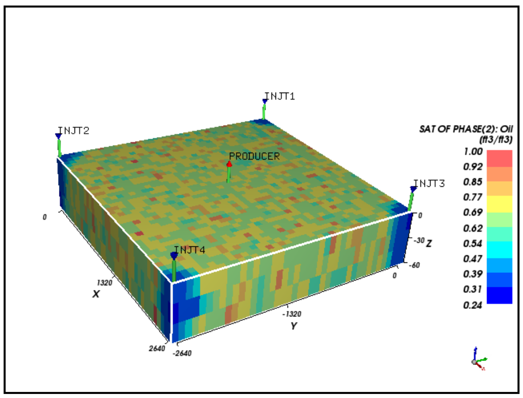
Figure 6-18: Oil Production Rate in Primary and Secondary Oil Recovery for Case G440T7 (Water Flooding Starts at Day 2,555)



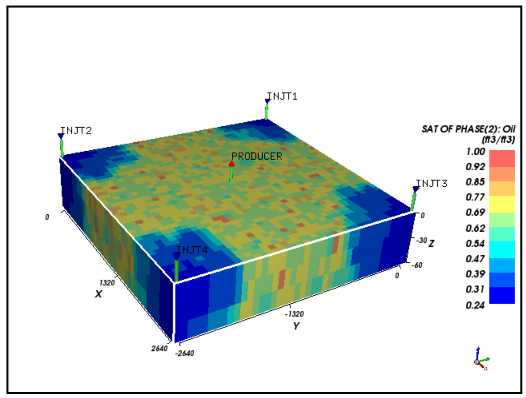
(1) At Day 0



(2) At Day 2,555

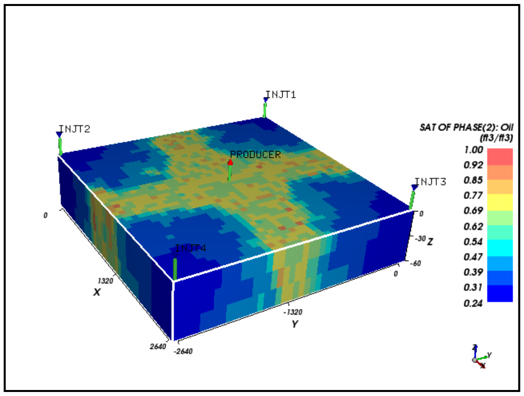


(3) At Day 2,595

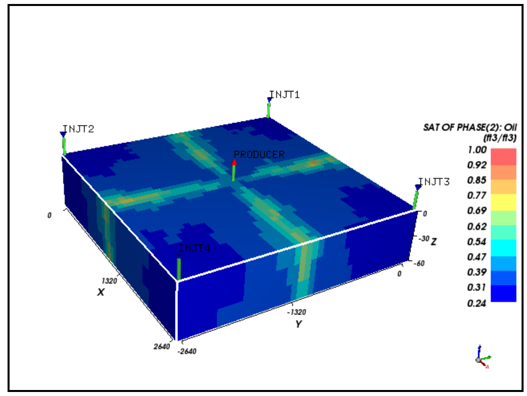


(4) At Day 2,995

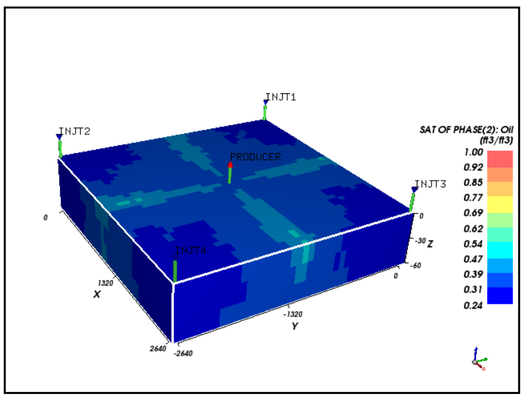
Figure 6-19(a): Reservoir Oil Saturation Change with Oil Production Time for Case G440T7 (Water Flooding Starts at Day 2,555)



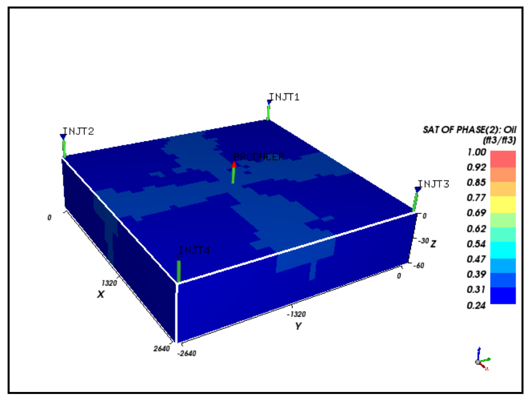
(5) At Day 3,515



(6) At Day 4,995

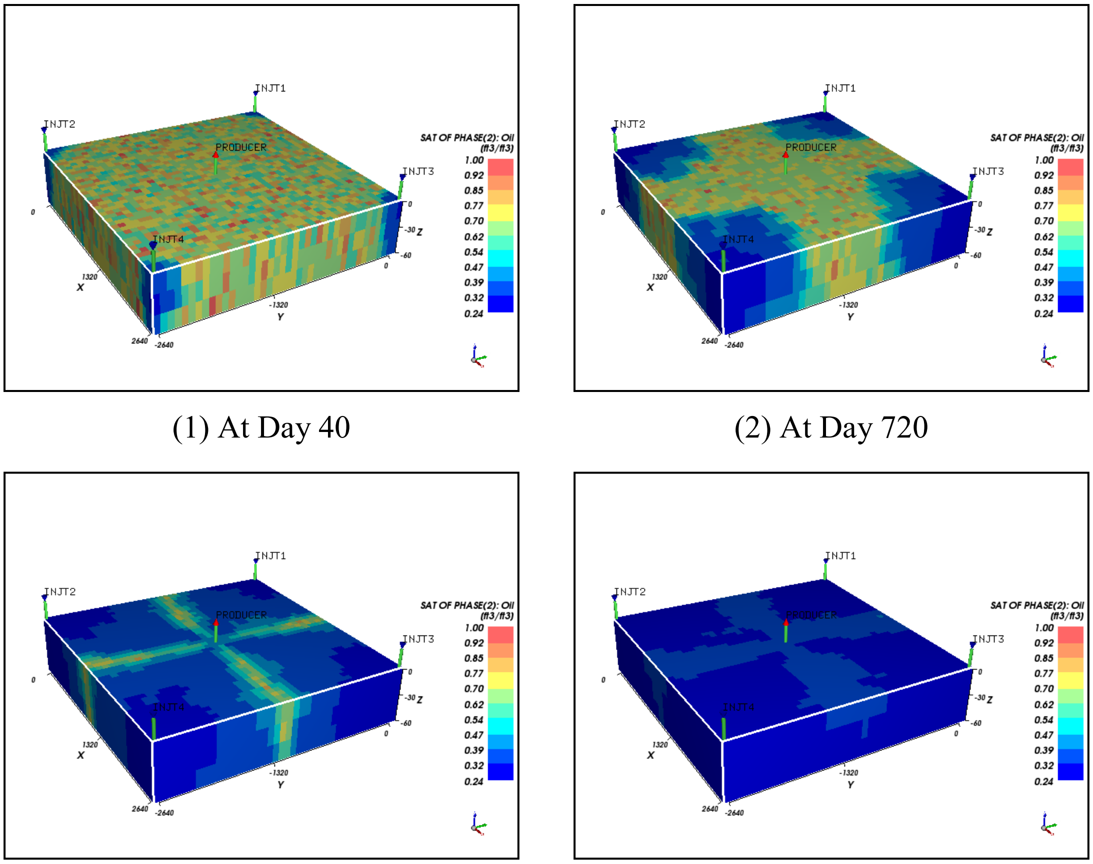


(7) At Day 7,555



(8) At Day 18,250

Figure 6-19(b): Reservoir Oil Saturation Change with Oil Production Time for Case G440T7 (Water Flooding Starts at Day 2,555) - Continued



(1) At Day 40

(2) At Day 720

(3) At Day 2,040

(4) At Day 14,200

Figure 6-20: Reservoir Oil Saturation Change with Oil Production Time for Case G440T0 (Water Flooding Starts at Time Zero)

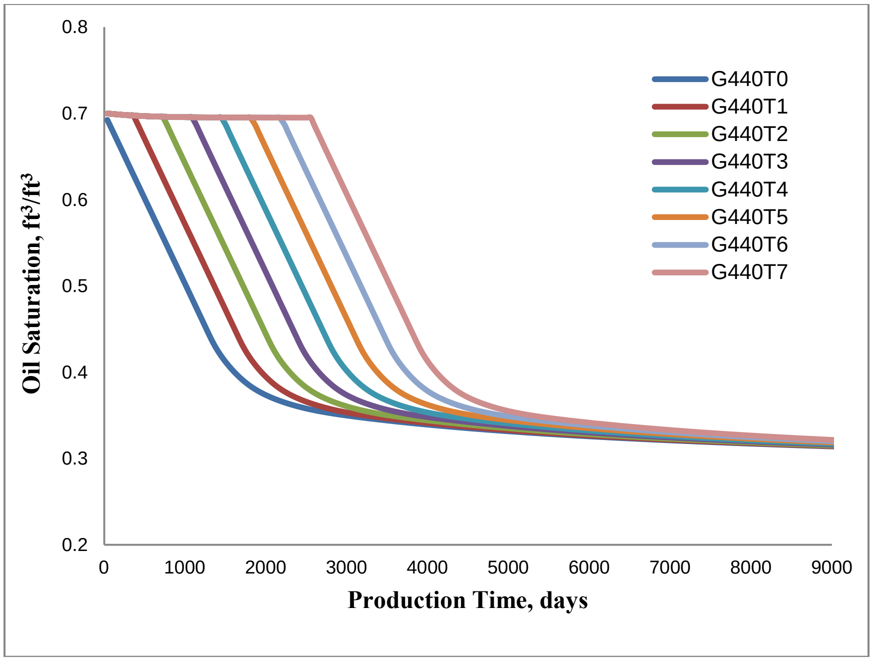


Figure 6-21: Average Reservoir Oil Saturation Change with Oil Production Time for Eight Cases of Different Water Flooding Switching Times

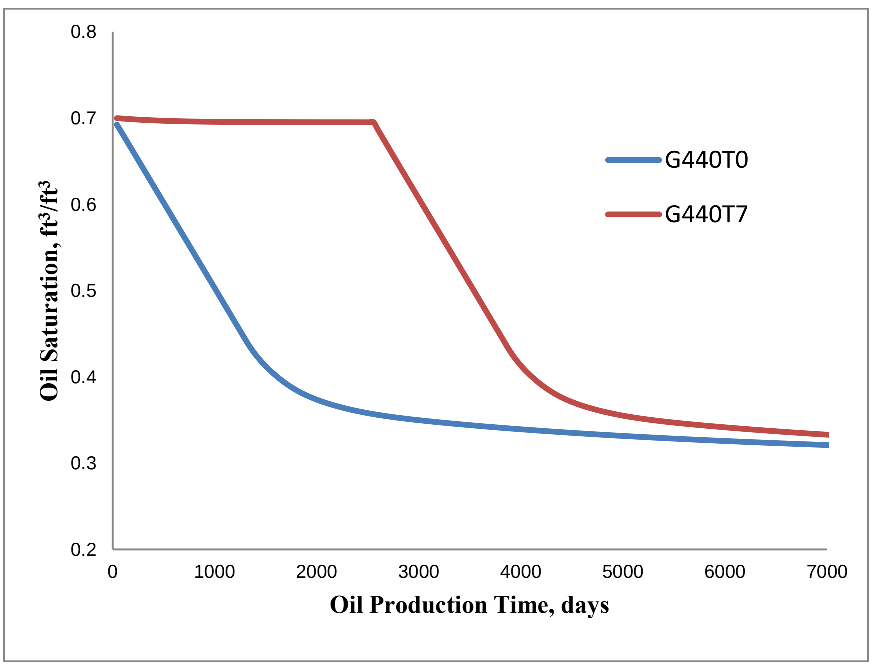


Figure 6-22: A Close Look at Average Reservoir Oil Saturation Change with Oil Production Time for Two Cases Starting Water Flooding at Different Times

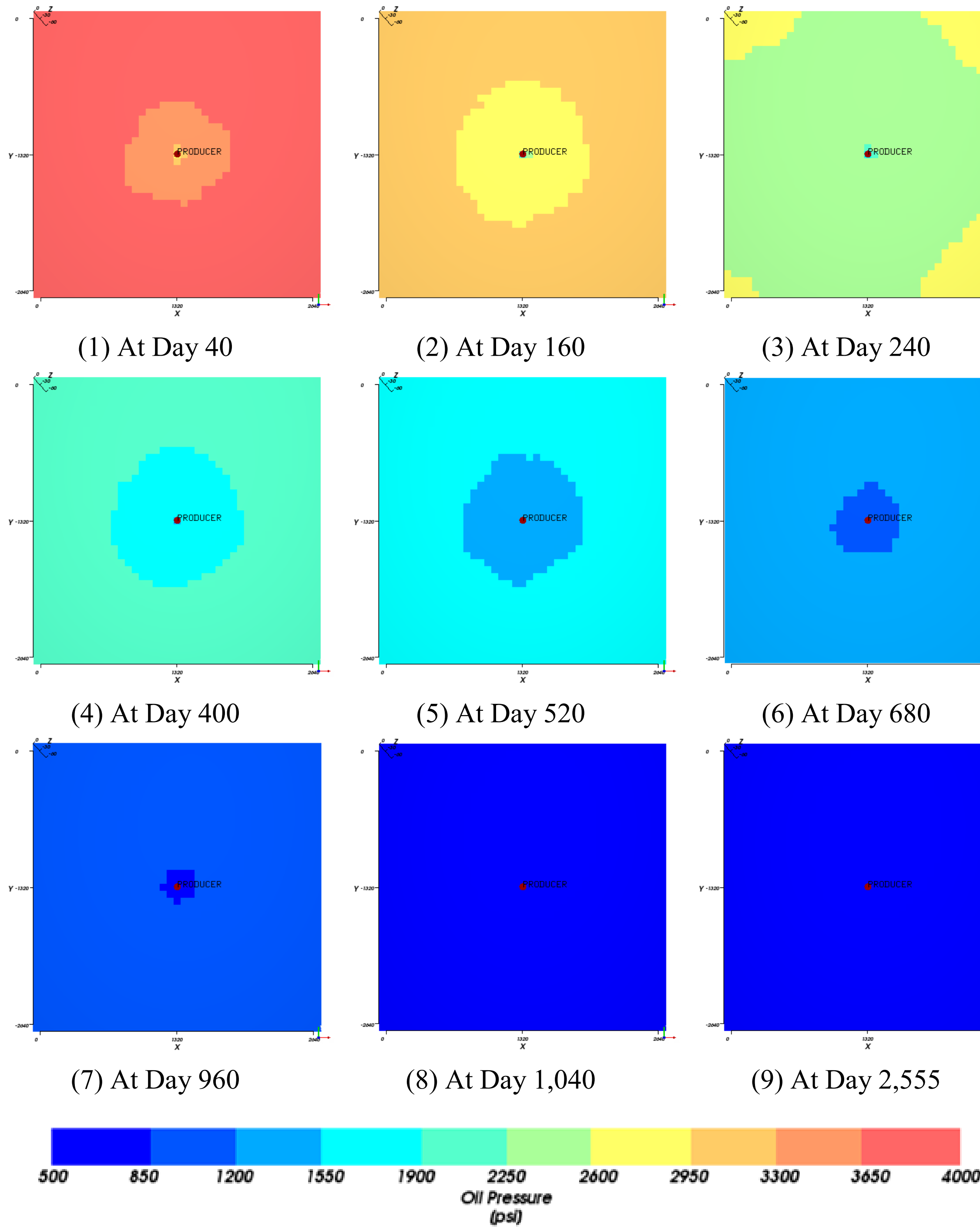
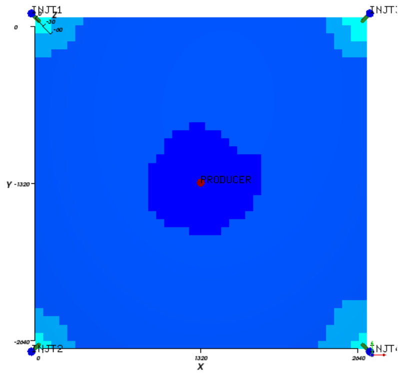
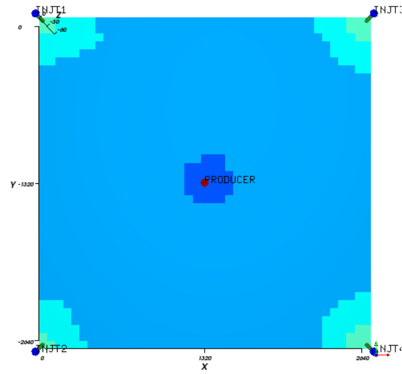


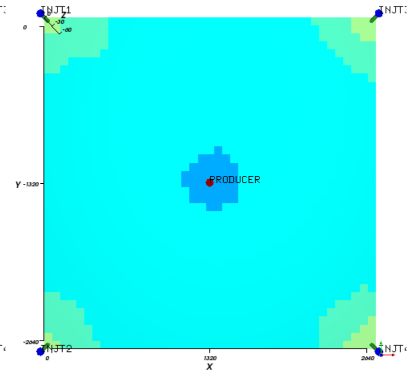
Figure 6-23(a): Reservoir Pressure Change with Oil Production Time for Case G440T7 (Water Flooding Starts at Year 7) - Primary Oil Production Stage



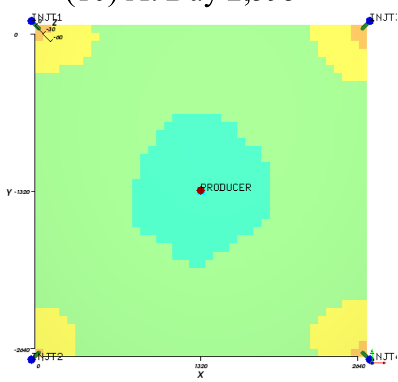
(10) At Day 2,595



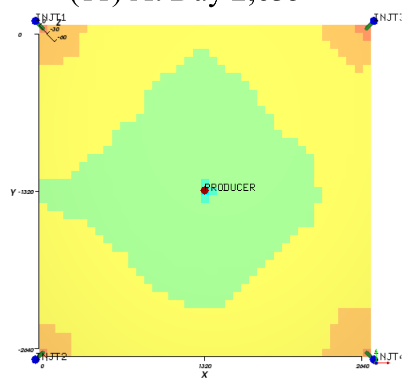
(11) At Day 2,635



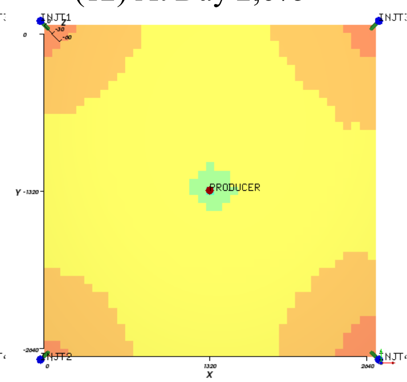
(12) At Day 2,675



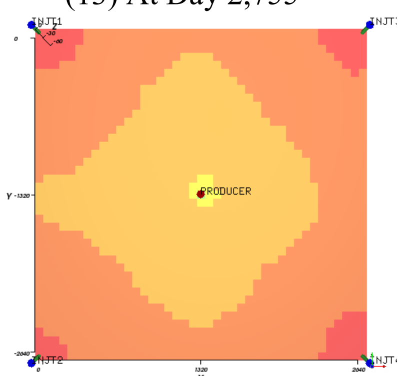
(13) At Day 2,755



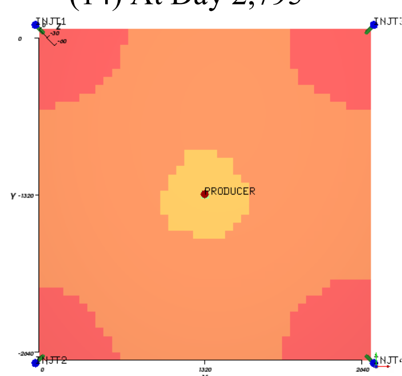
(14) At Day 2,795



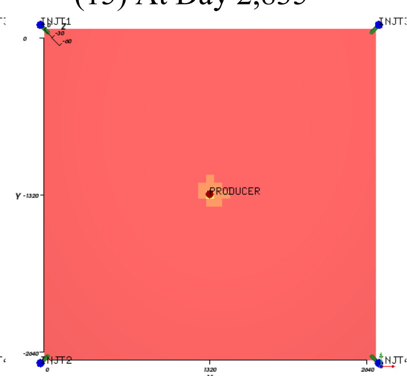
(15) At Day 2,835



(16) At Day 2,915



(17) At Day 2,955



(18) At Day 3,115

Figure 6-23(b): Reservoir Pressure Change with Oil Production Time for Case G440T7 (Water Flooding Starts at Year 7)-Water Flooding Oil Production Stage-1

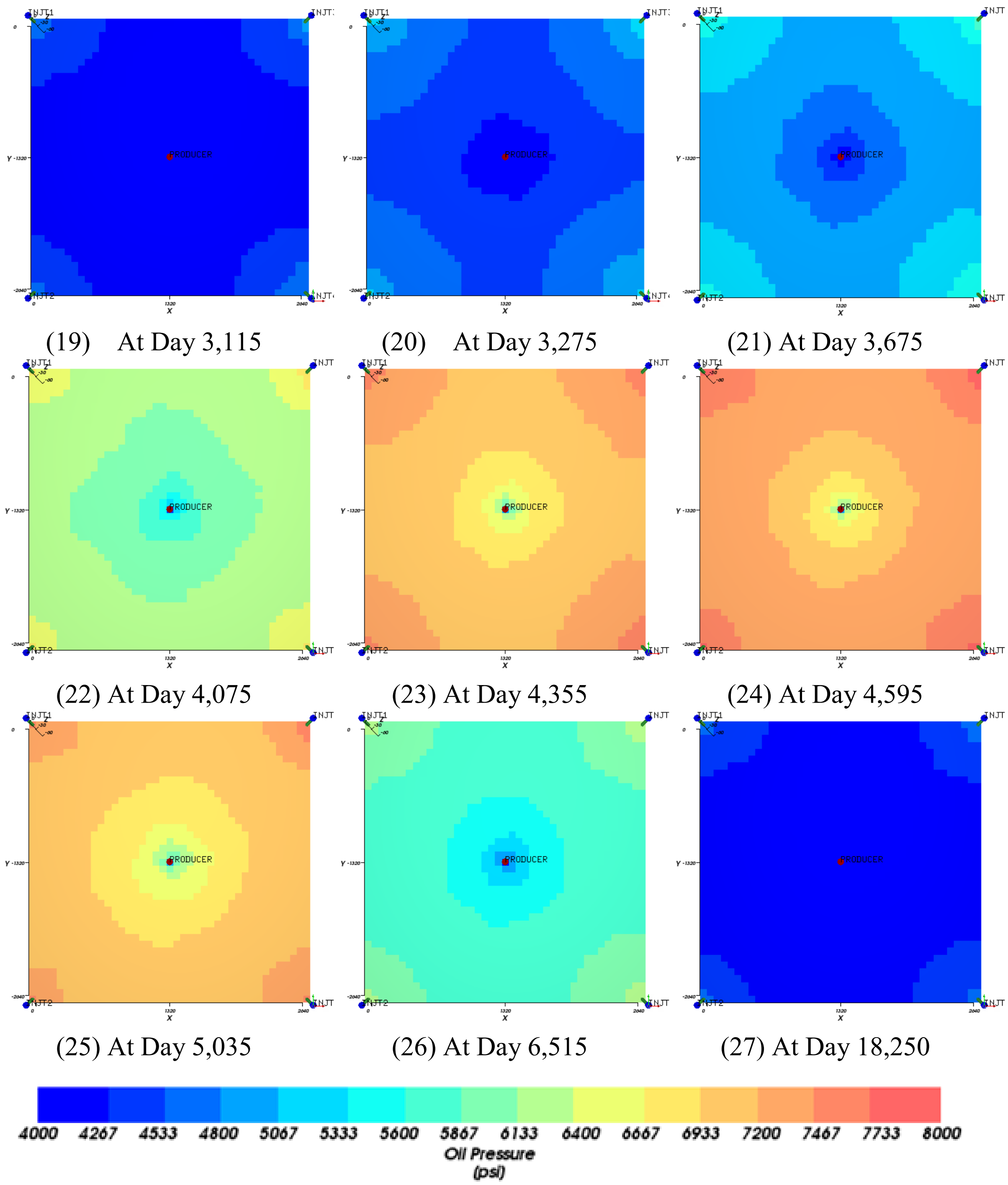


Figure 6-23(c): Reservoir Pressure Change with Oil Production Time for Case G440T7 (Water Flooding Starts at Year 7) - Water Flooding Oil Production Stage-2 Image (18) in Figure 6-23(b) is the same as Image (19) in Figure 6-23(c) but with different Pressure Scales.

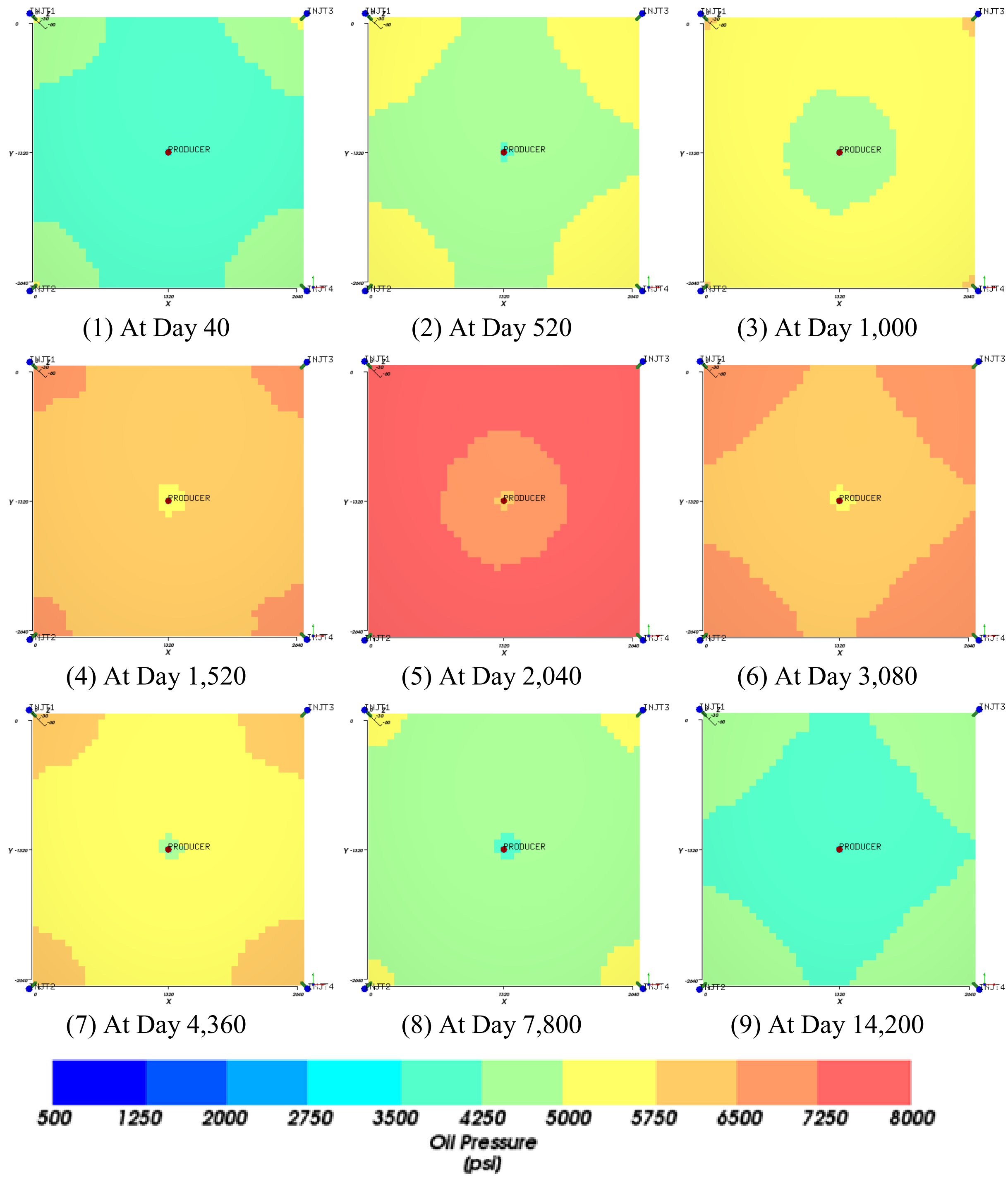


Figure 6-24: Reservoir Pressure Change with Oil Production Time for Case G440T0 (Water Flooding Starts at Time Zero)

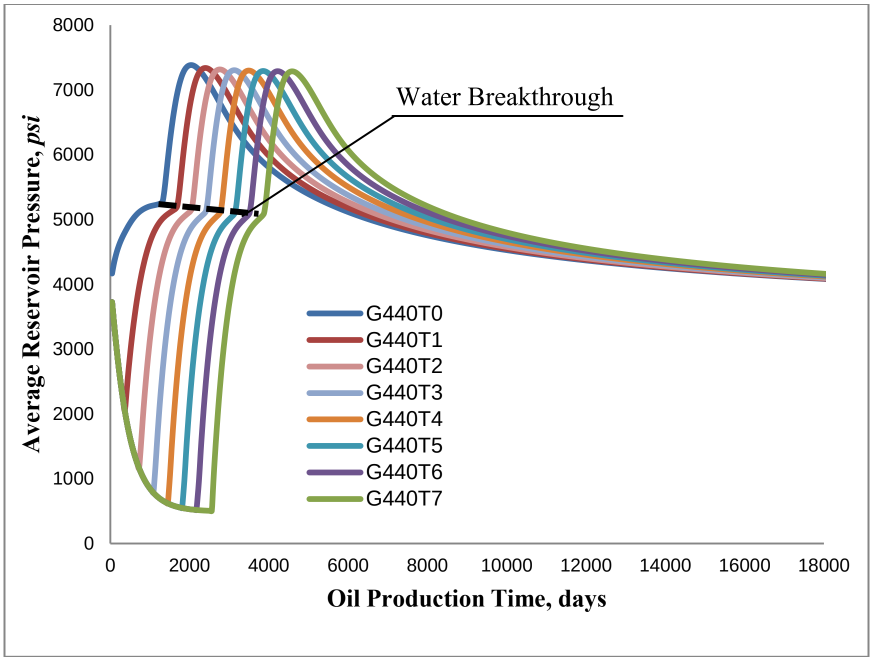


Figure 6-25: Average Reservoir Pressure Change with Oil Production Time for Eight Cases of Different Water Flooding Switching Times

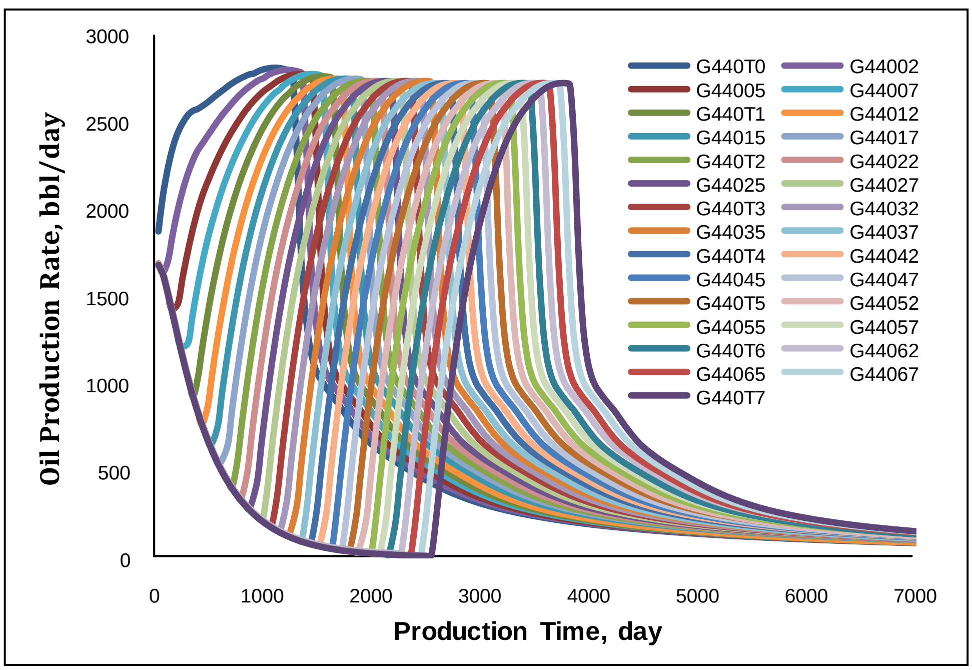


Figure 6-26: Oil Production Rate Change with Oil Production Time for 29 Cases of Different Water Flooding Switching Times

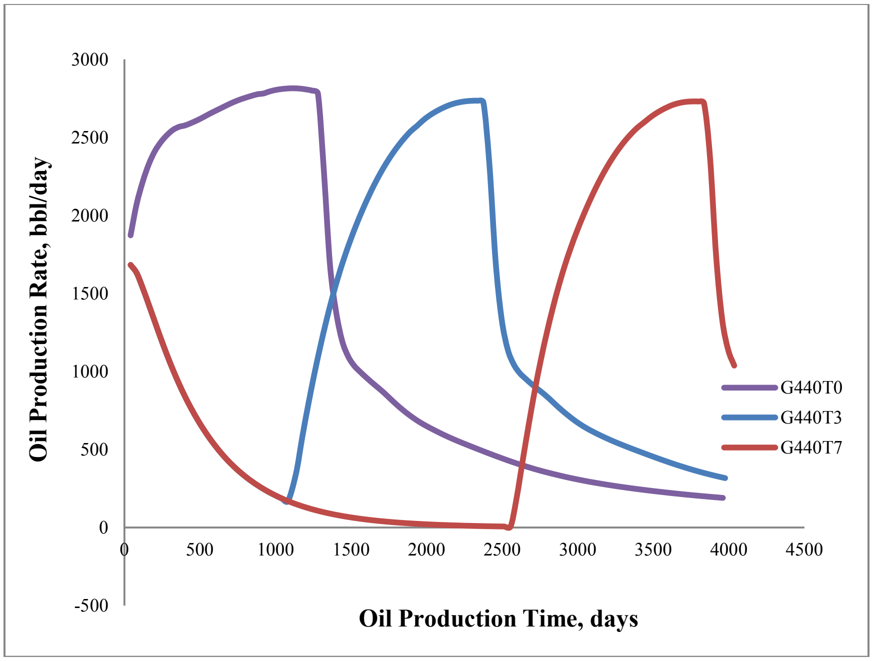


Figure 6-27: A Close Look at Oil Production Rate Change with Oil Production Time for Three Cases of Different Water Flooding Switching Times

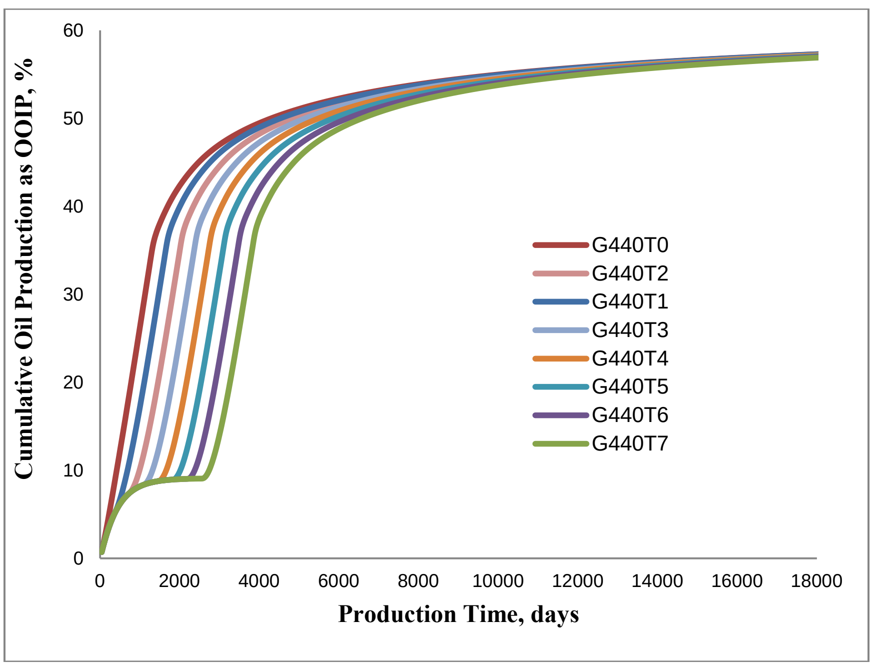


Figure 6-28: Cumulative Oil Production (as $\%\ OOIP$) with Oil Production Time for Eight Cases of Different Water Flooding Switching Times

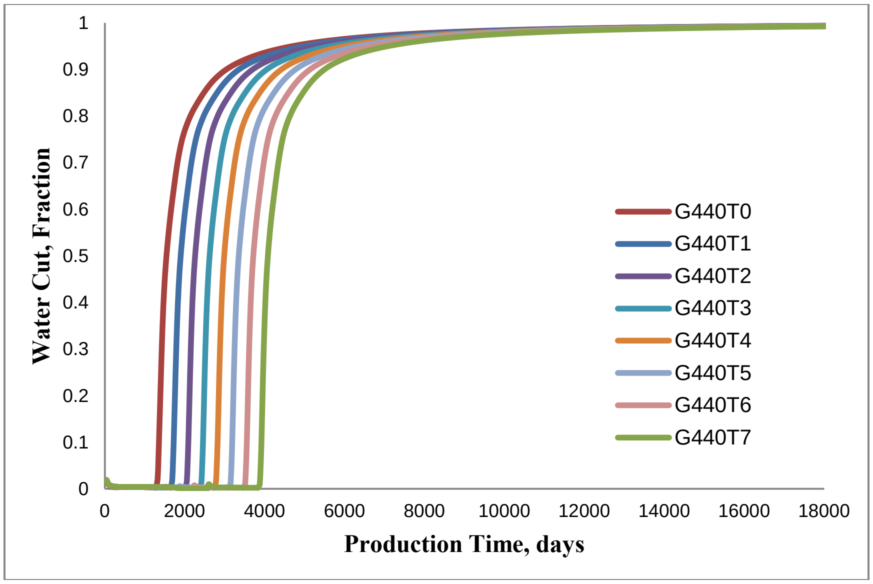


Figure 6-29: Water Cut Change with Oil Production Time for Eight Cases of Different Water Flooding Switching Times

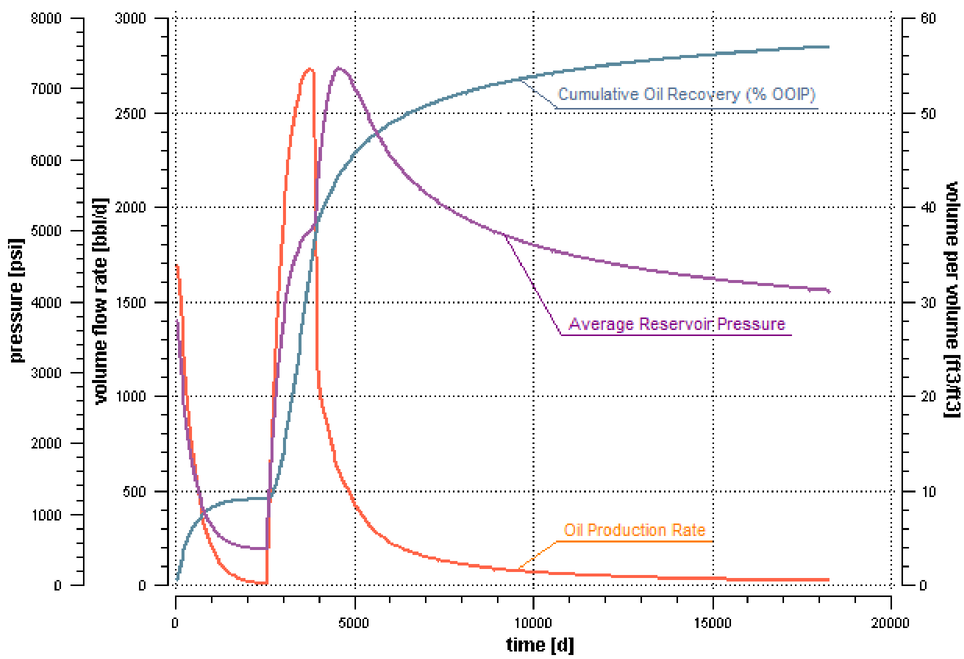


Figure 6-30: Oil Production Rate vs. Average Reservoir Pressure and Cumulative Oil Recovery (% $OOIP$) for Case G440T7 (Water Flooding Starts at Year 7)

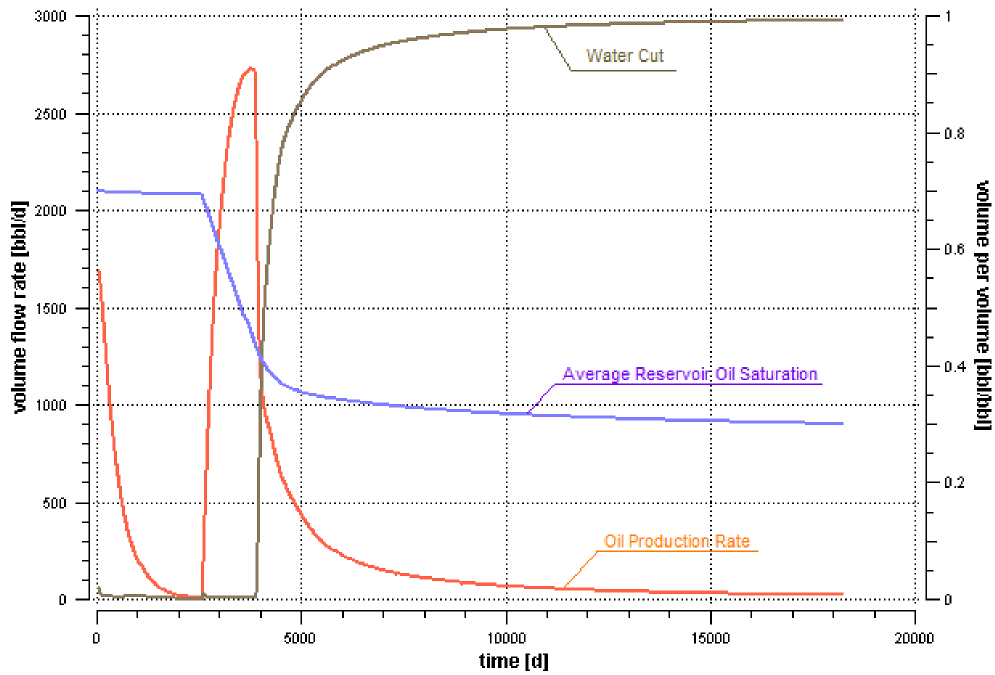


Figure 6-31: Oil Production Rate vs. Average Reservoir Oil Saturation and Water Cut for Case G440T7 (Water Flooding Starts at Year 7)

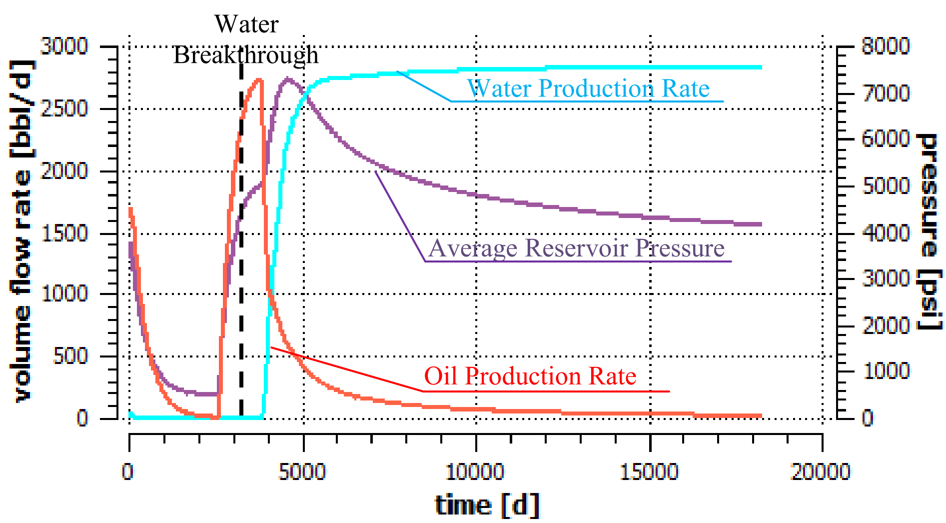


Figure 6-32: Oil Production Rate vs. Water Production Rate and Average Reservoir Pressure (Water Flooding Starts at Year 7)

CHAPTER 7: CASH FLOW PROJECTION FOR THE PETROLEUM EXPLORATION AND PRODUCTION ($E\&P$) INDUSTRY - THEORIES AND APPLICATIONS

The petroleum industry is commonly referred to as the oil and gas industry. Along with the increasing world-wide energy demands, the petroleum industry serves dominant functions in the world economy. It is also uniquely characterized and governed by a high level of risks, including both above-ground and underground risks; high level of technical complexity involved in exploration, production, and other segments; a large investment and long-term return on investment; and high levels of complexity in regulation, tax, and financial accounting rules (Koester, 1982). Thus, the economics of the oil and gas industry is quite complex and dynamic.

The true value of an oil and gas company is the underlying value of its oil and gas reserves. However, the value of an oil and gas reserve depends not only on how much and when the total oil and gas can be produced from the reserve and how much are the oil and gas can be sold, but also on many other factors. How to capitalize investment costs under different accounting rules and taxation laws, methods of depreciation, depletion, and amortization, which fiscal regime a company is under, and the capital structure used to finance the project, all exert impacts on the outcomes of cash flow calculations and the value of a project. This chapter briefly covers important theories, concepts, and rules that are related to the calculation of cash flows: costs in oil and gas exploration and production, full cost accounting and successful efforts accounting and their applications to the capitalization of oil and gas investment costs, depreciation theories and methods, fiscal regimes, and capital structure of financing projects.

7.1 Costs in the Petroleum Exploration and Production ($E\&P$) Industry

Today's oil and gas companies are involved in four types of functions: 1) exploration and production ($E\&P$); 2) refining and gas processing; 3) transportation and distribution; and 4) retail and marketing. An independent oil company is one that is involved in only $E\&P$ activities and a dependent or integrated oil company is one that is involved in $E\&P$ activities and at least one of the other three activities (Gallun, Stevenson, and Nichols, 1993). The $E\&P$ segment is also called "upstream" operations; and the other three segments are referred to as the "downstream" operations (Brock, Feiten, & Jennings, 1996).

For the upstream petroleum industry, the purpose is to discover new oil and gas reserves that can be produced profitably. There are four fundamental categories of costs associated with the petroleum upstream operations: 1) lease acquisition costs; 2) exploration costs; 3) development costs; and 4) production costs (Johnston & Johnston, 2006). Lease acquisition costs can be further classified as pre-acquisition costs and acquisition costs.

7.1.1 Pre-Acquisition and Acquisition Costs

Usually oil and gas companies do not own the oil and gas properties. In the United States, mineral rights are owned by individuals, the government, and other entities. An oil and gas company first needs to acquire mineral rights from the government or private land owner by entering oil and gas lease agreements to conduct the exploration and production activities.

For the purpose of just identifying a formation that may have oil and gas reserves, before signing a lease with a land owner, an oil company pays pre-acquisition costs for preliminary seismic studies on a target property, including the payment to the land owner for the right to conduct the seismic studies and the costs for seismic studies themselves.

Acquisition costs are paid in order for an oil company to obtain the mineral rights in potential oil and gas properties to undertake exploration and production activities. Acquisition costs include lease bonuses and lease purchase costs.

7.1.2 Exploration Costs

Defined by the Securities and Exchange Commission (SEC) in Reg. SX Rule 4-10 (a)(15), "Exploration costs are those costs incurred in (a) identifying areas that may warrant examination and (b) examining specific areas that possibly contain oil and gas reserves" (Brock, Feiten, & Jennings, 1996). Whether or not proved reserves exist on a property is determined through exploration. Exploration costs include seismic studies, geological and geophysical costs ($G\&G$), delayed rental, and drilling costs of exploratory wells.

An exploratory well is drilled in order to discover proved reserve in the area where the proved reserve is not known to exist yet. When it is determined that a proved reserve does not exist, the exploratory well is plugged and abandoned. The plugging and abandoning costs are also classified as exploratory costs. If it is determined that proved reserve exits, then all the costs to complete the well for producing oil and gas are classified as development costs.

7.1.3 Development Costs

Development costs are those costs spent on a producing system of wells and related equipment after it is determined that proved reserves do exist in a property so that the property is in producing conditions. Development costs cover many cost items, such as completing the exploratory well, drilling development wells, and surface equipment for production. Completing the exploratory well includes casing, cementing, perforating, and fracturing the well so that it is able to produce a proved reserve; a development well is drilled in an area and depth where a proved reserve has already been demonstrated to exist; the surface equipment includes the facilities for production, treating, gathering, and storing oil and gas, including christmas tree or pumping units, gathering lines, treatment facilities, and tanks.

7.1.4 Production Costs

Production costs include all costs for lifting oil and gas to the surface, and then gathering, treating, and storing them to make them ready for the market. Production costs include following cost items (Brock, Feiten, & Jennings, 1996): 1) labor to operate and maintain the wells, and the related equipment and facilities; 2) materials and supplies; 3) taxes and insurance on the property; and 4) severance or production taxes. Production costs can be classified into fixed costs and variable costs.

7.2 Full Cost Accounting and Successful Efforts Accounting in the Petroleum *E*&*P* Industry

In the financial accounting system in the United States, there coexist two different accounting methods for the petroleum industry to choose: full cost accounting and successful efforts accounting. Both accounting methods have been adopted throughout all sizes of oil and gas companies in the United States since the 1960s (Koester, 1982). Whether any costs incurred in pre-acquisition, acquisition, and exploration are expensed or capitalized depending on which accounting method is chosen. When a company capitalizes a cost, that cost is classified as an asset, which is recorded in the balance sheet and is later charged against production revenue in the form of depreciation, depletion, or amortization. On the other hand, if a cost is determined as an expense, that cost is immediately charged. Thus, with different accounting methods, the same company with the same project and the same oil and gas production profile may lead to very different asset values and different net incomes from its oil and gas production. The resulting project values are different accordingly.

There are two key areas of differences between the full cost and successful efforts accounting methods for the petroleum industry: 1) the treatment of the capitalization of the pre-acquisition, acquisition, and exploration costs; and 2) the size of the cost center for the distribution of the capitalized assets over the production revenues.

Based on a direct cause-and-effect relationship, the successful efforts accounting method capitalizes only those costs that directly result in future cash inflows. Thus, the successful efforts accounting considers only the costs in acquisition and exploration that directly result in proved reserves to be the necessary costs to discover oil and gas reserves. Any costs that are not related to specific proved reserves are considered as expenses. Under the full cost accounting method, on the other hand, all costs in pre-acquisition, acquisition, and exploration are considered necessary costs towards the eventuality of discovering oil and gas reserves, even those costs that do not find, or are not directly related to proved reserves, and are capitalized as oil and gas properties once they are incurred. Therefore, both successful and unsuccessful costs become parts of the total costs to be charged against future production revenues under the full cost accounting method.

All costs in acquisition, exploration, development, and production must be assigned to a cost center. Under the successful efforts accounting, one of three cost centers is used: the individual property or lease, the individual reservoir, and a field of a collection of several reservoirs (Koester, 1982 and Brock, Feiten, & Jennings, 1996). Under the full cost accounting method, the cost center covers the entire United States. For the companies which have international operations, their cost centers can be the individual countries, or a group of countries. The companies have bigger cost center under the full cost accounting method than under the successful efforts accounting method. With the defined cost center, all the capitalized costs incurred in pre-acquisition, acquisition, exploration, and development are spread, or amortized (this term will be discussed in later section of this chapter), against revenues from successful oil and gas production on a per barrel or per unit basis, or on the units-of-production basis, within the cost center.

Under the successful efforts accounting, all the pre-acquisition costs are expensed at the time they occur because preliminary seismic studies can only indicate which properties might have potential reserves, but cannot, by themselves, find new additional proved reserves. Under the full cost accounting method, all the pre-acquisition costs are capitalized and are then amortized over the production revenues in the entire cost center.

There are two steps to record the acquisition costs under the successful efforts accounting method. The acquisition costs are first held in an unproved property account. If later on it is determined that proved reserves do exist on the properties, the costs are declared to be assets and are capitalized. If it is resolved that the properties do not hold proved reserves, the costs are then removed from the unproved property account and charged as an expense. A company following the full cost accounting method capitalizes all acquisition costs.

Under the successful efforts accounting method, the exploration costs that are not related to discovering the proved reserves are expensed. Except for the costs of exploration wells, all exploration costs, including geological and geophysical ($G\&G$) costs and delay rentals, are expensed once they are incurred. Similar to the research costs, $G\&G$ costs are incurred to acquire information, and rarely directly lead to future cash inflows. Therefore, $G\&G$ costs are expensed regardless if they result in finding proved reserves (Koester, 1983). The costs of exploratory wells are expensed at the time when a company declares that there are no proved reserves being identified on the properties through the exploration wells; or capitalized if there exist proved reserves. That is, the only exploratory costs that are capitalized under the successful efforts accounting method are the costs of the exploratory wells that discover new reserves (Koester, 1982). The full cost accounting method capitalizes all the exploration costs.

Development costs are capitalized within the appropriate cost center under both the successful efforts and the full cost accounting methods, including the costs of dry hole development wells. Unlike the acquisition costs which are amortized over proved reserves, the development costs are amortized over proved developed reserves under the successful efforts accounting. Production costs are expensed under both the successful efforts and the full cost accounting methods.

Figures 7-1 and 7-2 are the overviews of the successful efforts and the full cost accounting methods. Table 7-1 contains a detailed comparison between these two accounting methods.

7.3 Depreciation Theories and Applications to the Petroleum $E\&P$ Industry

The petroleum exploration and production ($E\&P$) industry involves large amount of investments for long-term assets. Depreciation charges affect assets' values on balance sheet, taxable income and tax deduction, net income, cash flows, and values of projects. Thus in the petroleum $E\&P$ industry, correctly understanding depreciation theories, and applying depreciation theories and methods to the economic evaluations are important.

The subject of depreciation, including its importance, the causes and purposes of depreciation, and methods of depreciation, have been studied for decades in many academic disciplines, such as economics, accounting, and engineering, and in the application areas, such as in federal income tax deduction. This section covers the fundamentals about depreciation: why depreciation arises, definitions of depreciation, purposes of depreciation, and how to calculate depreciation.

7.3.1 Why Does Depreciation Arise?

Depreciation arises due to the difference in time length between the fiscal periods when the financial conditions and economic information, such as values, profits, and costs, are demanded to record and report and the service life of the assets which are used in production. If there was only one accounting period from the acquisition of an asset to the end of the asset's service life, that is, the information of the financial condition, such as revenues, operating costs, net income, is not desired at the intermediate time between the date of acquiring the asset and the date of discarding the asset, then the investment costs should be treated solely as an expense of operation for the whole production period with the same way as fuels consumed or raw materials used, and are regarded as revenue charges, or operating costs. There was no need for depreciation in such cases.

In reality, because of the business competition and the needs for obtaining and analyzing frequent and regular data regarding the financial conditions of a business, the accounting period, or fiscal period, can be a month, three months, six months, or a year, which may be much shorter than service lives of long-term assets.

To illustrate why depreciation arises, assume that there are only two accounting periods, Period 1 and Period 2. Period 1 and Period 2 are the same in time length, for example, one year. Period 1 is for investment only and Period 2 is for production only; the service life of the asset will end in Period 2 without any salvage value. There are no inventories, accruals, and deferred charges. All of the elements of production entering into the product are completely used up in Period 2. So the entire value of the asset invested in

Period 1 would be a charge to the product generated during Period 2. A complicated method is not involved to allocate the investment costs to Period 2.

Now assume Period 1 is one year's length and Period 2 covers two years' length and the net income and cash flow needs to be reported on a yearly basis. In this case, the investment costs need to be spread over two years into the operating costs. This is the case when depreciation arises: the allocation of the investment costs over two years to correctly reflect the operating costs in generate revenues using the invested asset each year in Period 2.

Further assume that Period 1 covers one year and Period 2 covers ten years. The investment costs in Period 1 need to be spread over the next ten years. How the investment costs are distributed over ten years so that each year bears the reasonable portion of the investment costs? Sound theoretical methods are desired to accomplish such a task.

In summary, if it were possible, the fiscal period to report the financial results would be long enough to cover all the natural service lives of the long-term assets, there would be no depreciation involved. The entire value of the investment would be a charge to products as part of the operating costs during the fiscal period. It is due to the shortening of the fiscal period to a year or less which generates the needs of spreading the investment costs over each fiscal period and developing theories and methods to match revenues with investment costs. The process is called depreciation, depletion, and amortization, which are discussed in detail in the following sections.

7.3.2 Definitions of Depreciation

From above discussions, it is clear that when the service lives of assets are longer than accounting periods, costs of assets should be spread to the accounting periods which benefit from the uses of the assets. And thus, the utilizing of long-term assets to generate revenues is recognized as part of operating costs.

In financial reports, the values of the assets are recorded in balance sheet from the time of investment to the time when assets are discarded. While assets are used and depreciation charges are taken, the values of the assets decrease accordingly. Taking depreciation charges and decreasing in assets' values are two sides of one coin. Definitions, theories, and methods of depreciation have been studied from the perspectives of both "sides" of the depreciation "coin". The conclusions drawn from each point of view are also different.

7.3.2.1 Definition of Depreciation as Cost Allocation

When viewed from accounting perspective, depreciation is defined as cost allocation: allocating the investment costs of an asset to each accounting period that benefits from using the asset.

When a long-term asset is purchased, the capitalized costs of the asset represent the prepaid costs for a bundle of future benefits. According to the matching principle of accounting, part of the capital expenditure of the asset should be allocated as a revenue expenditure, or expense, in the same accounting period that revenue is generated by using that asset. Depreciation describes the gradual conversion of the capitalized costs of the asset into expenses.

Depreciation is the process of allocating the prepaid costs over the productive or useful lives of assets using systematic and rational methods. In general, depreciation is used for building and equipment; depletion is used for natural resources; and amortization is used for intangible assets. In the petroleum industry, however, the term amortization is used uniquely. It may also refer to the total amount of depreciation, depletion, and amortization ($DD\&A$), as is discussed in detail in later sections. Since land is considered to have an unlimited life, typically there is no depreciation associated with the usage of land.

In financial statements, $DD\&A$ is a process of cost allocation, not evaluation. It does not refer to an asset's physical deterioration over time. Repairs do not eliminate the need for depreciation. It is not a process of determining the decline in values or determining the current market values of assets. The original costs of an asset minus the accumulated amount of $DD\&A$ of the asset is the book value or carrying value of the asset. Usually, the book value or carrying value of the asset is different from its current market value.

7.3.2.2 Definition of Depreciation as Value Changes of Assets

When viewed from the value changes of assets, deprecation means decreases in values of assets, either in tangible or intangible forms, to reflect the loss in the values of investments because of the service the assets have rendered. The value decrease of an asset is resulted from loss of its service life or other reasons. Physical decay (wear and tear), obsolescence, and inadequacy are direct causes of value decrease of an asset (Rittenhouse & Clapp, 1918).

Harold Hotelling, well-known in resource economics and in statistics, stated for the first time the now generally accepted one definition of depreciation in his paper of "A General Mathematical Theory of Depreciation" (Hotelling, 1925) that "Depreciation is defined simply as rate of decrease of value." According to this definition of depreciation, the mathematic formulation of depreciation is in the following form:

$$D(t) = -\frac{dV(t)}{dt},$$

where $D(t)$ is the depreciation charge at time t, and $V(t)$ is asset's value at time t. The total depreciation over a period of time $[a, b]$ is the difference between the values of the asset at the beginning $V(a)$ and at the end $V(b)$ of the period.

$$\int_a^b D(t)dt = -\int_a^b \frac{dV(t)}{dt}dt = V(a) - V(b)$$

The total depreciation of an asset over its whole useful life (*T*) is $[c - s(T)]$, where *c* is the original investment cost of the asset, $S(T)$ is the salvage value of the asset at the end of asset's service life. The average depreciation over the entire useful life (*T*) of an asset is $([c - s(T)]) \,/\, T.$ This is the same as straight line depreciation.

Because of the difference in definitions on depreciation, the theories, the methods, and amount of depreciation charges are different under different definitions. Based on the sound economic and accounting theory, cost allocation of depreciation aims to match revenues with the portion of investment costs. But it does not always adequately measure the value changes of the assets. It is unrealistic to expect that the true value decrease of an asset in a defined time period is the amount of the investment cost which contributes to the revenue generation in the same time period. For the petroleum economics analysis, the definition of cost allocation for depreciation is always used.

7.3.3 Fundamental Purposes of Depreciation

The fundamental purposes of depreciation can be viewed from the operation profit (or loss) standpoint and the valuation or balance sheet point of view. The former is the accountant's point of view and the later is the engineer's point of view. There are also other purposes of depreciation such as financing and tax deduction. These four fundamental purposes of depreciation are described in the following sections.

7.3.3.1 Accountant's Viewpoint of Depreciation Purpose

From the accountant's point of view, the purpose to charge depreciation is to determine the correct costs of doing business and to show the true costs in the income statement of operations.

Depreciation is part of production costs. Just as raw materials which form the product constitute costs of production, a portion of the service life of an asset which goes into each unit of the product should also bear the cost out of using the asset. Depreciation charge is to burden the product with a correct amount of cost from using the asset, thus, the net income of operation can be determined.

Under the accountant's viewpoint, the emphasis of depreciation is placed on the effort of showing the true costs against revenues, without much regard to the effect on the balance sheet and true valuation of the assets (Rittenhouse & Clapp, 1918).

7.3.3.2 Engineering Viewpoint of Depreciation Purpose

From the engineering point of view, the purpose of deprecation is to show the correct values of assets. Engineers attempt to determine an asset's actual present capability of rendering service in comparison with its original serviceability when the asset was new, with little regard to the purpose of assigning correct costs against revenues. By showing the true values of the asset at given dates and its periodic value changes, the periodic charges are applied as operating costs due to the asset's value decrease. This way, part of the revenues is unavailable for distribution among the stockholders and the value of the original capital invested remains intact (Rittenhouse & Clapp, 1918).

7.3.3.3 Depreciation as a Mean of Financing

Diverging from the answer for the fundamental question "why does depreciation arise?" there is another view of purpose for depreciation as solely a means of financing. Under this point of view, depreciation is a financing device to secure a sufficient amount of capital at the end of the service life of an asset to finance its replacement. During the intermediate period, the effect of depreciation charges is lost. There is no attempt made through the periodic deprecation charges to accurately state values of the asset. Whether or not a product generated in a given period is burdened with its just share of costs is not a concern either (Rittenhouse & Clapp, 1918).

7.3.3.4 Depreciation as a Mean of Tax Deduction

Depreciation charges are tax deductible. Tax deduction is a result of depreciation since depreciation is regarded as part of operating costs. In reality, tax purpose depreciation usually leads to relatively high deprecation charges and rapid income tax deduction in the early years of an asset's life. Thus, the income tax payments are deferred. When the purpose of depreciation is solely for tax deduction, it may lose sight of the fundamental reasons for charging depreciation, such as percentage depletion. The percentage depletion has been applied in the oil and gas industry since 1926 in the U.S. to encourage the ventures for oil and gas exploration. The amount of percentage depletion charges are not based on the original investment costs of assets, but based on production revenues. The accumulated total amount from periodic depreciation charges can even be higher than the original investment costs of the assets.

In summary, the four fundamental purposes, *i.e.*, cost allocation, valuation of assets, financing the replacement of assets, and tax deduction, should not contradict one another. When costs are allocated against revenues from an operation, the values of assets in balance sheet change accordingly. Since the depreciation charges make the portion of revenues unavailable to the stockholder, the amount of capital secured from depreciation can be used for financing. And since depreciation charges are part of operating costs, the total tax payments from income are deducted. However, when the emphasis of depreciation is laid on different purposes, the amount of depreciation charges can be much different. For the purpose of project evaluation for the petroleum $E\&P$ industry, in which very large amount of investment costs are involved, attention should be paid to understand the purposes and the applied methods of depreciation so that the true project values can be captured and right decisions can be made.

7.3.4 How to Calculate Depreciation-Methods of Depreciation in General

Theoretically, from the cost allocation point of view, the ideal method of charging depreciation is the one through which each unit of the product bears its just proportion of assets' costs. Three factors must be taken into consideration under most methods of calculating depreciation:

1) original cost of the asset,

2) salvage value of the asset, and

3) estimated service life of the asset.

Original cost is the real full cost of an asset in position ready to use. Salvage value, which is also called scrap value or residue value, is the estimated value of an asset at the time it is discarded, either removed from the position and ready for sale or other methods of disposal. The estimated service life is the time during which the asset is used for service. It is also called the effective working life of an asset. There are many expressions of service life, including units of calendar time, such as years; or units of service time, such as working hours; or units of output, such as tones. Except for the original cost of the asset, two out of the three factors of determining depreciation charges are estimations.

The methods of depreciation may be classified broadly under four categories:

1) proportional methods,

2) variable percentage methods,

3) compound interest methods, and

4) miscellaneous methods.

In this chapter, the first two categories of depreciation methods are described.

7.3.4.1 Proportional Methods of Depreciation

Under the proportional methods of depreciation, the periodic depreciation charge is proportional to some fixed base value of the asset. There are mainly three proportional methods of depreciation:

a) straight line,

b) working hours, and

c) service output.

(a) Straight Line Method of Depreciation

Straight line method of depreciation assumes that the loss of the asset's value in each period is proportional to the length of its service life. Thus, the same time length of service should allocate the same depreciation charge as operating cost. This method spreads the depreciation charge evenly over the periods of the service life of an asset. This method is usually used where the time elements of depreciation are the controlling factors of depreciation. Under the straight line method of depreciation, the periodic depreciation is calculated with the following equation:

$$Depreciation\ Expense = \frac{(Original\ Costs - Salvage\ Value)}{Estimated\ \text{Asset}'s\ Useful\ Life}.$$

In this equation, $(Original\ Cost - Salvage\ Value)$ is called the depreciable cost. $(1\ /\ (Estimated\ \text{Asset}'s\ Useful\ Life\ ))$ is the straight line depreciation rate.

(b) Working Hours Method of Depreciation

Under the working hours method of depreciation, the service life of an asset is estimated and expressed in terms of service units instead of calendar units of time. In a given accounting period, the service time (in hours) of the asset is recorded and compared with the estimated length of the service life of the asset (also in working hours) to derive the proportion of the depreciation which must be charged to that accounting period. With this method, the rate of depreciation per working hour can be applied directly to the product and an equitable distribution of the depreciation cost over product can be derived. If the service rendered by the asset is the controlling factor of depreciation, then the working hours method should give satisfactory results of depreciation charges.

(c) Service Output Method of Depreciation

The service output method of depreciation is also called the production units depreciation or units-of-production depreciation. This method is based on the assumption that depreciation is solely the results of using the asset and that the passage of time plays no role in the depreciation process. There is a direct relationship between the amount of depreciation and the units of output in each accounting period. Under this method, the estimated useful life of an asset is measured by the expected total units of future production or output of the asset. Accordingly, the depreciation that should be charged in a given accounting period is directly proportional to the units of output in that accounting period so that every unit of output is burdened an equal share of the total depreciation charge. The higher the actual production in an accounting period, the higher the depreciation expense. When there is no production, the depreciation expense is zero.

Under the units-of-production method of depreciation, the depreciation expense is estimated with the following equation:

$$Depreciation\ Expense = \frac{(Original\ Costs - Salvage\ Value)}{Estimated\ Total\ Production} \times Actual\ Production\ ,$$

where $[((Original\ Costs - Salvage\ Value))\ /\ (Estimated\ Total\ Production)]$ is the depreciation rate per unit of production.

The units-of-production method of depreciation is used for the cost allocation of natural resources. When a natural resource is used up, its capitalized costs are spread over the production periods when revenues are generated. This process is called depletion as described in the later section.

7.3.4.2 Variable Percentage Methods of Depreciation

There are two parts in the variable percentage methods of depreciation: one is the base for depreciation, the other is the percentage to be charged off in each accounting period. For the estimation of depreciation under the variable percentage methods, if the percentage is fixed, the base varies; if the base is fixed, the percentage varies. Two variable percentage methods are discussed in this section:

a) fixed percentage of diminishing value method, and

b) changing percentage of cost less salvage value method (or sum of expected life periods method).

(a) Fixed Percentage of Diminishing Value Method

The fixed percentage of diminishing value method of depreciation is a time basis rather than an output basis depreciation. Under this method, the periodic depreciation is charged as a fixed proportion of the book value of the asset by the end of the previous period. The book value decreases, and thus the base for the depreciation decreases, along the service time. With this method, the periodic depreciation charges are higher in the beginning years and progressively diminish toward the end of the service life of an asset.

Compared to the straight line method of depreciation, the fixed percentage of diminishing value method of depreciation is an accelerated method of depreciation. This method recognized that many assets are more efficient, provide more and better service, and may generate more revenues, in the early years of their useful lives, and that fast-changing technologies cause some assets to become obsolescent and lose service value rapidly. For those assets, allocating more depreciation to earlier years than to later years is consistent with the matching principle of accounting, that is, more depreciation is charged in years when higher amount of revenues are generated (Needles & Powers, 2004). One application of the fixed percentage of diminishing value method of depreciation is called declining balance depreciation method.

Under the declining balance depreciation method, depreciation expenses are computed by applying a fixed rate higher than the straight line depreciation rate to the asset's net book value. Though many fixed rates can be applied, such as 1.5 times or 1.75 times of the straight line depreciation rate, the most common rate is a percentage equal to twice of the straight line depreciation rate. When the fixed rate is twice of the straight line depreciation rate, the method is called the double declining balance method. The following is the equation to calculate the depreciation expenses under the double declining balance method:

$$Depreciation\ Expense = \frac{2}{Estimated\ Asset's\ Useful\ Life} \times Net\ Book\ Value$$

$$= 2 \times \frac{(Original\ Cost - Accumulated\ Depreciation)}{Estimated\ Asset's\ UsefulLife},$$

where $\left({}^{2}/_{Esstimated\ Asset's\ Useful\ Life}\right)$ is the fixed percentage, which is twice of the straight line depreciation rate.

Under the fixed percentage of diminishing value method of depreciation, the net book value of an asset other than the depreciable cost (the difference between the original cost and the salvage value) is used in calculating the depreciation expenses. In other words, the estimated salvage value is not taken into account in estimating the depreciation expenses. However, since the total accumulated depreciation should not exceed the total depreciable cost, in the year when calculated depreciation expenses exceed the amount necessary to lower the book value to the estimated salvage value, the depreciation expense should be adjusted so that the asset’s book value exactly equals its salvage value. There should be no future depreciation taking place in subsequent years. That is, the depreciation expense in the last year is limited to the amount which reduces the asset's book value to its salvage value. On the other hand, when the book value of the asset becomes smaller and smaller, the depreciation expenses decrease. Most tax codes allow double declining balance depreciation to switch to straight line deprecation when the straight line depreciation charge is higher than the charge of double declining balance depreciation.

(b) Changing Percentage of Cost Less Salvage Value Method

The changing percentage of cost less salvage value method of depreciation is also a service time basis depreciation. Under this method, the depreciation base remains fixed as the original investment cost minus salvage value. However, the rate of depreciation decreases from period to period. In a given accounting period, the depreciation rate is determined by a fraction of which the denominator is the sum of the expected service life of the asset viewed from the beginning of each accounting period, and the numerator is the expected future service life of the asset viewed from that accounting period. For example, if the expected total service life of an asset is five accounting periods, then at the beginning of each successive accounting period, the expected service life of the asset is five, four, three, two, and one, respectively. The common denominator of the depreciation rate for each accounting period is the sum of five, four, three, two, and one (5+4+3+2+1), which is 15. The numerator of the depreciation rate for each accounting period is five, four, three, two, and one, respectively. The depreciation rate in each accounting period is therefore 5/15, 4/15, 3/15, 2/15, and 1/15, respectively.

Another name for the changing percentage of cost less salvage value depreciation method is called sum of the years digits depreciation (SYD). Compared to the fixed percentage of diminishing value method, the changing percentage of cost less salvage value method charges more depreciation during the early periods of the service life and less during the later periods.

7.3.5 Applications of Depreciation Methods to the Petroleum $E\&P$ Industry

Most of the methods mentioned above in calculating depreciation charges are applied to the petroleum exploration and production ($E\&P$) industry, including straight line depreciation, units-of-production depreciation, decline balance depreciation, and sum of the years digits depreciation (SYD). In reality, there are even more depreciation methods than the abovementioned methods of depreciation applied in the petroleum $E\&P$ industry. However, not every method of depreciation is allowed to be used in every situation. The applications of deprecation methods to the petroleum *E&P* industry are governed by strict accounting rules and complicated and changing taxation laws.

As mentioned above, there are three categories of cost allocation for long-term assets: depreciation, depletion, and amortization. Generally speaking, depreciation is used for the cost allocation for tangible long-term assets, such as plants and equipment; depletion is used for the cost allocation for natural resources; and amortization is used for the cost allocation for long-term intangible assets, such as patents and copyrights. However, in the petroleum $E\&P$ industry, even though the term $DD\&A$ is widely used referring to depreciation, depletion, and amortization, the single term “amortization” is also used referring to all the three categories of cost allocation for the original investment costs (Koester, 1982).

For the petroleum $E\&P$ industry, in the process of allocating assets' costs to match the revenues generated from using the assets, there are two sets of accounting rules to follow: financial reporting and tax reporting rules. Financial reporting follows the generally accepted accounting principles ($GAAP$); tax reporting follows the Internal Revenue Code. The purpose of financial reporting is to provide a company’s economic information. The objective of the International Revenue Code on depreciation is to reduce the tax liability for a company and to encourage certain business activities that are considered to benefit society. “The federal tax code that relates to oil and gas producing companies has two primary characteristics. It is extremely complex, and it is quite substantially different from the basic rules which are followed for financial accounting for oil and gas producing companies” (Koester, 1982). Some tax methods of depreciation are not acceptable for financial reporting under $GAAP$.

In the petroleum $E\&P$ industry, for the purpose of financial reporting under $GAAP$, only the units-of-production method of depreciation is allowed to use. In this application, the depreciation is called cost depletion. For the purpose of tax accounting, many methods of depreciation, depletion, and amortization are used according to the definitions and classifications of tangible or intangible assets for the petroleum $E\&P$ costs, types of the petroleum companies, *i.e.*, major or independent companies, and even the oil and gas production rates. The details about the methods of depreciation, depletion, and amortization in the petroleum $E\&P$ industry, under both the financial reporting and tax accounting purposes, are discussed in next section.

7.4 DEPRECIATION, DEPLETION, AND AMORTIZATION IN THE PETROLEUM $E\&P$ INDUSTRY

7.4.1 Cost Depletion for the Financial Accounting Purpose

Depletion is a process of allocating costs of natural resources to the units produced and sold. This process is also called cost depletion in the petroleum exploration and production ($E\&P$) industry. For the purpose of financial reporting, in the petroleum $E\&P$ industry, only the units-of-production method of depreciation is used for cost depletion, allocating all the capitalized investment costs in pre-acquisition, acquisition, exploration, and development over the total production revenues in the defined cost center. When for the purpose of financial reporting, the three terms, *i.e.*, depreciation, depletion, and cost depletion can be used interchangeably in the petroleum $E\&P$ industry.

In reality, there are two parts involved in this cost allocation process for natural resources. First, natural resources, such as oil and gas reserves, are converted to inventory. As a natural resource is produced and converted to inventory, the book value of the asset in the balance sheet is proportionally reduced. Second, according to the matching principle, when the natural resource in the inventory is sold and revenues are generated, depletion expenses occurred. The depletion of the natural resource matches inventory increase. Depletion expenses match revenues from selling the natural resource. If the total amounts of natural resource produced in a given accounting period are not sold out in the same accounting period, only those units that are sold in the same accounting period can be recorded as a depletion expense, while the units that are not sold are considered inventory. In other words, periodic depletion of natural resource refers to the units produced, but a depletion expense includes only the units sold. Periodic depletion does not equal the depletion expense when units produced do not equal units sold in a given accounting period. In the following discussions, it is assumed that units of oil and gas produced equal the units of oil and gas sold in any accounting periods. In other words, inventory is assumed zero in any accounting period.

Under this assumption, according to the units-of-production method of depreciation, the depletion expense per unit of oil and gas produced is determined by dividing the costs of the natural resource minus salvage value by the estimated number of units available to be produced. The amount of the depletion expense for each accounting period is then computed by multiplying the depletion expense per unit by the number of units produced in the same accounting period.

Thus, under the units-of-production method of depreciation, the periodic depreciation expense can be calculated with one of the following two equations (Brock, Klingstedt, and Jones, 1982):

$$Depletion\ Expense =$$

$$\frac{Original\ Capitalized\ Costs - Salvage\ Value}{Estimated\ Total\ Reserve} \times\ Production\ for\ Period$$

or

$$Depletion\ Expense =$$

$$\frac{Book\ Value - Salvage\ Value}{Estimated\ Beginning\ Reserve} \times\ Production\ for\ Period.$$

When a natural resource is newly developed, the total available units to be produced can be estimated; and the costs for depletion include all the original investment costs less the salvage value. At the intermediate time, the cost for depletion is the asset's book value minus the salvage value; and the available units for production are the remaining reserve at the beginning of each accounting period.

For successful efforts accounting, the capitalized acquisition costs are depleted over proved reserves; and the capitalized exploration and development costs are depleted over proved developed reserves. For full cost accounting, all the capitalized costs in acquisition, exploration, and development are depleted over the proved reserves. For successful efforts accounting, the cost center is one property, one reservoir, or a collection of reservoirs. For full cost accounting, the cost center is entire United States, or a group of countries for the companies involved in the international petroleum $E\&P$ activities.

7.4.2 Depreciation, Depletion, and Amortization in the Petroleum $E\&P$ Industry for Tax Deduction Purpose

7.4.2.1 Cost Items

As discussed before, in the petroleum exploration and production $(E\&P)$ industry, under the successful efforts accounting method, whether or not the incurred costs are capitalized depends on whether or not the costs result in the discovery of proved reserves. Under the full cost accounting method, whether or not costs are capitalized is determined by the criterion whether or not the costs are used to discover proved reserves. However, under tax accounting, none of the above criteria is applied. For the tax purpose accounting, the total costs occurred in lease acquisition, exploration and development are divided into two categories: 1) leasehold costs, and 2) drilling and development costs. Leasehold costs cover the costs for non-drilling activities, which are discussed in detail later in this section. The drilling and development costs are further classified as: a) intangible drilling and development costs (IDC), and b) tangible drilling and development costs (TDC) or lease and well equipment costs (Brock, Klingstedt, & Jones, 1982). Under tax accounting, the criterion to determine the capitalization of drilling and development costs is whether or not costs are tangible or intangible. There are significant differences in cost allocation for tax deduction purpose under U.S. tax code and for financial reporting purpose under $GAAP$.

The definitions of tangible and intangible assets under tax purpose cost allocation for the petroleum $E\&P$ industry are much different from the definitions under $GAAP$ in general. Under $GAAP$ and in financial reports, tangible assets are those long-term assets which have physical substance, such as buildings and equipment. Tangible assets are also called fixed assets (Libby, Libby, and Short, 2004). Intangible assets are long-term assets that have no physical or material substance but have values based on rights or privileges they offer to the company, such as licenses, copyrights, and patents (Libby, Libby, and Short, 2004).

For the petroleum $E\&P$ industry, under tax deduction purpose, whether or not drilling and development costs are tangible or intangible depends on whether or not the assets have salvage values. "IDC is defined as expenditures for drilling and development that in themselves do not have a salvage value and are incident to and necessary for drilling of wells and the preparation of wells for the production of oil and gas" (Gallun, Stevenson, and Nichols, 1993), no matter if the assets have material substances or not. Tangible costs are defined as those costs which lead to assets with salvage values. Specifically, all surface equipment downstream from the christmas tree is classified tangible, and so are the installation costs of the equipment; everything down hole from the christmas tree, if has a salvage value, is tangible, while the cost of installing it is intangible; casing, tubing, and the christmas tree are considered tangible, while the costs of installing those items are intangible; most down hole costs that are not associated with tubing or casing are intangible; the cost of drilling the bore is intangible; the cost of cementing the well is intangible; the cost of logging the well is intangible; the cost of perforating and fracturing the well is intangible (Koester, 1982 and Koester, 1983). Typically, IDC accounts for 60% to 70% of the total drilling and development costs. In other words, for every dollar of TDC, \$1.50 or \$2.00 of IDC is spent.

Under tax purpose accounting, the definitions and classifications of tangible and intangible costs only cover the exploration drilling and development stages up to the point of production. The costs of acquiring the property itself are not included in the tangible nor intangible costs. They are treated separately in the cost item called leasehold costs. Leasehold costs cover the non-drilling activity costs during the acquisition and exploration stage, including pre-acquisition costs, shooting rights, lease bonus, seismic studies, and other geological and geophysical ($G\&G$) costs. These cost items are capitalized and added into depletion basis (Koester, 1983).

In summary, for the tax purpose accounting, costs incurred in the petroleum $E\&P$ activities are broadly classified into three cost items: leasehold costs, tangible drilling costs (TDC), and intangible drilling costs (IDC). Each cost item is treated differently for the cost allocation under tax deduction accounting. Sections 7.4.2.3, 7.4.2.4, and 7.4.2.5 provide the details on the cost allocation for these three cost items.

7.4.2.2 Cost Center

For the petroleum companies using full cost accounting method, their cost center is entire United States. Successful efforts companies use a lease, reservoir, or field for their cost center. For tax purpose accounting, the cost center is usually the individual property. If a property proves to be unsuccessful and can be qualified for impairment under the tax rules, all costs associated with that property, even the tangible costs, may be expensed in the year impairment occurred.

7.4.2.3 Depreciation for Tangible Assets - Modified Accelerated Cost Recovery System (MACRS)

For the petroleum $E\&P$ industry under tax purpose accounting, all tangible assets are depreciated using the Modified Accelerated Cost Recovery System ($MACRS$) with following three specific criteria:

1) seven year assets' life

2) double decline balance depreciation switching to straight line depreciation

3) first year half-year convention

The $MACRS$ depreciation discards the concepts of estimated useful life and salvage value of an asset. Under $MACRS$, depreciation is computed: 1) on the original costs of assets; 2) over a period of years described by the law for all assets of similar types; 3) using the method of double declining balance depreciation with half-year convention for most assets other than real estate. Under $MACRS$, assets' depreciable lengths of time are often shorter than the estimated useful lives used for cost allocation in financial statements. $MACRS$ enables a company to allocate most of the assets' costs early in the depreciation process. $MACRS$ can only be used for tax purpose accounting. It cannot be used for financial reporting purpose.

With the first year half-year convention, all assets are assumed to be in service in the middle of the first year regardless of when the assets are actually placed in service during the year. Therefore, it is allowed to take half of the first year depreciation in the year when assets start to serve.

7.4.2.4 Amortization and Expense for Intangible Costs

For tax accounting purpose, the intangible drilling and development costs (IDC) is treated differently for the major and independent petroleum companies. For independent companies, all IDC is expensed when they are incurred, no matter the costs are associated with exploratory or development wells, with successful wells or dry wells. The expenses are treated as loss forward for tax benefits. Major oil companies can expense 70 percent of their IDC in the year incurred for successful domestic wells and 100 percent for unsuccessful domestic wells. The remaining 30 percent of IDC is amortized over five years using straight line method beginning the month the costs are incurred.

7.4.2.5 Depletion for Leasehold Costs

As mentioned above, under the generally accepted accounting principles $(GAAP)$, companies following both full cost and successful efforts accounting methods use the units-of-production method to allocate capitalized assets. However, for tax purpose accounting, petroleum companies have two options for the cost allocation of the capitalized leasehold costs: (a) cost depletion, and (b) percentage depletion. The depletion deduction taken is the larger one between percentage depletion and cost depletion for the companies which are qualified for percentage depletion.

(a) Cost Depletion

Cost depletion under tax deduction purpose is calculated the same way as the one for the financial accounting purpose. The capitalized leasehold costs are depleted with the units-of-production method of depreciation.

(b) Percentage Depletion

In the United States, percentage depletion was added in the Revenue Act of 1926 for the purpose of encouraging the oil and gas exploration ventures. Percentage depletion is taken on the basis of gross revenues from the oil and gas production. From 1926 to 1969, oil and gas companies were allowed to take 27.5 percent of gross revenues as depletion deduction for more than four decades. In 1969, the percentage was reduced to 22 percent of gross revenues (American Petroleum Institute, 1972). In 1982 and 1983, the rate was further reduced to 18 percent and 16 percent, respectively (van Rensburg, 1999). From 1984 to present, the percentage depletion rate has been kept 15 percent. The percentage depletion is limited to 100 percent of the net income from a property (McIntyre, 1996).

The major oil and gas companies gave up the percentage depletion in 1975 (van Rensburg, 1999). Now only the independent oil and gas companies with a certain production rate limit are qualified to take percentage depletion on their leasehold costs.

Since percentage depletion deductions are not based on the actual capitalized leasehold costs, the cumulated depletion amount from percentage depletion is allowed to exceed the original investment (Koester, 1983). Even though the application of percentage depletion may encourage further exploration efforts, it may also inaccurately reflect the real value of reserves in a given field.

In addition, "for the purpose of undertaking economic evaluations of petroleum or mineral properties which may produce for twenty or more years into the future, it should be understood that depletion allowances – and particularly percentage depletion – are likely to change over time." (van Rensburg, 1999) Understanding the uniqueness of percentage depletion provides guidance in selecting cost allocation methods in economic evaluations for the petroleum $E\&P$ projects.

In summary, under tax deduction purpose, when a property is not impaired, the cost allocation of leasehold costs, tangible drilling and development costs (TDC), and intangible drilling and development costs (IDC) is performed according to the following rules: 1) the leasehold costs are capitalized and depleted with cost depletion for major oil and gas companies, and with cost depletion or percentage depletion for independent oil and gas companies; 2)TDC are depreciated with $MACRS$ for both major and independent oil and gas companies; 3) for independent oil and gas companies, IDC are fully expensed as loss forward; 4) for major oil and gas companies, when wells are successful, 30 percent of IDC is capitalized and amortized with the straight line depreciation method over 5 years, while 70 percent of IDS is expensed as loss forward; 5) for major oil and gas companies, when wells are unsuccessful, all IDC is expensed as loss forward. However, when a property is declared impaired, all the costs associated with the property may be expensed. Table 7-2 summarizes the cost allocation for both the major and independent oil and gas companies on un-impaired oil and gas properties.

7.5 Fiscal Systems for the Petroleum $E\&P$ Industry

"Governments have no control over the gifts of nature, but they do control taxes" (Johnston, 1994). The profit a petroleum company ultimately receives depends on how the production revenues are divided between the government and the petroleum company. Unlike economic theory, which focuses on the benefits derived from labor and capital, rent theory deals with how benefits are divided among the laborers, owners of the capital, and landowners through wages, profit, and rent (Johnston, 1994). Rent is the portion of the benefits which goes to the landlord for the use of the land. Governments attempt to capture as much economic rent as possible through bonuses, royalties, production sharing, and taxes. With signature bonuses, governments capture the benefit at the time of transferring the mineral rights; with royalties, production sharing, or tax, governments capture the benefits during oil and gas production time. Bonuses and royalties are risk-free for governments, while production sharing and tax involve risks for governments (Johnston, 1994).

Figure 7-3 shows the detail on how the revenues from petroleum production are allocated. The negotiation between petroleum companies and governments to split the production revenues is usually according to one of the two basic fiscal systems or fiscal regimes: the concessionary system and the contractual system.

The concessionary system is called a royalty/tax system. Under the concessionary system, the government or land owner transfers title of the minerals to petroleum companies; and the petroleum companies are then subject to the payments of royalties and taxes to the land owner. Usually, a concessionary system has three components: 1) royalty, 2) deduction, and 3) tax. The royalty is a percentage of the gross revenues from the selling of produced oil and gas. In the United States, royalty rate ranges from 10% to 20%; the typical federal royalty rate in the United States is $1/8$ (12.5%) for onshore projects and $1/6$ (16.67%) for offshore projects (Kaiser & Pulsipher, 2004). The deduction includes operating costs, interests on loans, loss carry forward, and $DD\&A$. In the United States, state or federal tax is determined as a percentage of taxable income.

There are two forms of contractual agreements in the contractual system: service contracts and production sharing contracts (PSC). Under a contractual system, the government retains the mineral rights; and oil and gas companies receive a share of production or revenues from the produced oil and gas according to either a production sharing contract (PSC) or a service contract.

There are typically four basic components in a production sharing contract: 1) royalty, 2) cost recovery, 3) profit oil, and 4) tax. Same as in the concessionary system, the royalty is a percentage of the gross revenues of the oil and gas sold. Cost recovery includes operating costs, interests on loans, unrecovered costs carried forward, and $DD\&A$. There is a limit on the cost recovery under a production sharing contract. Profit oil, which is the difference between net revenue and cost recovery, is then divided into contractor profit oil and government profit oil according to the agreement. Net revenue minus government profit oil is the base for income tax. Figure 7-4 shows how net income is calculated under two fiscal regimes: the concessionary system and the production sharing contracts (PSC) , one type of contracts in the contractual system.

7.6 CAPITAL STRUCTURE AND COST OF CAPITAL FOR THE PETROLEUM $E\&P$ INDUSTRY

For a public petroleum company, when there is no internal capital, such as retained earnings, to be considered to invest an oil and gas project, the project can be financed through external funds from capital market: borrowing money from creditors (liabilities or debts) and issuing stocks (equity) in stock market. The mixture of liabilities and stockholders' equity is called capital structure (Libby, Libby, and Short, 2004). Capital structure can range from pure debts to pure equity. Capital structure affects cost of capital, discount rate, and value of a firm or a project differently according to the market assumptions, i.e., perfect market or imperfect market.

According to the Modigliani-Miller theorem (Modigliani and Miller, 1958), under the perfect market assumption, that is, no tax, no transaction costs, no bankruptcy costs, and absence of arbitrage, the value of a firm is not affected by whether it is financed by equity or liabilities and it is irrelevant to the dividend policy of the company. The authors for the Modigliani-Miller theorem, Franco Modigliani and Merton Howard Miller, were awarded the Nobel Prize in Economics, for which the Modigliani-Miller theorem was largely responsible (Grinblatt and Titman, 2002).

In reality, the assumptions underlying the Modigliani-Miller theorem are not always held, such as the existence of corporate tax and bankruptcy costs. The value of a company is affected by the decision of financing and the capital structure when market is imperfect. So it is very important, in making decisions on capital structure, to understand the distinctions between liabilities and stockholders' equity, and advantages and disadvantages in using these two different types of financing resources with the relaxing of the perfect market assumption.

There are two important differences between liabilities and equity (Grinblatt and Titman, 2002). First, liabilities claims are senior to equity claims. According to the law, creditors have the right over stocker holders and must be paid in full before stockholders can receive anything. If a company fails to pay its debts, either the principal or interest, when they are due, the creditors have the legal right to force the company to sell its assets to meet the required debt payments (Needles and Powers, 2004). Thus, it is riskier for a company to borrow money to finance a project than issue equity. Second, interest payments on liabilities are tax deductible but dividends on equity are not. That is, debt financing is cheaper than equity financing when financial distress and bankruptcy costs are not considered .

The advantages for a company to use equity over long-term liabilities, which typically require payments more than one year in the future, are as followings: 1) it is less risky to finance projects with equity than with liabilities; 2) it does not need to be paid back; 2) the dividends on equity are usually paid only if the company has earned sufficient net income and the board of directors decides to pay the stockholders; 3) when a company does not pay a cash dividend, the cash generated by profitable operations can be reinvested in its operations (Libby, Libby, and Short, 2004).

The advantages of using long-term liability over equity include: 1) the holders of common stock maintain control of the business entity; 2) the company receives tax benefits from interest payments; 3) if a company is able to earn more on its assets than interest on debts, the excess will increase the earnings for stockholders.

The disadvantages for a company to borrow long-term debts over issuing equity to finance a project include: 1) cash is required to make periodic interest payments whether the company has net income or loss; 2) cash is required to pay back principal at the maturity date of debts, which results in a negative impact on cash flows; 3) if the company fails to make timely interest and principal payments, it can be forced into bankruptcy by creditors. Hence, for a company whose operations are subject to ups and downs can be in danger by issuing debts.

It is challenging for a company to make an optimal capital structure decision. There are two existing theories regarding the choices of capital structure: the static capital structure theory, or trade-off theory, and the dynamic capital structure theory, or the pecking order theory. According to the trade-off theory of capital structure, companies optimize their capital structure and maximize their values by balancing the benefits and costs of using debts. Increase in the ratio of debt to equity enhances the benefits of tax deduction for a taxpaying company; however, at the same time, the risks of financial distress and bankruptcy costs increase. Hence, a company would increase its debt financing until reaching a point at which the marginal value on tax benefit equals the expected marginal cost of financial distress or bankruptcy costs due to the debt financing (Beyer, 2010). According to the pecking order theory of capital structure, companies prefer internal funds over external funds, and prefer debt to equity capital when external financing is required (Myers and Majluf, 1984). In other words, there exists a specific hierarchy in the financing process of a company. With the accessible sources of financing, a company would choose to use retained earnings first, then debts, and then equity.

With the result on capital structure decision, the cost of capital for the project can be calculated with the weighted average cost of capital (WACC) method (Grinblatt and Titman, 2002). That is,

$$Cost\ of\ Capital = w_E \times Cost\ of\ Equity + w_D \times Cost\ of\ Debt\ , \qquad (7-1)$$

or

$$r_C = w_E \times \overline{r_E} + \ w_D \times \overline{r_D}, \qquad (7-2)$$

where r_C : cost of capital,

$\overline{r_E}$: cost of equity,

$\overline{r_D}$: cost of debts, tax benefit on interest payments included,

w_E: percentage of equity in capital structure,

w_D : percentage of debts in capital structure,

$(w_E + w_D) = 1.$

In the traditional net present value (NPV) evaluations, the cost of capital is used as discount rate to calculate the NPV for a project. As is shown in Eq. (7-1) or Eq. (7-2), when cost of equity, $\overline{r_E}$, is not the same as cost of debts, $\overline{r_D}$, changing in capital structure, w_E or w_D, will result in different cost of capital, discount rate, NPV, or value of the project.

In summary, capital structure does affect cost of capital, discount rate, and value of a project in the real world (Bierman, 2003). Capital structure decision is one of the major financial decisions a company has to make. Thus, for the purpose of evaluating petroleum $E\&P$ projects, in which large capital investment are involved, the selection of appropriate capital structure should be taken into consideration. Understanding the relationship between the capital structure in financing a project and the estimated value of the project, distinctions between different sources of financing and the uniqueness of petroleum $E\&P$ projects, will facilitate the decision on capital structure for project evaluations in the petroleum $E\&P$ industry.

7.7 Cash Flow Projections for the Petroleum $E\&P$ Industry

7.7.1 Definition of Cash Flow

While there are three broad categories of cash flow, that is, cash flow from operations, cash flow from financing, and cash flow from investments, cash flow from operations is the main concern for the purpose of economic evaluations for petroleum projects. From the above discussions, the cash flow projections should take many factors into consideration, including: fiscal regimes, capital structure, accounting systems, methods for depreciation, depletion, and amortization, costs involved in acquisition, exploration, development, and production.

Dutch Jewish philosopher Baruch Spinoza once said that many arguments started from the difference in definitions for the same subject (GOOD TV Broadcasting Corp., 2010). Greek philosopher Socrates said that we should try our life-time best to find out the correct definition for every important subject for understanding (GOOD TV Broadcasting Corp., 2010).

In the paper "Defining Free Cash Flow," Mills, Bible, and Mason (2002) pointed out that "While companies, analysts, and investors realize the importance of cash flow, there is no real consensus on the definition of cash flow or free cash flow. There are many ways to define cash flow and free cash flow, resulting in problems of consistency and comparability." In this study, the definition of cash flow from operations for the upstream petroleum $E\&P$ industry under the concessionary fiscal regime is defined as follows (van Rensburg, 1999):

Cash Flow from Operations in Current Dollars

Production (Q)
Price (P)
Gross Revenue ($Q \times P$)
$-$ Royalties
$-$ Operating costs (excluding $DD\&A$)
$-$ Ad valorem and severance taxes
$-$ State income tax
$-$ Amortization (including depreciation)
$=$ Taxable income before depletion
$-$ Cost or percent depletion
$-$ Loss forward
$=$ Taxable income
$-$ Income tax
$=$ Net Income (if negative, loss forward to next year)
$+$ Depletion
$+$ Amortization (including depreciation)
$+$ Loss forward
$+$ Return of working capital
$+$ Net salvage (land)
$+$ Net salvage (plants and equipment)
$=$ Cash flow in current dollars

Even though it looks like, from the above calculation process, cash flow is from adding noncash expenses to net income when there is no salvage value nor changing in working capital, there is a fundamental difference between net income and cash flow in concepts. Net income measures the performance of a business operating on one “slice” of the project life called an “accounting period.” The capitalized investment costs are not shown as a whole in any accounting period through net income. Since net income already captures all the costs and benefits information, it can be independently used to measure the business performance in one single accounting period; and to compare the performance in different accounting periods for the same project, for different projects in same accounting period, and even for different projects in different companies doing different business. On the other hand, cash flow from one accounting period cannot be used to measure the costs and benefits relationship of a business operating in that accounting period. The absolute amount of cash flow in one accounting period may not necessary mean good business performance in that accounting period. For example, negative net income may become positive cash flow just by adding back $DD\&A$ or by collecting account receivable.

In general, cash flow from one accounting period should not be used to measure the business performance of a project or to compare the performance of different projects because the cost information is not complete. Cash flow needs to be used from the whole life of a project, that is, from its capital investment time to the end of the project life; and it is used to measure how much is gained from the investment through a project. Cash flow is indeed important for running a business. Shortage in cash flow during the middle of project may cause bankruptcy if a company’s debt payments cannot be satisfied even though there is a positive net income. Adding back $DD\&A$ to net income for cash flow not just because

$DD\&A$ is non-cash cost items, it is because capitalized investment costs are already allocated in the form of $DD\&A$ in a given accounting period in net income. If $DD\&A$ was not added back, the part of capitalized investment costs would be double counted. Theoretically, both net income and cash flow can be used in project evaluations. With net income, the capitalized investment costs should not be included again; with cash flow, the $DD\&A$ charge should be added back to calculate cash flow.

Correctly understanding the fundamental difference between net income and cash flow and the relationship between net income and cash flow are very important to conduct project economic evaluations, including real options evaluations, especially when there are switching options or stop options.

7.7.2 Assumptions of the Synthetic Petroleum $E\&P$ Company and the Project

In this study, the economic evaluations are undertaken for a synthetic $U.S.$ public independent petroleum $E\&P$ company. Following are the additional assumptions for the company and the project. 1) The company only has business in the exploration and production of oil and gas. Thus, the beta for the company is the one for oil and gas assets of the company. 2) The project is pure equity financed so that there is no bankruptcy risk even at the time when the petroleum production fluctuates up and down dramatically. 3) The company adopts the successful efforts accounting method. 4) The company is not qualified to take percentage depletion for tax deduction purposes. 5) The cost center for this project is the single lease for the reservoir.

For the project evaluation purpose, it is further assumed that there is no inventory or inventory costs. All the oil produced in any accounting period is sold in the same accounting period. The quality of the produced oil is close to the West Texas Intermediate (WTI) so that it can be sold at the same price as WTI anytime. The wellhead prices of the produced oil are the same as market prices by neglecting the transportation and other related costs.

7.7.3 Determination of Costs in Petroleum Exploration, Development, and Production

There are five sources to obtain and analyze the cost data for petroleum exploration, development, and production described as follows:

1) industry data from Texas American Resource Company in the form of responding the survey which is designed by the author of this dissertation as shown in Appendix B, and communications through emails between the dissertation author and the company's chief operating officer Dr. Tim Taylor (Taylor, 2010) and Operations Engineer Mr. Jake Klein (Klein, 2010)
2) academic data from the University of Texas at Austin in the form of responding the survey and email communications between the dissertation author and Dr. Paul M. Bommer (Bommer, 2010)
3) government data, including American Petroleum Institute (API) Joint Association Survey on Drilling Costs (JAS) (American Petroleum Institute, 2003), and communications through emails between the dissertation author and API Energy Market Statistics Analyst Mr. Raghavan Narayanan (Narayanan, 2010)

4) public accessible government data from U.S. Energy Information Administration (EIA) (U. S. Energy Information Administration, 2003, 2010b, and 2010c)
5) cost data from "Integration of the $NETL$ Oil and Gas Modeling Systems - Costs /Constraints Design Document" (U. S. Department of Energy, National Energy Technology Laboratory, 2003)

Appendix C contains the cost data for the acquisition, exploration, and development stages from three of the above mentioned five data sources. While cost data from different sources are compared, it is found out that the cost data from different sources usually do not agree with each other. For example, for a U.S. onshore exploration oil well at about 4,300 ft, the most updated (2008) average cost from API office (Narayanan, 2010) is $1,951,000 /well, while the cost from the industry data (2010) is only $325,000/well, as shown in Appendix C. Hence, further analyses are conducted on the available cost data in order to understand the reasons which account for the differences in the existing cost data and to choose the appropriate cost data for cash flow projections.

In the $NETL$ Oil and Gas Modeling Systems, there are complete data on the drilling and completion costs, additional costs for converting primary recovery to secondary recovery, variable and fixed operating costs, based on the region and well depth. However, the cost data are on the normalized oil price of $30/bbl. The formula may fit the costs in and before 2002 but it is too low to be used as current costs.

JAS data are commercial data from an industry survey. Figures 7-5 and 7-6 show the change of average oil well drilling cost from 1986 to 2008 in the form of cost/well and cost/foot, not specifying whether the well is onshore or offshore, an exploration or development well. Both figures show that drilling costs increased dramatically after year 2000. According to the 2002 JAS data, drilling cost for a U. S. onshore exploration oil well was \$296,000/well. The cost increased to \$1,951,000/well in 2008. This increase in drilling cost matches the general trend of drilling cost change shown in Figures 7-5 and 7-6.

EIA provides historical data of lease equipment costs (production facility costs) and operating costs. Those data are very useful for cost analysis in general, such as, cost changes with time, cost changes with oil and gas prices, and cost changes with well depth. However, a lot of data is confined to a lease of 10 production wells and 11 water injection wells. Figure 7-7 shows the cost change of water injection wells in West Texas. The trend matches the general changing pattern of drilling costs.

To determine a set of investment and operating costs for cash flow projections and real options analysis, the data from the above five sources are used to obtain the general trend of cost change, to identify the reasonable reference costs, and to guide the necessary adjustments to the existing cost data.

First, from both industry and academic data, it is observed that the cost of a water injection well, including drilling and completion, is very close to the cost of the exploration oil well plus the cost of completing the exploration oil well. EIA data contain water injection well cost (11 wells together) from 1986 to 2008 for wells in West Texas at various well depths. Figure 7-8 shows the comparison between the U. S. oil well cost and the water injection well cost in West Texas in the same time period from 1986 to 2008.

Unsurprisingly, the oil well cost is higher than the water injection well cost. After year 2000, the difference becomes much more significant. Comparing with the $NETL$ drilling and completion cost formula for a primary producer in the Mid-Continent from the 2000 JAS data, for a 4,000 foot depth oil well, the drilling and completion cost was \$186K/well. An EIA water injection well cost in year 2000 was \$226K/well. These two cost data are very close to each other.

Two sets of cost data are estimated and applied for cash flow projections: low costs and high costs, as showed in Cash Flow Data (1) and Cash Flow Data (2) in Table 7-4, Table 7-5, and Appendix C. For the low cost data, the cost for drilling and completing a water injection well is set as a reference cost for the drilling and completion cost for a primary oil producer. That is, the total cost for a primary production well, including both drilling and completion, at present time is estimated as \$751,545, of which \$451,545 is estimated as exploration drilling cost, and \$300,000 as completion cost. The high cost set uses JAS' most current exploration and development costs. For a 4,000 foot onshore oil well, the exploration cost is \$1,951,000 and the development cost is \$1,617,000 (Narayanan, 2010). Table 7-4 contains the detail cost data for acquisition, exploration, and development. From the EIA index (U. S. Energy Information Administration, 2010c), between 1994 and 2009, the changing rate of the water injection well cost is between -19% and 42% with an average of 4.97%, as shown in Figure 7-9. An annual escalation rate of 4.97% on the cost for water injection well and facilities is applied in the cash flow model.

For the oil production costs, the academic and EIA data are both very low. From the academic data, the cost for primary and water flooding oil recovery is \$8,000 per month and \$16,000 per month, respectively. EIA provides historical direct annual operating costs for primary and water flooding oil recovery in a lease with 10 producing wells and 11 water injection wells based on oil production rate of 90 bbl/day per well in West Texas (U. S. Energy Information Administration, 2010c). Converting the operating costs to cost/ bbl oil produced, it is from \$0.28/bbl to \$0.71/bbl for primary oil recovery, and from \$0.90/bbl to \$2.40/bbl for water flooding, during the period from 1986 to 1999, as shown in Figure 7-10. The cost data match the results from the $NETL$ operating cost formula for the cost in the year of 2000. Figure 7-11 shows the ratio of the direct annual operating costs of water flooding to primary oil production; it is between 2.4 and 2.5. The above analyzing results are used as guidance in determining the variable operating costs and the minimum operating costs of both the primary and water flooding oil recovery for the low and high cost cases, as shown in Table 7-5.

7.7.4 Cost of Capital for the Synthetic Petroleum $E\&P$ Project

The cost of capital for the synthetic petroleum $E\&P$ project is calculated with the weighted average cost of capital ($WACC$) as described in Eq. (7-1) or Eq. (7-2). According to the capital structure assumption made in section 7.7.2 for this synthetic project, the project is pure equity financed to prevent bankruptcy in case of downside petroleum production, that is,

$$w_E = 100\%\,,\ \text{and}\ \ w_D = 0\%\,.$$

Then Eq. (7-1) becomes

$$Cost\ of\ Capital = Cost\ of\ Equity,$$

that is,

$$r_C = \overline{r_E}.$$

Cost of equity is calculated with the Capital Asset Pricing Model ($CAPM$) (Duffie, 1992), that is,

$$\overline{r_E} = r_f + \beta_E\left(\overline{r_M} - r_f\right), \qquad (7-3)$$

where

r_f : risk-free rate of return,

$\overline{r_M}$: expected rate of return for market portfolio,

$(\overline{r_M} - r_f)$: market risk premium,

$$\beta_E = \frac{cov(r_E\ , r_M)}{var(r_M)},$$

r_M : rate of return for market portfolio.

When $\beta_E > 1$, then the individual stock is riskier than the market portfolio; and

$\beta_E < 1$, then the individual stock is less risky than the market portfolio.

(a) Beta ($\boldsymbol{\beta_E}$) for the Synthetic Petroleum $\boldsymbol{E\&P}$ Company

According to the assumptions made in section 7.7.2 for this synthetic petroleum $E\&P$ company, the company is an independent company and it only has business in the exploration and production of oil and gas. Thus, the beta (β_E) for the company is the beta for oil and gas assets of the company. The beta for this company is then estimated with the reference of the beta of the Apache independent oil and gas company, that is,

$\beta_E = 1.16$ (Apache Corporation, 2010).

(b) Expected Return for the Market Portfolio

Standard & Poor's 500 ($S\&P$ 500) is used as the market portfolio. The stocks included in the $S\&P$ 500 are traded on one of the two largest American stock market exchanges: the $NYSE$ and the $NASDAQ\ OMX$. The historical annual return of the $S\&P$ 500 from 1960 to 2009 is very volatile, as shown in Figure 7-12 ($CAGR$ of the Stock Market, 2010). The average annual return of $S\&P$ 500 from 1989 to 2009 is calculated as the expected annual return for market portfolio, that is,

$\overline{r_M} = 11.20\%$ ($CAGR$ of the Stock Market, 2010).

(c) Risk-free Rate of Return

Figure 7-13 shows the historical annual return for the 20-year Treasury bond from January 3, 2000 to September 30, 2010. The highest return is 6.97% and the lowest is 2.86%. The most current return is 3.38% (U. S. Department of the Treasury, 2010). The average return from January 3, 2000 to September 30, 2010 is calculated and used as a risk-free rate of return, that is,

$r_f = 4.97\%$.

(d) Cost of Equity for the Synthetic Petroleum $E\&P$ Company

According to the Beta (β_E) for the independent oil and gas company, the expected rate of return on a market portfolio $(\overline{r_M})$, and risk-free rate r_f, the cost of equity for the synthetic petroleum $E\&P$ company is calculated using Eq. (7-3):

$$\overline{r_E} = 4.97\% + 1.16 \times (11.20\% - 4.97\%) = 12.20\% .$$

(e) Cost of Capital and Risk Premium for Market Portfolio and for the Project

By assumption, the project is equity financed. Then the cost of capital equals the cost of equity, that is,

$$r_C = \overline{r_E} = 12.20\% .$$

The risk premium for market portfolio, or market risk premium, which is $\left(\overline{r_M} - r_f\right)$, is calculated as follows:

$$\left(\overline{r_M} - r_f\right) = 11.20\% - 4.97\% = 6.23\% .$$

The risk premium for the project is:

$$(r_C - r_f) = 12.20\% - 4.97\% = 7.23\% .$$

7.7.5 Fiscal Regime for the Cash Flow Projection for the Synthetic Petroleum $E\&P$ Project

According to the assumptions described in section 7.7.2, the synthetic petroleum $E\&P$ company and the project is in the United States. Thus the cash flow is calculated under the concessionary fiscal system. Table 7-3 contains the details of the fiscal regime in the United States, such as royalty and production tax, and the selected data used in this dissertation.

7.7.6 Calculation for Depreciation, Depletion, and Amortization

For the synthetic petroleum $E\&P$ project, the depreciation, depletion, and amortization, for the cost allocation of the capitalized acquisition, exploration, and development costs, is calculated using the financial accounting method. As described in sections 7.2 and 7.4.1, cost depletion is applied to the capitalized acquisition cost using the units-of-production method on the proved reserve, which is equal to the original oil in place ($OOIP$); cost depletion is also used for the capitalized exploration and development costs using the units-of-production method on the proved development reserve, which is equal to the cumulative recovery of $OOIP$ at 99% water cut.

7.7.7 Calculating Flow and Results of Cash Flow Forecasting for the Synthetic Petroleum $E\&P$ Project

Figures 7-14 (a) through (d) show parts of the calculating flow and results from one case of cash flow forecasting for the synthetic petroleum $E\&P$ project. Figure 7-14(a) includes fixed data on reservoir such as original oil in place ($OOIP$), fiscal regime such as royalty, risk adjusted discount rate, and risk free discount rate. It also shows the calculation of operating costs, including the minimum operating costs, for both primary oil recovery and water flooding oil recovery. Figure 7-14(b) contains initial investment costs and the calculation for the costs of water injection well and facility at different water flooding switching times. Figure 7-14(c) shows the schedule of depreciation, depletion, and amortization ($DD\&A$) for the initial investment and water flooding well and facility costs; it also shows the calculation for the remaining proved reserve and remaining proved developed reserve. Figure 7-14(d) includes the details for the calculation of cash flows according to the definition described in section 7.7.1 and for the calculation of NPV.

Table 7-1: Comparison of Successful Efforts and Full Cost Accounting Methods for the Petroleum $E\&P$ Industry

Cost Items	Successful Efforts Accounting	Full Cost Accounting
Pre-acquisition Costs	Expensed	Capitalized
Acquisition Costs	Capitalized if lead to proved reserves; otherwise expensed	Capitalized
Costs for Seismic Studies for Exploration Purposes	Expensed	Capitalized
G & G Costs for Exploration Purposes*	Expensed	Capitalized
Delayed Rentals	Expensed	Capitalized
Successful Exploratory Drilling Costs	Capitalized	Capitalized
Unsuccessful Exploratory Drilling Costs (Including Plugging and Abandoning Costs)	Expensed	Capitalized
Completing Costs for Successful Exploration Wells	Capitalized	Capitalized
Successful Development Drilling and Completing Costs	Capitalized	Capitalized
Unsuccessful Development Drilling Costs	Capitalized	Capitalized
Surface Facility Costs for Production	Capitalized	Capitalized
Production Costs	Expensed	Expensed

Source: Fundamentals of Oil and Gas Accounting (Gallun, Stevenson, and Nicoles, 1993)
*$G\&G$ Costs: Geological and Geophysical Costs

Table 7-2: Tax Purposes Cost Allocation for the Petroleum $E\&P$ Industry (for Unimpaired Properties)

	Major Oil and Gas Companies	Independent Oil and Gas Companies
Leasehold Costs	Cost depletion	Cost depletion, or percentage depletion for qualified companies
Intangible Drilling and Development Costs (IDC)	Successful Domestic Wells: 70% expensed, 30% amortization over 5 years with straight line method Unsuccessful Domestic Wells: 100% expensed	Expensed
Tangible Drilling and Development Costs (TDC)	Depreciated According to $MACRS*$	Depreciated According to $MACRS*$

$*$ $MACRS$: Modified Accelerated Cost Recovery System

Table 7-3: Fiscal Regime in the U. S. and the Selected Data Used in Dissertation

	U. S. Fiscal Regime	Data Used in Dissertation
Type of Fiscal Regime	Concession	Concession
Royalty, % of Gross Revenue	10-20; Usually 1/8 for onshore production, 1/6 for offshore production	12.5
Cost Recovery, %	100	100
Income Tax, % on Taxable Income	35	35
Production Tax, % of Net Revenue	1.25-12.5 (4.6 in Texas)	5
State Texas, % on Taxable Income	California: 9.6 Texas: 0 Oklahoma: 4.5	0

Source: 1) Brashear Group LLC, 2004
2) Fiscal System Analysis: Concessionary and Contractual Systems Used in Offshore Petroleum Arrangements (Kaiser and Pulsipher, 2004)

Table 7-4: Costs of Acquisition, Exploration, and Development for Cash Flow Model

Cost Items*	Cash Flow Data (1)	Cash Flow Data (2)
Capitalized Acquisition Cost	$160,000	$160,000
Capitalized Exploration Cost	$511,545**	$1,951,000
Capitalized Development Cost for One Production Well (Completion and Production Facility)	$412,500	$1,617,000
Capitalized Water Injection Cost for Four Wells (Drilling, Completion, and Facility)	$3,141,180	$14,272,000
Total Investment Cost without Water Injection	$1,084,045	$3,728,000
Total Investment Cost with Water Injection	$4,225,225	$18,000,000

*Appendix C contains the details of the data sources.

**The cost includes seismic and $G\&G$ studies for exploration purpose.

Table 7-5: Operation Costs ($OPEX$) from Different Sources and the Cost Data Used in Cash Flow Model

	$OPEX$ for Primary Oil Production	$OPEX$ for Water Flooding (5-Spot) Oil Production	Minimum Cost for Primary Oil Production	Minimum Cost for Water Flooding (5-Spot) Oil Production
Academic Data	$8,000/month	$16,000/month		
EIA Data	$0.71/bbl	$2.40/bbl		
Cash Flow Data (1)	$1.00/bbl	$3.00/bbl	$2,000/month	$6,000/month
Cash Flow Data (2)	$30.00/bbl	$50.00/bbl	$6,000/month	$1,8000/month

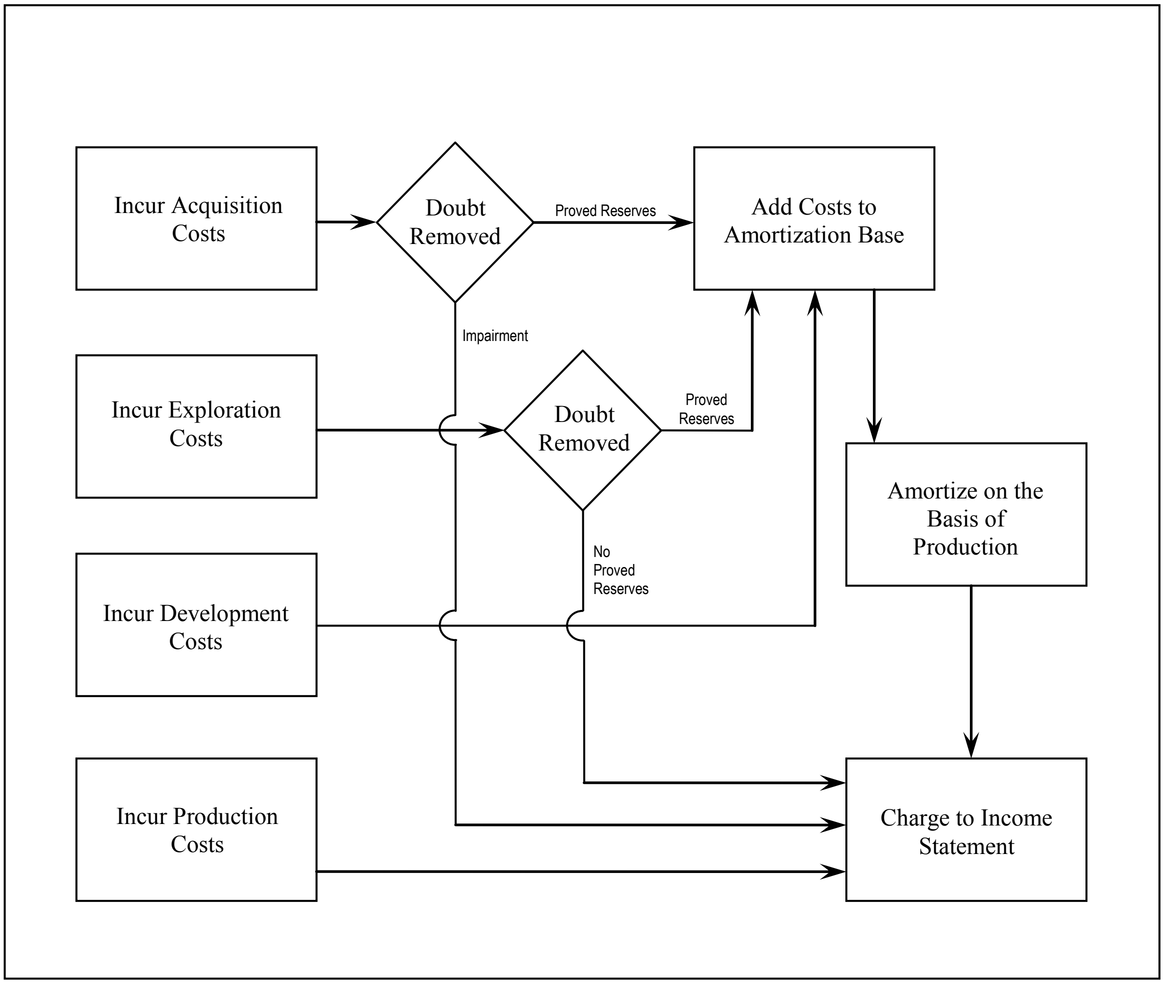


Figure 7-1: Overview of the Successful Efforts Accounting Method for the Capitalization of Acquisition, Exploration, Development, and Production Costs in the Petroleum $E\&P$ Industry

Sources: Handbook on Oil and Gas Accounting (Koester, 1982)

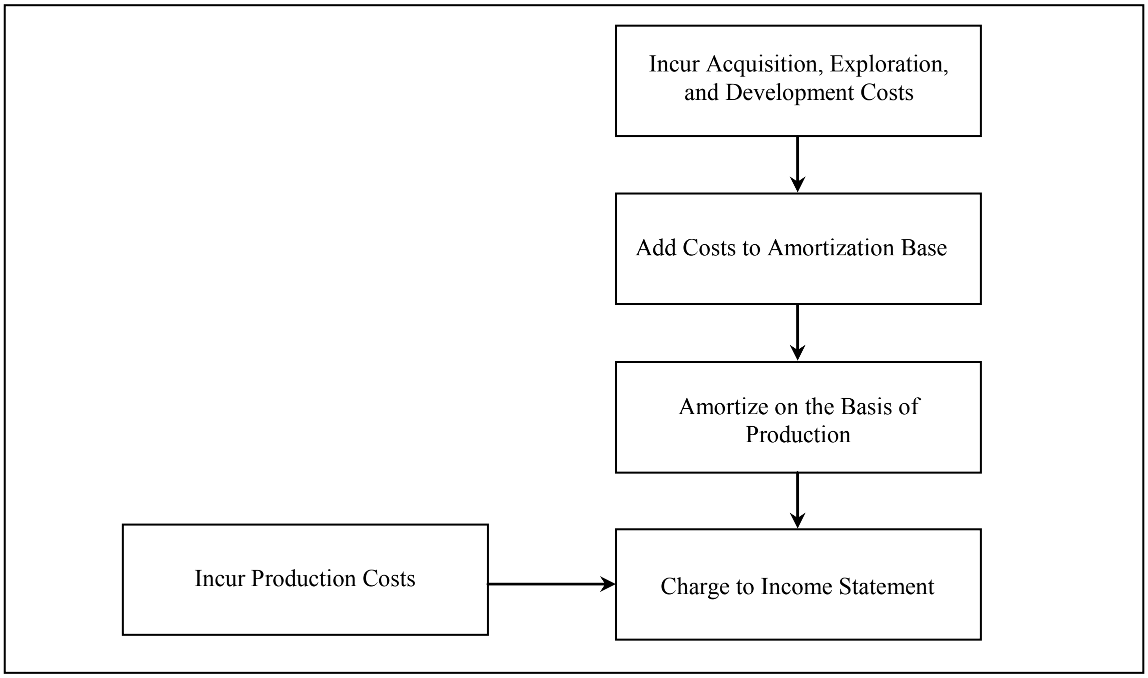


Figure 7-2: Overview of the Full Cost Accounting Method for the Capitalization of Acquisition, Exploration, Development, and Production Costs in the Petroleum $E\&P$ Industry

Sources: Handbook on Oil and Gas Accounting (Koester, 1982)

<table>
<tr><td rowspan="5">Production (Gross) Revenues</td><td rowspan="2">Total Profit</td><td>Rent</td><td>Government Take
• Bonuses
• Royalties
• Production-Sharing
• Taxes
• Government Participation</td></tr>
<tr><td rowspan="4">Contractor Entitlement</td><td>Contractor Take (Net cash flow to operator)</td></tr>
<tr><td rowspan="3">Cost Recovery</td><td>Operating Costs</td></tr>
<tr><td>Development Costs</td></tr>
<tr><td>Exploration Costs</td></tr>
</table>

Figure 7-3: Allocations of Revenues from Petroleum Production

Source: International Petroleum Fiscal Systems and Production Sharing Contracts (Johnston, 1994)

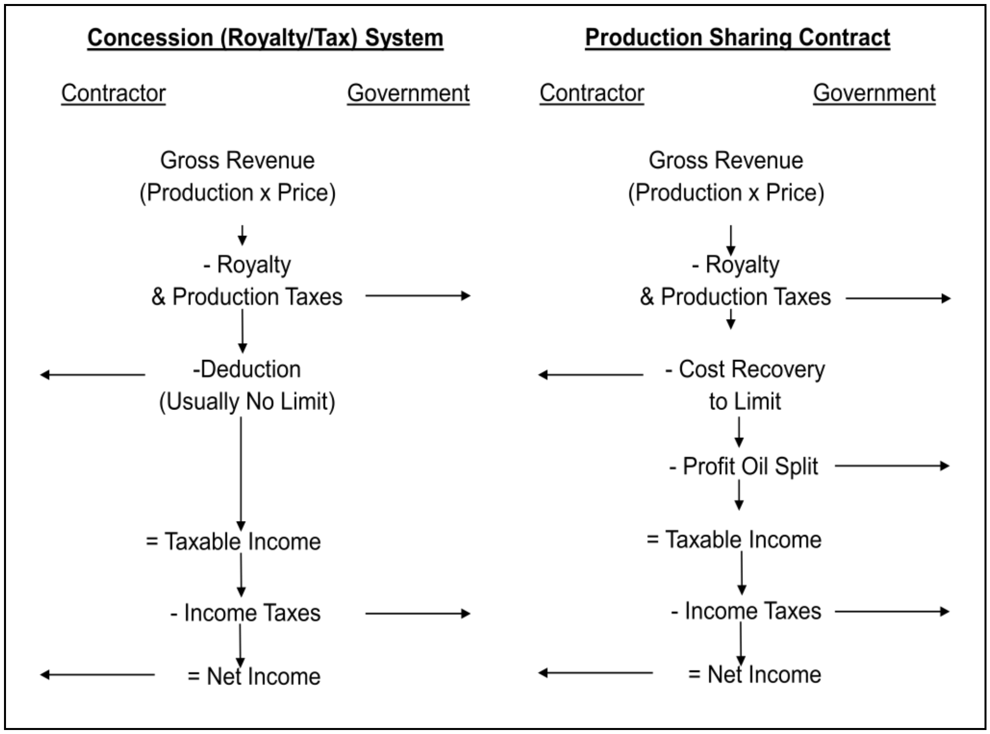


Figure 7-4: Flow Chart to Calculate Net Income under the Concessionary System and the Production Sharing Contracts (PSC)

Source: Brashear Group LLC, 2004

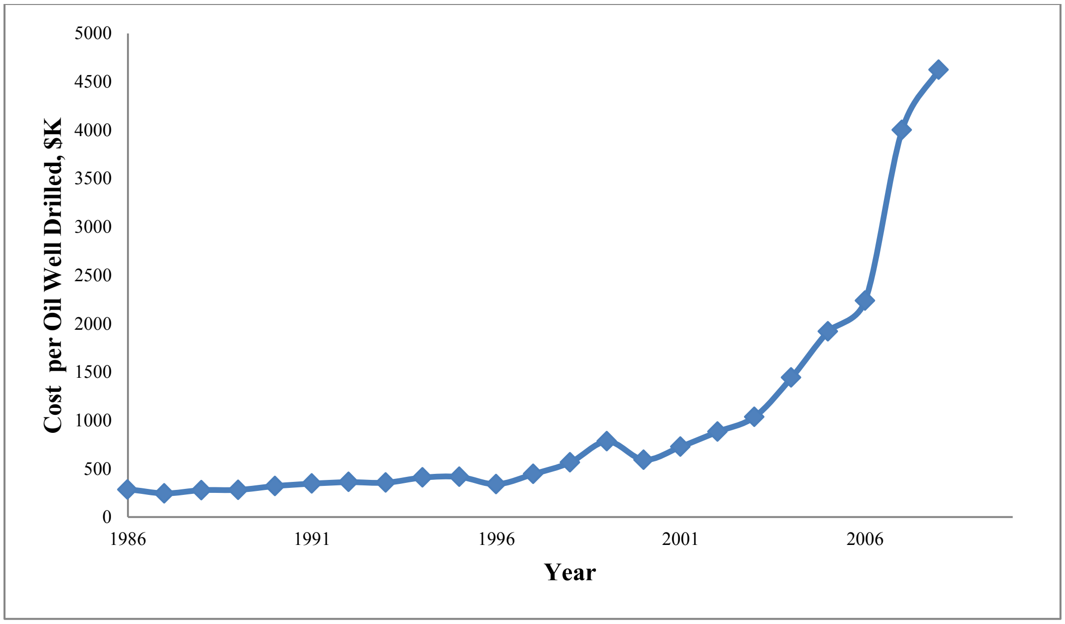


Figure 7-5: Cost Changes with Time per Oil Well Drilled from 1986 to 2008
Source: U. S. Energy Information Administration, 2010b

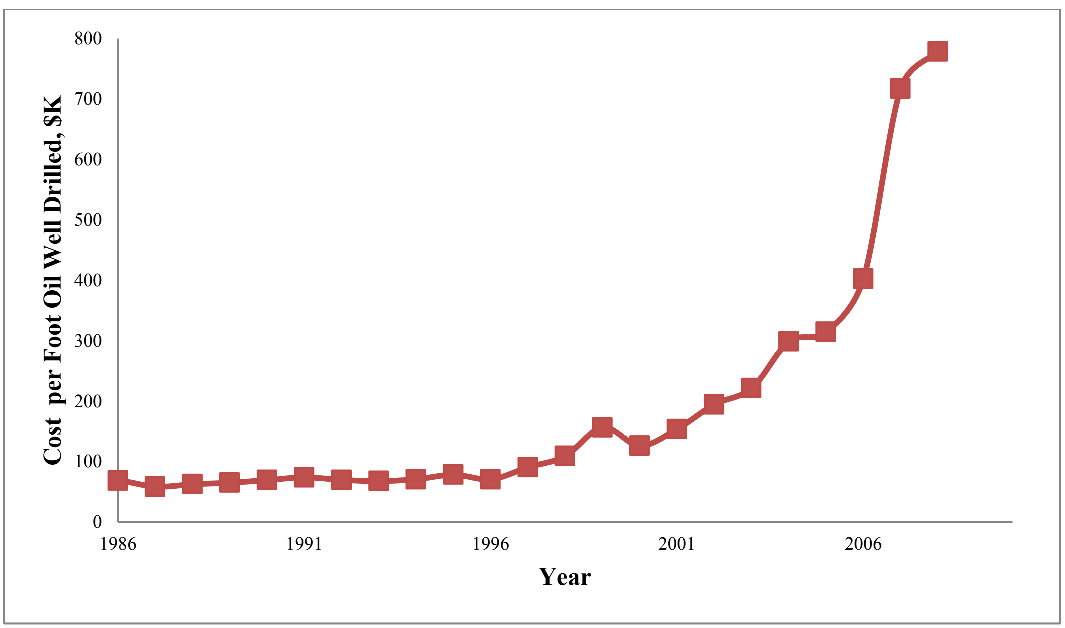


Figure 7-6: Cost Changes with Time per Foot Oil Well Drilled from 1986 to 2008
Source: U. S. Energy Information Administration, 2010b

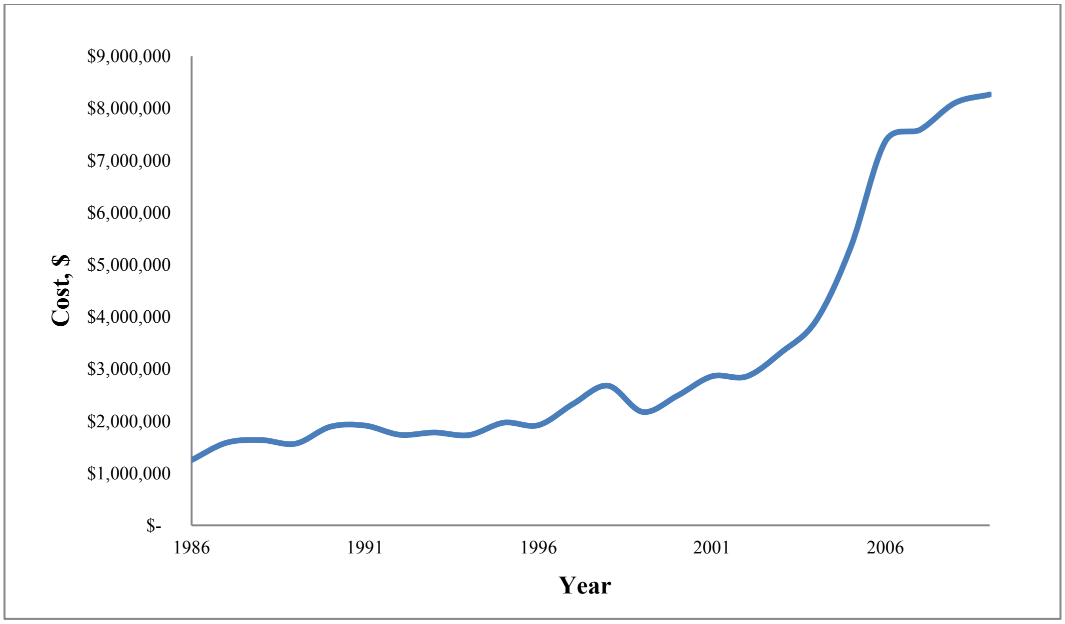


Figure 7-7: Water Injection Well Cost Change from 1986 to 2008 (11 Wells in West Texas at 4000 ft)
Source: U. S. Energy Information Administration, 2003 and 2010c

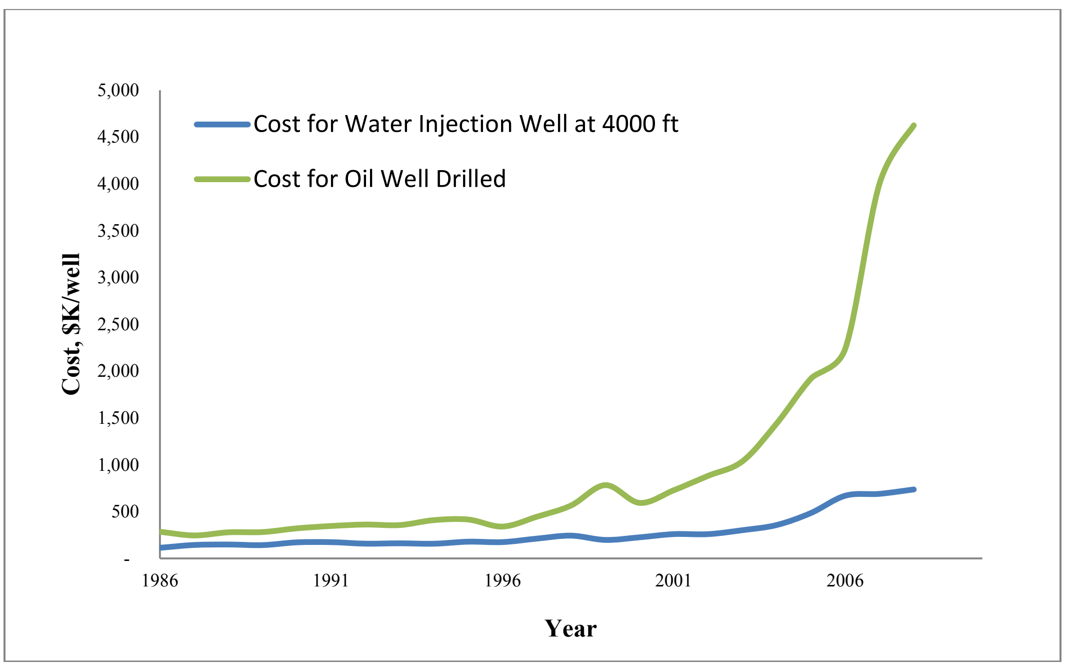


Figure 7-8: Comparison between Oil Well Cost and Water Injection Well Cost in West Texas from 1986 to 2008
Source: U. S. Energy Information Administration, 2003 and 2010c

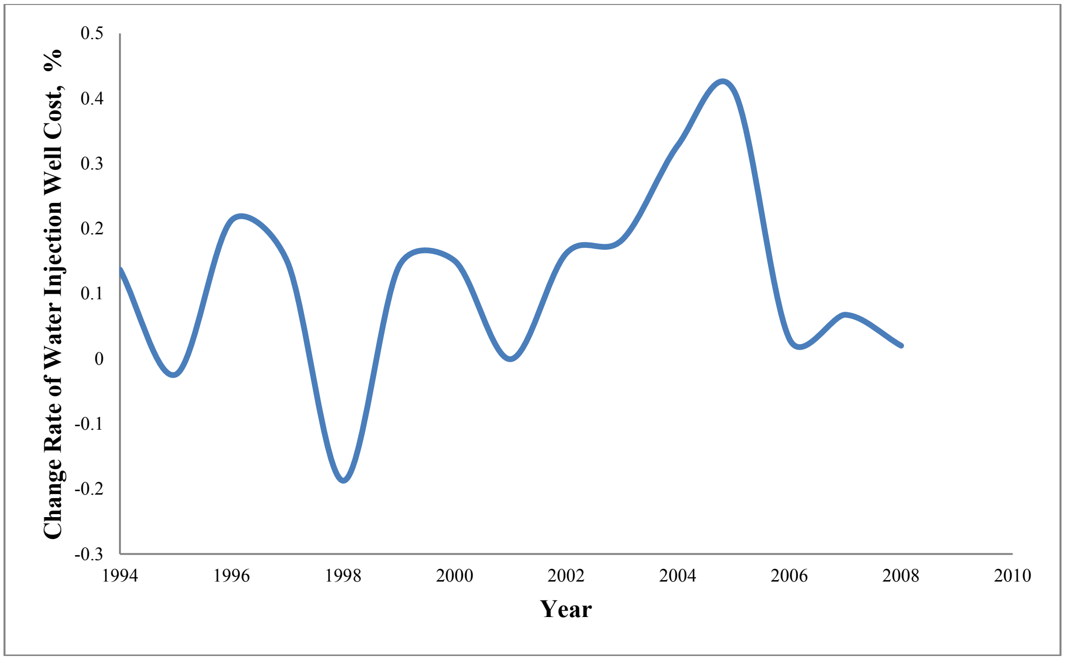


Figure 7-9: Change Rate of Water Injection Well Cost from 1994 to 2009
Source: U. S. Energy Information Administration, 2010c

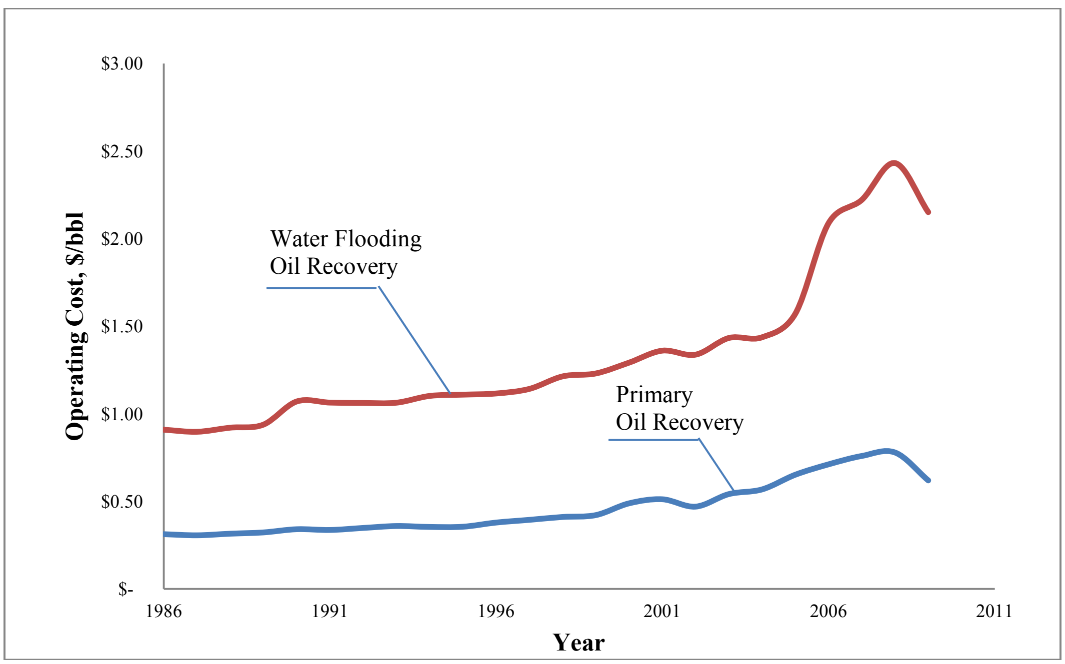


Figure 7-10: West Texas Direct Annual Operating Cost Changes with Time for Primary/Water Flooding Oil Recovery at 4000 ft per bbl Oil Produced
Source: U. S. Energy Information Administration, 2003 and 2010c

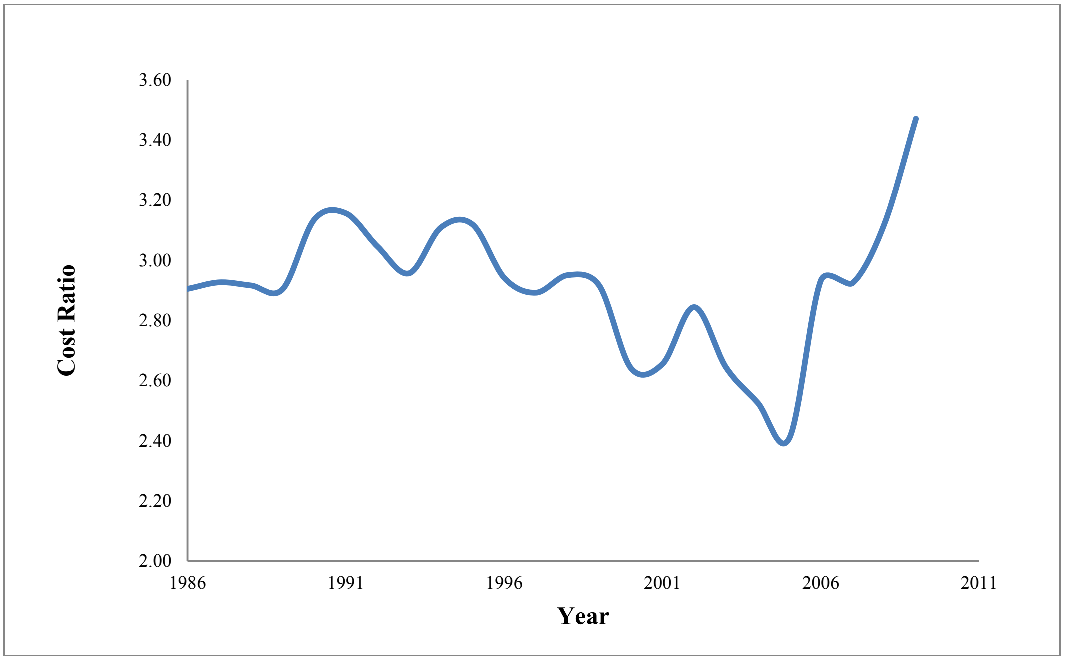


Figure 7-11: Ratio of Operating Cost of Water Flooding to Primary Oil Recovery for 10 Oil Producers (90bbls of Oil Produced per Day per Well) in West Texas
Source: U. S. Energy Information Administration, 2003 and 2010c

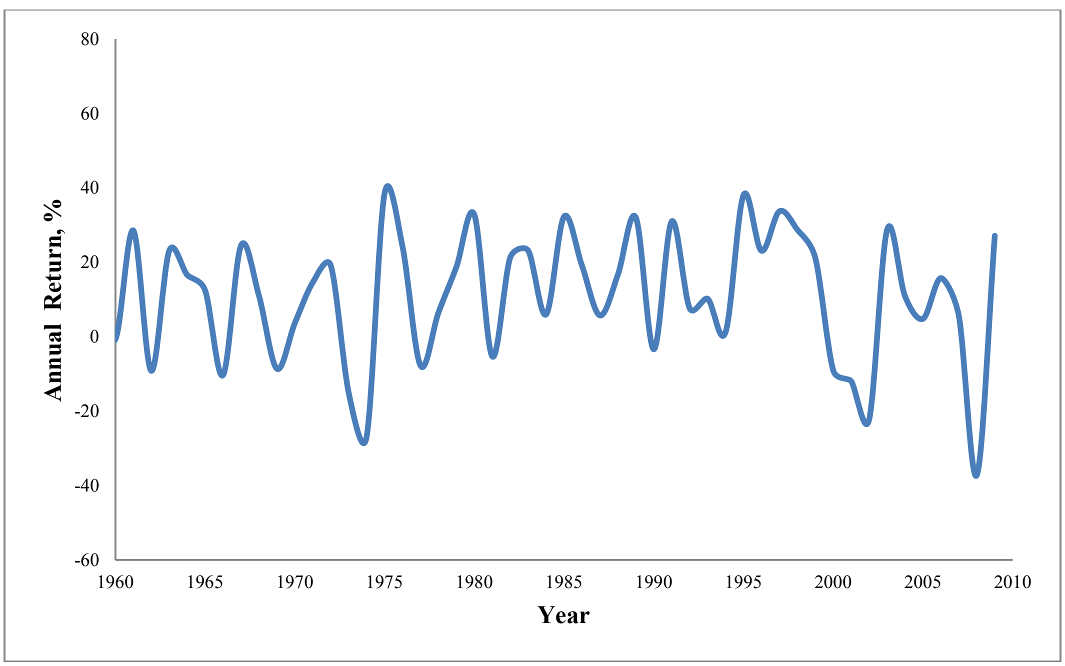


Figure 7-12: *S&P* 500 Historical Annual Return from 1960 to 2009
Source: CAGR of the Stock Market, 2010

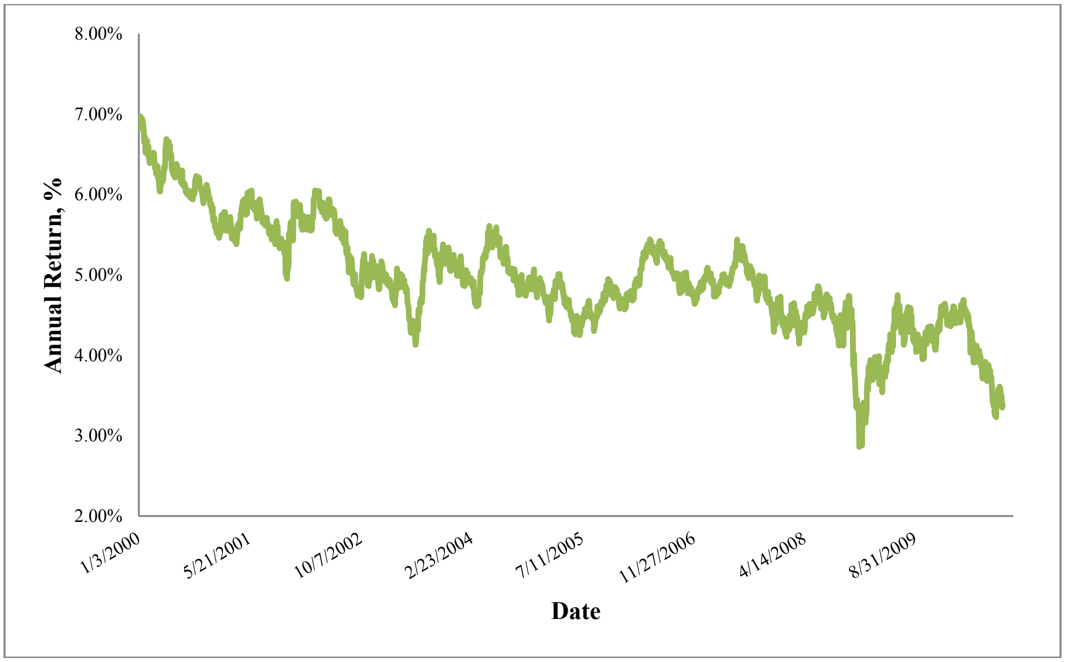
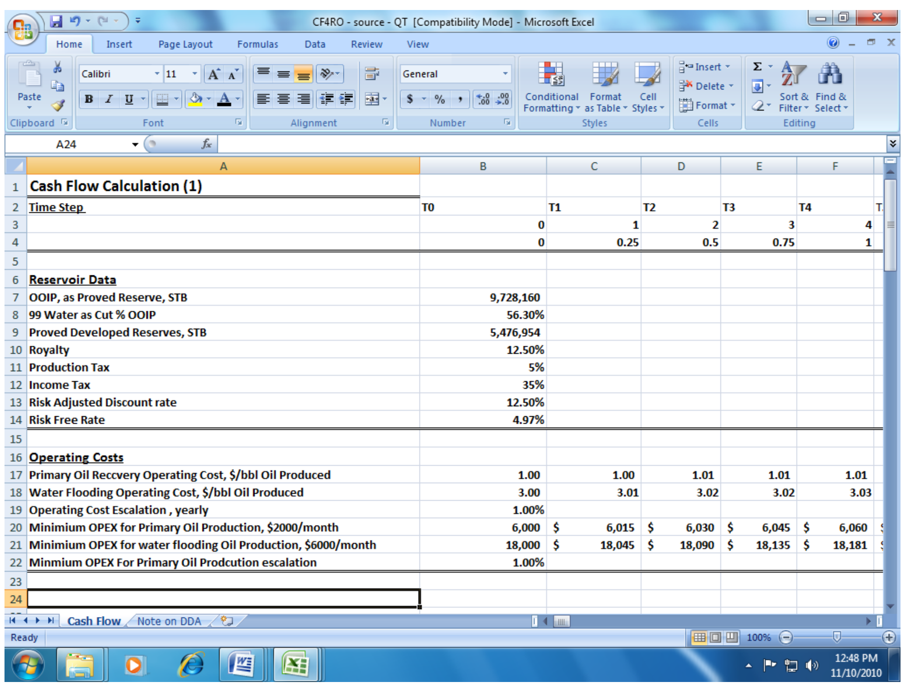
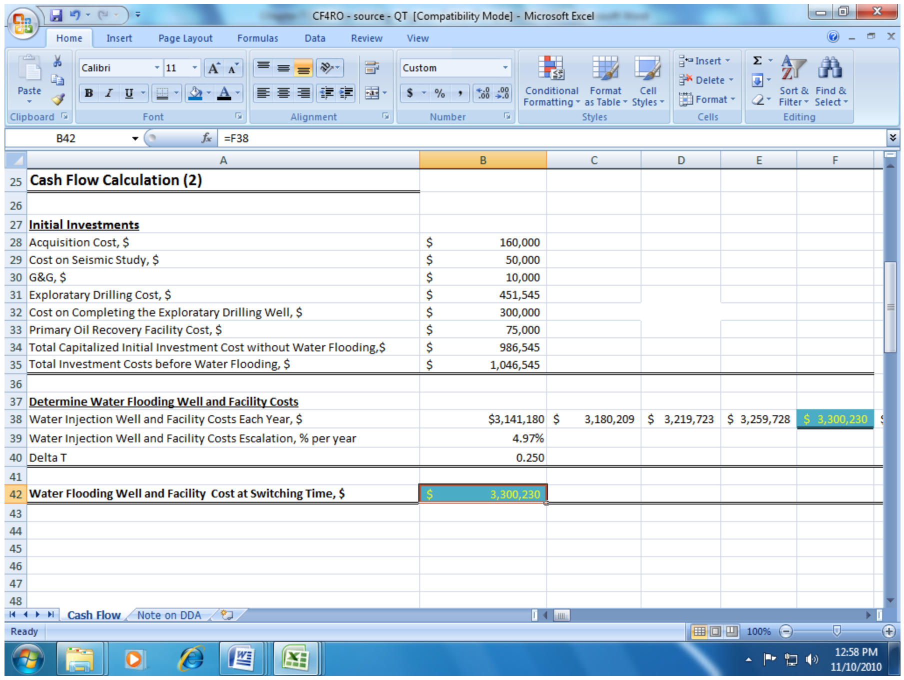


Figure 7-13: 20-Year Treasury Bond Annual Return
Source: United State Department of the Treasury, 2010



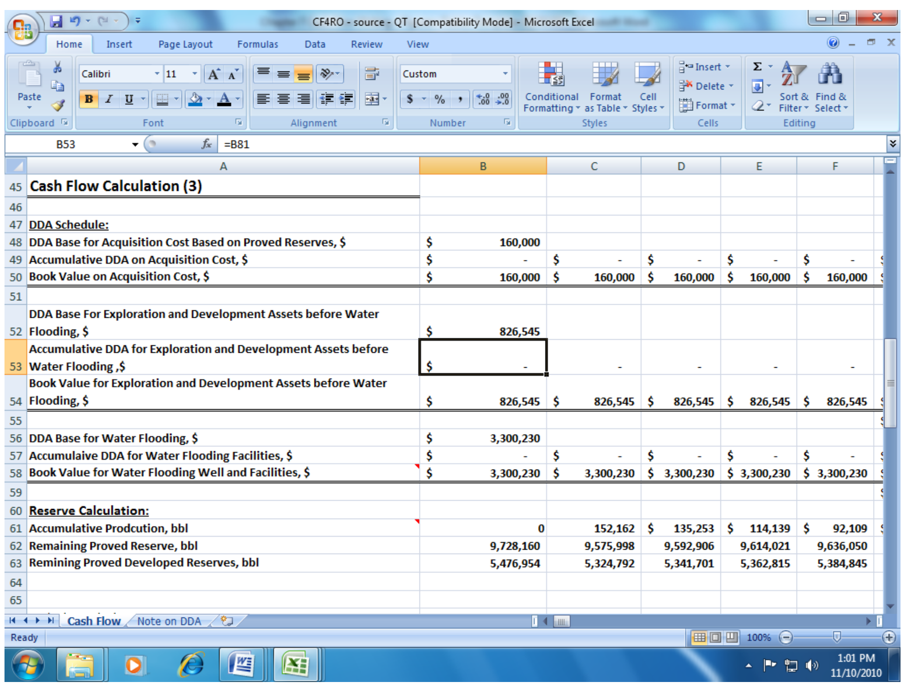
	A	B	C	D	E	F
1	**Cash Flow Calculation (1)**					
2	Time Step	T0	T1	T2	T3	T4
3		0	1	2	3	4
4		0	0.25	0.5	0.75	1
5						
6	**Reservoir Data**					
7	OOIP, as Proved Reserve, STB	9,728,160				
8	99 Water as Cut % OOIP	56.30%				
9	Proved Developed Reserves, STB	5,476,954				
10	Royalty	12.50%				
11	Production Tax	5%				
12	Income Tax	35%				
13	Risk Adjusted Discount rate	12.50%				
14	Risk Free Rate	4.97%				
15						
16	**Operating Costs**					
17	Primary Oil Reccvery Operating Cost, $/bbl Oil Produced	1.00	1.00	1.01	1.01	1.01
18	Water Flooding Operating Cost, $/bbl Oil Produced	3.00	3.01	3.02	3.02	3.03
19	Operating Cost Escalation , yearly	1.00%				
20	Minimium OPEX for Primary Oil Production, $2000/month	6,000	$ 6,015	$ 6,030	$ 6,045	$ 6,060
21	Minimium OPEX for water flooding Oil Production, $6000/month	18,000	$ 18,045	$ 18,090	$ 18,135	$ 18,181
22	Minmium OPEX For Primary Oil Prodcution escalation	1.00%				
23						
24						

Figure 7-14(a): Cash Flow Calculation (1) - Data on Reservoir, Fiscal Regime, Discount Rates, and Operating Costs



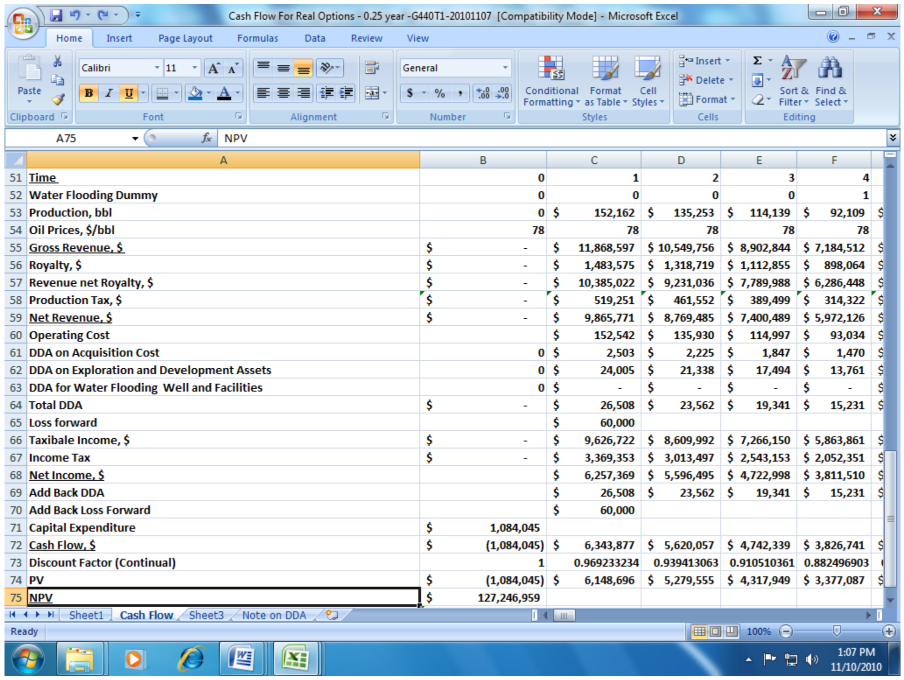
	A	B	C	D	E	F
25	**Cash Flow Calculation (2)**					
26						
27	**Initial Investments**					
28	Acquisition Cost, $	$ 160,000				
29	Cost on Seismic Study, $	$ 50,000				
30	G&G, $	$ 10,000				
31	Exploratary Drilling Cost, $	$ 451,545				
32	Cost on Completing the Exploratary Drilling Well, $	$ 300,000				
33	Primary Oil Recovery Facility Cost, $	$ 75,000				
34	Total Capitalized Initial Investment Cost without Water Flooding,$	$ 986,545				
35	Total Investment Costs before Water Flooding, $	$ 1,046,545				
36						
37	**Determine Water Flooding Well and Facility Costs**					
38	Water Injection Well and Facility Costs Each Year, $	$3,141,180	$ 3,180,209	$ 3,219,723	$ 3,259,728	$ 3,300,230
39	Water Injection Well and Facility Costs Escalation, % per year	4.97%				
40	Delta T	0.250				
41						
42	**Water Flooding Well and Facility Cost at Switching Time, $**	$ 3,300,230				

Figure 7-14(b): Cash Flow Calculation (2) - Initial Investment and Costs of Water Flooding Well and Facility at Different Water Flooding Switching Times



	A	B	C	D	E	F
45	**Cash Flow Calculation (3)**					
46						
47	**DDA Schedule:**					
48	DDA Base for Acquisition Cost Based on Proved Reserves, $	$ 160,000				
49	Accumulative DDA on Acquisition Cost, $	$ -	$ -	$ -	$ -	$ -
50	Book Value on Acquisition Cost, $	$ 160,000	$ 160,000	$ 160,000	$ 160,000	$ 160,000
51						
52	DDA Base For Exploration and Development Assets before Water Flooding, $	$ 826,545				
53	Accumulative DDA for Exploration and Development Assets before Water Flooding ,$	$ -	-	-	-	-
54	Book Value for Exploration and Development Assets before Water Flooding, $	$ 826,545	$ 826,545	$ 826,545	$ 826,545	$ 826,545
55						
56	DDA Base for Water Flooding, $	$ 3,300,230				
57	Accumulaive DDA for Water Flooding Facilities, $	$ -	$ -	$ -	$ -	$ -
58	Book Value for Water Flooding Well and Facilities, $	$ 3,300,230	$ 3,300,230	$ 3,300,230	$ 3,300,230	$ 3,300,230
59						
60	**Reserve Calculation:**					
61	Accumulative Prodcution, bbl	0	152,162	$ 135,253	$ 114,139	$ 92,109
62	Remaining Proved Reserve, bbl	9,728,160	9,575,998	9,592,906	9,614,021	9,636,050
63	Remining Proved Developed Reserves, bbl	5,476,954	5,324,792	5,341,701	5,362,815	5,384,845

Figure 7-14(c): Cash Flow Calculation (3) -*DD&A* Schedule and Remaining Reserve



	A	B	C	D	E	F
51	Time	0	1	2	3	4
52	Water Flooding Dummy	0	0	0	0	1
53	Production, bbl	0	$ 152,162	$ 135,253	$ 114,139	$ 92,109
54	Oil Prices, $/bbl	78	78	78	78	78
55	Gross Revenue, $	$ -	$ 11,868,597	$ 10,549,756	$ 8,902,844	$ 7,184,512
56	Royalty, $	$ -	$ 1,483,575	$ 1,318,719	$ 1,112,855	$ 898,064
57	Revenue net Royalty, $	$ -	$ 10,385,022	$ 9,231,036	$ 7,789,988	$ 6,286,448
58	Production Tax, $	$ -	$ 519,251	$ 461,552	$ 389,499	$ 314,322
59	Net Revenue, $	$ -	$ 9,865,771	$ 8,769,485	$ 7,400,489	$ 5,972,126
60	Operating Cost		$ 152,542	$ 135,930	$ 114,997	$ 93,034
61	DDA on Acquisition Cost	0	$ 2,503	$ 2,225	$ 1,847	$ 1,470
62	DDA on Exploration and Development Assets	0	$ 24,005	$ 21,338	$ 17,494	$ 13,761
63	DDA for Water Flooding Well and Facilities	0	$ -	$ -	$ -	$ -
64	Total DDA	$ -	$ 26,508	$ 23,562	$ 19,341	$ 15,231
65	Loss forward		$ 60,000			
66	Taxibale Income, $	$ -	$ 9,626,722	$ 8,609,992	$ 7,266,150	$ 5,863,861
67	Income Tax	$ -	$ 3,369,353	$ 3,013,497	$ 2,543,153	$ 2,052,351
68	Net Income, $		$ 6,257,369	$ 5,596,495	$ 4,722,998	$ 3,811,510
69	Add Back DDA		$ 26,508	$ 23,562	$ 19,341	$ 15,231
70	Add Back Loss Forward		$ 60,000			
71	Capital Expenditure	$ 1,084,045				
72	Cash Flow, $	$ (1,084,045)	$ 6,343,877	$ 5,620,057	$ 4,742,339	$ 3,826,741
73	Discount Factor (Continual)	1	0.969233234	0.939413063	0.910510361	0.882496903
74	PV	$ (1,084,045)	$ 6,148,696	$ 5,279,555	$ 4,317,949	$ 3,377,087
75	NPV	$ 127,246,959				

Figure 7-14(d): Cash Flow Calculation (4) - Cash Flow and Net Present Value (NPV)

CHAPTER 8: APPLICATION OF REAL OPTIONS IN THE PETROLEUM $E\&P$ INDUSTRY

8.1 THE NEED FOR REAL OPTIONS EVALUATION METHOD IN THE PETROLEUM $E\&P$ INDUSTRY

After an oil field is discovered and developed, there are three major stages in the production of petroleum, *i.e.,* primary, secondary, and tertiary production. In primary production, oil is produced by natural drives in the reservoir. Statistically, when a reservoir approaches the end of its primary production, 25% to 95% of the oil in the reservoir may remain there. Some portion of the remaining oil is recovered through various secondary oil recovery methods. Water flooding is the least expensive and most widely used secondary oil recovery method.

As shown in Figure 6-28, for the reservoir described in Chapter 6, primary oil production only recovers about 10% of the original oil in place ($OOIP$). The oil production rate becomes very low when the reservoir pressure decreases to close to the oil production bottom hole pressure (*BHP*). However, the time that oil is produced with a low oil production rate can last many years. As shown in Figure 8-1, after about four years of primary oil production, the oil production rate decreases to less than $50bbl/day$. The production time when the oil production rate is less than $5bb/day$ can last as long as 10 years. When should be the time to switch from primary to secondary recovery? Should the switching time be at the end of the primary oil recovery when the oil production rate is very low, at an earlier time when oil production rate is still high, or even at the beginning of the primary oil production?

The industry may use the criteria called "economic limit" to make the decision on when to switch from primary to secondary oil recovery. When the benefit from the oil production is less than the cost, the current production option stops. "Some leases will expire if positive cash flow is not received for 30 or 60 days" (Taylor, 2010). This is a passive action to some extent. In addition, when the negative cash flow occurs, is it because of the low oil price, or because of the low oil production rate? If it is because of the low oil production rate, it can be overcome by switching to water flooding earlier.

Another method is to exhaust all possibilities of water flooding switching times, simulate the corresponding oil production profiles, calculate the net present values, and then select the switching time with the highest NPV value. However, this method only works when the future oil prices are known and will not be different from the expected prices. For example, if the best water flooding switching time is at the 4^{th} year according to the highest NPV evaluation, when approaching the 3^{rd} year, the actual oil price is very low, should the water flooding still start at the 4^{th} year? On the other hand, when at the 3^{rd} year, the oil price is very high, should the company still wait for the 4^{th} year to start the water flooding?

For an oil and gas company, how can it make the best from a developed reservoir by arranging the oil production according to specific reservoir conditions and the changing oil prices and costs so that the total value the company receives from the natural resource can be maximized?

The best solution to this problem is the real options method - applying option pricing theory to the evaluation and decision making. The real options method takes oil price fluctuation, oil production changes with different water flooding switching times, and managers' flexibility of exercising switching according to oil prices and production states into consideration and discovers the best time of switching. Thus, the maximum value of producing the oil reservoir can be captured.

This chapter uses the synthetic reservoir described in Chapter 6 and the oil price models discussed in Chapter 3 and Chapter 4 to perform the real options evaluation on when to switch from primary to secondary (water flooding) oil recovery. This application provides a scientific approach to find the best switching time and the value of the project conditioning on the decision to start water flooding at the best switching time at different price states.

8.2 Methods of Real Options Evaluation

Option pricing theory starts from financial options. In a financial market, an option is a right, not an obligation, to buy or sell an asset, which is called underlying, at the exercise price on or before the expiration time of the option. If an option is allowed to be exercised before the expiration time, it is called an American option. If an option can only be exercised at the expiration time, it is called a European option. A call option gives the holder of the option the right to buy the underlying asset. A put option gives the holder of the option the right to sell the underlying asset.

A real option is a right, not an obligation, to take an action at a specified cost on or before a predetermined time. Analogy to a financial option, the specified cost is the exercise price for the real option; the predetermined time is the expiration time of the real option.

The value of a financial option can be determined with the Black-Scholes option pricing model. For example, the value of a European call option with an underlying asset S, such as a stock, is calculated through the following Black-Scholes formula (Black and Scoles, 1973, and Hull, 2000):

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2),$$

$$d_1 = \frac{ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T},$$

where C is the current value of the call option, S_0 is the current value of the underlying asset S, r is risk-free rate, K is the option exercise price, T is time to option expiration, σ is the volatility of the underlying asset, and $N(.)$ is the cumulative normal distribution function.

The Black-Scholes option pricing model is based on the assumptions that the underlying follows the geometric Brownian motion (GBM) process; there is no dividend payment; the volatility σ of the underlying process is constant; the risk-free rate is constant; and the option is a European option, which means the options can only be exercised when it is mature (Hull, 2000).

Even though the water flooding switching option can mimic the European option through a bundle of European options, the other assumptions, such as the GBM process and constant volatility of the project value, are difficult to satisfy. In reality, as it has been shown in Chapter 6, when the oil production rate changes so dramatically, it is very unlikely that the project value would evolve according to the GBM process. So the evaluation of water flooding switching does not use the Black-Scholes model in this study.

Another method of pricing stock options is the binomial model with a "replicating portfolio." Assume that the stock price follows a binomial process in the time interval $[t,\ t+\Delta t]$. During this time interval, the stock price changes from S to Su with probability q, and from S to Sd with probability $(1-q)$; the bond price increases from B to $e^{r\Delta t}B$, where r is the risk-free interest rate. The price of holding Δ shares of stocks and bonds with value B is $(S\Delta + B)$ at the beginning time. It will be $(Su\Delta + e^{r\Delta t}B)$ with probability q and $(Sd\Delta + e^{r\Delta t}B)$ with probability $(1-q)$ at the ending time of $t+\Delta t$. Assume that the current value of the call option of the underlying stock is C, the value of the call option will be either C_u, which equals $Max(0, Su-K)$, when the stock price is high with probability q, or C_d, which is $Max(0, Sd-K)$, with probability $(1-q)$ when stock price is low. K is the exercise price of the option.

Let the "replicating portfolio" exactly mimic the payoffs to the call option, that is,

$$Su\Delta + e^{r\Delta t}B = C_u,$$

$$Sd\Delta + e^{r\Delta t}B = C_d,$$

$$S\Delta + B = C.$$

Then

$$\Delta = \frac{C_u - C_d}{S(u-d)},$$

$$B = \frac{C_d u - C_u d}{(u-d)e^{r\Delta t}},$$

$$C = [qC_u + (1-q)C_d]e^{-r\Delta t},$$

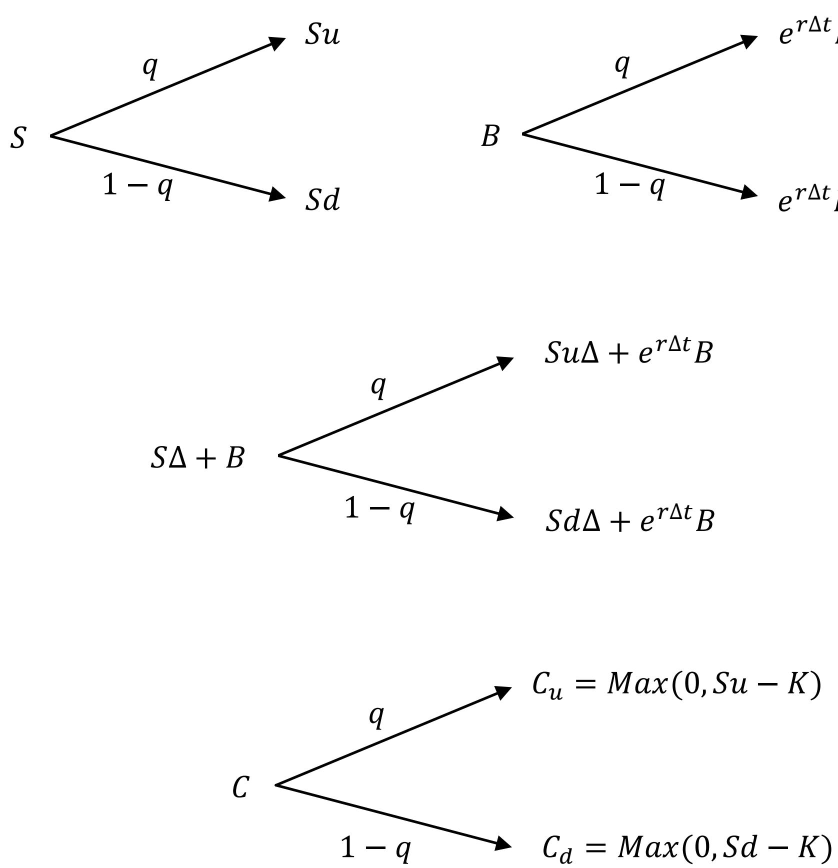
where

$$q = \frac{e^{r\Delta t} - d}{u - d},$$

$$1 - q = \frac{u - e^{r\Delta t}}{u - d},$$

$$d < e^{r\Delta t} < u\,.$$

The following chart demonstrates the pricing of stock options with binomial model:



With the knowledge about the underlying process, u and d, together with the exercise price K, C_u and C_d can be determined. With u, d, and risk-free rate r, q can be obtained. Then the option price C can be calculated with C_u, C_d, q, and r. The binomial model is very flexible and useful to value the stock options. However, it is not intuitive if applied to the water flooding switching options. It may not be able to find a traded asset, which can be used with bonds to exactly mimic the payoffs of the value of the water flooding switching.

In this study, the value of the flexibility in water flooding switching time is not directly calculated from any of the above formulae. It is through the comparison of the values of the project with and without the flexibility in water flooding switching time. Oil prices are separated from oil production. In other words, there is no need to model the cash flow or project value process. Oil prices are first modeled with GBM and mean reversion models; then the binomial lattice method is applied to the GBM and mean reversion price models; and then for each binomial lattice of oil prices, with the oil production profile, a cash flow lattice is generated. The value lattice of the project is then calculated step by step with the cash flow lattice. The following sections present the details about the process of calculating the values of the flexibility in water flooding switching time.

8.3 Binomial Lattice for a Stochastic Process

A stochastic price process can be reconstructed with a binomial lattice developed by Cox, Ross, and Rubinstein (1979) for the GBM model, and by Nelson and Ramaswamy (1990) for the mean reversion model. At any time interval $[t, t + \Delta t]$, a stochastic process of a random variable X with X_t at the beginning of the time interval t, will either move up to X_t^+ with a probability of q_t, or move down to X_t^- with a probability of $(1-q_t)$, at the end of the time interval $t+\Delta t$. In a risk-neutral world, as is described in section 8.4, X_t^+, X_t^-, and q_t can be calculated from the parameters in the stochastic process. q_t is then called risk-neutral probability or martingale equivalent probability.

With a general stochastic process described as

$$dX = \alpha(X,t)dt + \sigma(X,t)dW\,, \tag{8- 1}$$

where W is the Wiener process, X_t^+, X_t^-, and q_t can be calculated as

$$X_t^+ \equiv X_t + \sigma(X,t)\sqrt{\Delta t}, \tag{8- 2}$$

$$X_t^- \equiv X_t - \sigma(X,t)\sqrt{\Delta t}, \tag{8- 3}$$

$$q_t \equiv \frac{1}{2}\left(1 + \frac{\alpha(X,t)}{\sigma(X,t)}\sqrt{\Delta t}\right). \tag{8- 4}$$

When the above risk-neutral probabilities are applied to the up and down movements of a random variable, the expectations of future values of the random variable can be discounted using the risk-free discount rate.

8.3.1 Binomial Lattice for the GBM Oil Price Model

For the stochastic price P evolving according to the GBM process

$$dP = \mu P dt + \sigma P dW, \tag{8 − 5}$$

where μ and σ are assumed constant in the process, let

$$X = LnP, \tag{8 − 6}$$

then

$$dX = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dW, \quad (8-7)$$

$$X_t^+ = X_t + \sigma\sqrt{\Delta t}, \quad (8-8)$$

$$X_t^- = X_t - \sigma\sqrt{\Delta t}, \quad (8-9)$$

$$q_t \equiv \frac{1}{2}\left[1 + \frac{(\mu - \frac{\sigma^2}{2})}{\sigma}\sqrt{\Delta t}\right]. \quad (8-10)$$

Since $X = LnP$, then

$$Ln(P_t^+) \equiv LnP_t + \sigma\sqrt{\Delta t} \,,$$

$$Ln(P_t^-) \equiv LnP_t - \sigma\sqrt{\Delta t} \,,$$

$$P_t^+ = e^{\sigma\sqrt{\Delta t}} P_t = P_t u \,, \quad (8-11)$$

$$P_t^- = e^{-\sigma\sqrt{\Delta t}} P_t = P_t d \,, \quad (8-12)$$

where

$$u = e^{\sigma\sqrt{\Delta t}} \,, \quad (8-13)$$

$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u} \,. \quad (8-14)$$

According to Eq. 8-10, along the price evolution following the GBM stochastic process, the risk-neutral probabilities for price up and down movements are the same since μ and σ are assumed constant.

8.3.2 Binomial Lattice for the One-factor Mean Reversion Oil Price Model

For a stochastic price P evolving according to the one-factor mean reversion process

$$\frac{dP}{P} = \eta(Ln\bar{p} - LnP)dt + \sigma dW \,, \qquad (8-15)$$

where η, $\bar{p}$, and σ are assumed constant in the process, let

$$X = LnP, \qquad (8-16)$$

then

$$dX = \eta\left(Ln\bar{p} - \frac{\sigma^2}{2\eta} - LnP\right)dt + \sigma dW,$$

$$dX = \eta(\bar{x} - X)dt + \sigma dW, \qquad (8-17)$$

where

$$\bar{x} = Ln\bar{p} - \frac{\sigma^2}{2\eta}\,. \qquad (8-18)$$

Then according to Nelson and Ramaswamy (1990),

$$X_t^+ = X_t + \sigma\sqrt{\Delta t}, \qquad (8-19)$$

$$X_t^- = X_t - \sigma\sqrt{\Delta t}, \qquad (8-20)$$

$$q_t \equiv \left\{\begin{array}{ll} \frac{1}{2}\left[1 + \frac{\eta(\bar{x} - X_t)}{\sigma}\sqrt{\Delta t}\right] & if \quad 0 \leq \frac{1}{2}\left[1 + \frac{\eta(\bar{x} - X_t)}{\sigma}\sqrt{\Delta t}\right] \leq 1 \\ 0 & if \quad \frac{1}{2}\left[1 + \frac{\eta(\bar{x} - X_t)}{\sigma}\sqrt{\Delta t}\right] \leq 0 \\ 1 & if \quad \frac{1}{2}\left[1 + \frac{\eta(\bar{x} - X_t)}{\sigma}\sqrt{\Delta t} \geq 1\right] \end{array}\right\}, (8-21)$$

or

$$q_t = max\left\{0, min\left[1, \frac{1}{2}\left(1 + \frac{\eta(\bar{x} - X_t)}{\sigma}\sqrt{\Delta t}\right)\right]\right\}. \qquad (8-22)$$

Again, since

$$X = LnP,$$

$$LnP_t^+ \equiv LnP_t + \sigma\sqrt{\Delta t}\,,$$

$$LnP_t^- \equiv LnP_t - \sigma\sqrt{\Delta t}\,,$$

then,

$$P_t^+ = e^{\sigma\sqrt{\Delta t}}P_t = P_t u\,, \qquad (8-23)$$

$$P_t^- = e^{-\sigma\sqrt{\Delta t}}P_t = P_t\, d\,, \qquad (8-24)$$

$$u = e^{\sigma\sqrt{\Delta t}}\,, \qquad (8-25)$$

$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}\,. \qquad (8-26)$$

Since X_t changes along the price evolution process, the risk-neutral probabilities for price up and down movements change accordingly for the one-factor mean reversion price model.

8. 4 Risk-neutral World and Risk-neutral Probability

In financial mathematics, there are two parallel worlds: the physical world and the risk-neutral world. In the physical world, investors are risk averse; investors demand high return when bearing high risk. The performance of an asset return related to risk can be measured by the Sharpe ratio.

$$Sharpe\ Ratio = \frac{Excess\ Return\ of\ Asset\ or\ Portfolio}{Standard\ Deviation\ of\ Asset\ or\ Portfolio\ Return}$$

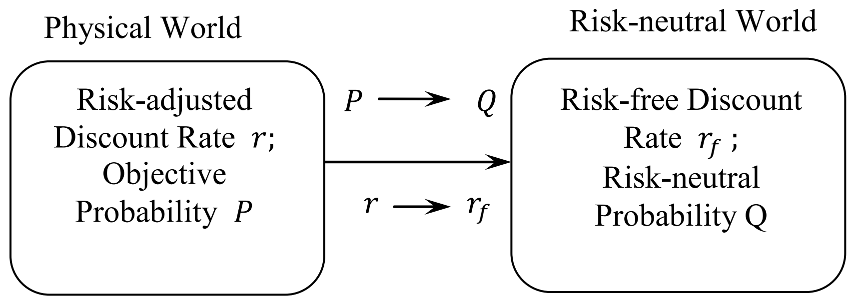
$$= \frac{\bar{r} - r_f}{\sqrt{var(r)}}$$

The higher the Sharpe ratio, the higher the excess return per unit of risk. Generally, investors choose a project, among many investment opportunities, with the highest Sharpe ratio. In the risk-neutral world, investors are risk indifferent and demand only a risk-free rate of return.

In the physical world, the evaluation of an investment with uncertainty involves three parts: 1) future outcomes; 2) probability measure (denoted as P) for future events; and 3) discounting the expected future outcomes, which is calculated from 1) and 2), at the risk-adjusted discount rate. In this case, the probability measure P is objective. The measure of risk is incorporated in the risk-adjustment discount rate such as using the capital asset pricing model ($CAPM$).

In the risk-neutral world, risk is incorporated into the probabilities. The objective probabilities are adjusted so that the expected value under the adjusted probabilities can be discounted using the risk-free rate. The adjusted probabilities are called risk-neutral probabilities, or equivalent martingale probability measure Q.

These two worlds, *i.e.*, the physical world and the risk-neutral world, can be connected when there are no arbitrage opportunities in a complete market.



Under the no arbitrage assumption, there is one unique set of risk-adjusted probabilities, *i.e.*, risk-neutral probabilities, to give the same value when future expected outcomes under risk-neutral probabilities are discounted with risk-free rate as the value discounted using risk-adjusted discount rate with the objective probabilities for future events. Duffie (1992) stated this very concisely "…the absence of arbitrage is equivalent to the existence of an equivalent martingale measure."

The following example is used to illustrate the probability change mentioned above. Assume that there is a stock with a price of \$100/share at time t_1; suppose that there will be 80% chance that the stock price is \$120/share and 20% chance of \$70/share at time t_2. There is one year time lag between time t_1 and time t_2. The risk adjusted discount rate is 10%; the risk-free rate is 5%. For strictness, synthetic probabilities are used instead of risk-neutral probabilities to illustrate this example. The following approach can find out the synthetic probability q under which the risk-free discount rate can be applied:

$r^+ = (120 - 100)/100 = 20\%,$

$r^- = (70 - 100)/100 = -30\%,$

$$qr^{+} + (1-q)r^{-} = r_f \, ,$$

$$q = \frac{r_f - r^{-}}{r^{+} - r^{-}} = \frac{5\% - (-30\%)}{20\% - (-30\%)} = 70\% \, .$$

With this probability, the present value for future stock prices under the synthetic probabilities is

$$(\$120/share \times 70\% + \$70/share \times 30\%)/(1+5\%) = \$100/share.$$

With real probability, the present value of the expected future stock prices under risk-adjusted discount rate is

$$(\$120/share \times 80\% + \$70/share \times 20\%)/(1+10\%) = \$100/share.$$

That is, with the synthetic probabilities, 70% for price of \$120/share, and 30% for price of \$70/share, the present value of the expectation for the future prices discounted at the risk-free rate is the same as the present value of future prices under physical probability of 80% for price of \$120/share, 20% for price of \$70/share, and discounted at the risk adjusted discount rate of 10%.

But if the risk-free rate is used but still under the objective probabilities, then

$$(\$120/share \times 80\% + \$70/share \times 20\%)/(1+5\%) = \$105/share \neq \$100/share.$$

Or, if the risk-adjusted discount rate is used but under the synthetic probabilities, then

$$(\$120/share \times 70\% + \$70/share \times 30\%)/(1+10\%) = \$95.5/share \neq \$100/share.$$

Comparing the two worlds, the physical and the synthetic, the price states are the same, both [\$120/bbl, \$70/bbl]; the probabilities of the two price states are different, [80%, 20%] under physical world, but [70%, 30%] under synthetic world; the expected values at time t_2 are different because of the different probabilities (probability change cause the change of mean). However, the discounted values from time t_2 to time t_1 are exactly the same under two matched discount rates and probabilities.

Converting one probability distribution to another is called probability measure change. The following section discusses the rigor mathematic theory on the probability measure change.

8. 5 GIRSANOV'S THEOREM

Girsanov's theorem provides the mathematical approach to transform one probability measure into another. When applying Girsanov's theorem to finance, it tells how to change a real probability distribution to a risk-neutral probability distribution, or called equivalent martingale probability distribution, so that a risk-free rate can be applied to value assets and derivatives.

For a stochastic process such as a price process

$$dP_t = \alpha_t P_t dt + \sigma_t P_t dW_t \,, \tag{8-27}$$

$$\frac{dP_t}{P_t} = \alpha_t dt + \sigma_t dW_t \,, \tag{8-28}$$

W_t is a Brownian motion under the probability measure $P,$ such that

$$E^P(W_t) = W_0 = 0,$$

$$Var^P(W_t) = t.$$

And the density function for W_t is

$$f^P(W_t) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{1}{2}\left(\frac{W_t}{t}\right)^2}.$$

In Eq. (8-28), add the right side of the equation with

$$(\alpha_t - r)dt - (\alpha_t - r)dt.$$

Then

$$\frac{dP}{P_t} = \alpha_t dt + \sigma_t dW_t + (\alpha_t - r)dt - (\alpha_t - r)dt,$$

$$\frac{dP_t}{P_t} = rdt + \sigma_t dW_t + (\alpha_t - r)dt$$

$$= rdt + \sigma_t dW_t + \sigma_t \frac{(\alpha_t - r)}{\sigma_t} dt$$

$$= rdt + \sigma_t (dW_t + \frac{\alpha_t - r}{\sigma_t} dt)$$

$$= rdt + \sigma_t d\left(W_t + \frac{\alpha_t - r}{\sigma_t} t\right).$$

Let

$$W_t^Q = W_t + \frac{\alpha_t - r}{\sigma_t} t\,, \qquad (8-29)$$

and

$$\gamma = -\frac{\alpha_t - r}{\sigma_t}\,, \qquad (8-30)$$

then

$$W_t^Q = W_t - \gamma t,$$

and then

$$\frac{dP_t}{P_t} = rdt + \sigma_t dW_t^Q. \qquad (8-31)$$

Compared with W_t, W_t^Q has a mean of $-\gamma t$ and variance of t under the original probability measure of P. That is

$$E^P(W_t^Q) = E^P(W_t - \gamma t) = E^P(W_t) - \gamma t = -\gamma t\,,$$

and

$$f^P(W_t^Q) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{1}{2}\left(\frac{W_t^Q - \gamma t}{t}\right)^2}.$$

Therefore, W_t is a Brownian motion under probability measure P. But W_t^Q is not a Brownian motion under probability measure P, since other than at time zero, the expectation of W_t^Q is not zero. A new probability measure Q is desired so that under which

$$E^Q(W_t^Q) = W_0^Q = 0\,,$$

$$Var^Q(W_t^Q) = t\,,$$

$$f^Q(W_t^Q) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{1}{2}\left(\frac{W_t^Q}{t}\right)^2}.$$

W_t^Q is a Brownian motion under probability measure Q. W_t is not a Brownian motion under probability measure Q, since other than at time zero, the expectation of W_t under probability measure Q is not zero, but changes with time. That is

$$E^Q(W_t) = E^Q(W_t^Q + \gamma t) = E^Q(W_t^Q) + \gamma t = \gamma t\,.$$

By the transformation of Eq. (8-29), the process of dP_t/P_t, which originally has a drift rate of α_t under probability measure of P, has a drift rate of r under probability measure Q once the probability measure Q is determined so that W_t^Q is a Brownian motion. With the transform of probability measure from P to Q, the drift rate of the process changes to a risk-free rate automatically. Girsanov's theorem states that there exists a probability measure Q under which W_t^Q is a Brownian motion. And when there is no arbitrage opportunity, the probability measure Q is unique.

For many years, the petroleum industry could not accept that a risk-free rate was used to discount the values from cash flows when real options were included in the decision making. The argument was based on the Sharpe ratio. From the above analysis of probability transformation, it is clear that risk is not neglected or unrecognized when converting the probability measure from one into another. For example, oil prices in the future can still be as high as $100/bbl or as low as $40/bbl. But when a different set of probabilities is assigned to future oil price states, the present value of future cash flows based on oil prices will be different. Future price states will not be altered by the change in probability measure. The change in probability measure changes the likelihood of the occurrence of each price state so that the discounted expectations of future values will be the same with a different discount rate.

Attention needs to be paid to the fact that a probability measure needs to match the discount rate. In the binomial lattice method of reconstructing the stochastic process, the risk-neutral probability is calculated with Equations (8-4), (8-10), and (8-21). Then the risk-free rate needs to be used to discount the future events which are governed by the risk-neutral probability distribution.

8. 6 REAL OPTIONS EVALUATION METHOD FOR THE WATER FLOODING SWITCHING TIME FLEXIBILITY

For the production of the synthetic reservoir studied in this dissertation, assume that there are only two production stages: primary and secondary (water flooding) oil recovery. The total production time is 25 years when the water cut in production flow reaches 97 percent, according to the reservoir simulation results in Chapter 6. The project is evaluated at the third quarter of year 2010.

Water flooding can start at time zero; and at year seven, no matter what state the oil price is at, water flooding switching starts if it does not start earlier. Water flooding switching is an irreversible process. That is, once water flooding starts, it cannot switch back to primary oil production. It is assumed that the managerial decision on water flooding switching is revised every three months and it is long enough to build all the water flooding facilities in three months. If water flooding does not start at time zero, it can start by the end of three months, or six months, etc., until reaching the end of year seven. Therefore, there are 28 decision making opportunities. Figure 8-2 illustrates the decision making process of water flooding switching with eight switching opportunities. Both the geometric Brownian motion (GBM) and one-factor mean reversion price models are used to value the water flooding switching for the purpose of comparison.

The 25 years' oil production time is divided into 100 yearly quarters with three months in one yearly quarter. From time zero to the end of project life, the time series is defined as T_0, T_1, T_2, …, T_{99}, T_{100}. The investment costs are made in T_0; all the exploration and development activities are conducted in T_0; and all the primary production facilities are built in T_0. The water flooding well and facilities are completed one time lag ahead of the starting time of water flooding. For example, if water flooding starts in T_3, the water flooding wells are drilled and completed, and the related facilities are build in T_2.

The amount of oil sold at the oil price of $P(T_i)$, which is the quarterly price for the oil produced from the synthetic oil reservoir, is the total amount of oil produced from the reservoir from the beginning of time T_i, which is the end of time T_{i-1}, to the end of time T_i. Therefore, the gross revenue in T_i is equal to the price $P(T_i)$ multiplied by the amount of oil produced from the beginning of time T_i to the end of time T_i. Based on the assumption made in Section 7.7.2, that the oil produced in the synthetic oil reservoir can be sold at the same prices as those of the West Texas Intermediate (WTI), the historical WTI oil prices and the parameters calibrated from the historical WTI oil prices are used to forecast the future oil price $P(T_i)$ produced from the synthetic oil reservoir.

In Chapters 3 and 4, parameters for the GBM and one-factor mean reversion price models are obtained from daily, weekly, monthly, and yearly historical WTI oil prices. To forecast the quarterly future price distributions for the oil produced in the synthetic oil reservoir, the annualized parameters from the WTI monthly oil prices are used to closely represent the model parameters for the quarterly oil prices.

8.6.1 Establishment of the Lattices for Probabilities, Prices, and Cash Flows

The establishment of the lattices for probabilities, prices, and cash flows is a forwarding process from T_0 to T_{100}. The lattices are built in Microsoft Excel. In Excel, probability, price and cash flow lattices are triangles with 101 columns from row 1 to row 101.

According to Equations (8-5) through (8-14), the probability and price lattices can be calculated for the GBM price model. In the model, P_0 is the WTI oil price at the third quarter of 2010, which is \$76.05/bbl; μ (annualized) equals 0.16247; and σ (annualized) equals 0.3217. Both μ and σ are parameters calibrated from the historical WTI monthly prices from January 2000 to April 7, 2010, as shown in Chapter 3, Section 3.4.

According to Equations (8-15) through (8-26), the lattices of oil prices, the logarithm of oil prices, which is $Ln(P)$, and probabilities, are calculated for the one-factor mean reversion price model. Figures (8-3), (8-4), and (8-5) are illustrations for the partial binomial lattices, at a three-month time step, of oil prices, the logarithm of oil prices, and probability for the one-factor mean reversion oil price model, starting from the third quarter of 2010. The parameters for the one-factor mean reversion price model are calibrated from the historical WTI monthly prices from January 2004 to May 28, 2010, as described in Chapter 4, Section 4.3. That is, $\eta = 0.9235$, $\sigma = 0.3512$, and $\bar{p} = \$78/\text{bbl}$. Both η and σ are annualized.

The parameters used in the GBM and the one-factor mean reversion price models for real options evaluations for the synthetic petroleum project are summarized in Table 8-1, based on parameters calibrated from the historical WTI monthly oil prices. u and d are calculated according to Eq. (8-13) and Eq.(8-14) for the GBM price model and according to Eq. (8-25) and Eq. (8-26) for the one-factor mean reversion price model. $\bar{x}$ is calculated according to Eq. (8-18) for the one-factor mean reversion price model.

Based on the price and probability lattices, the cash flow lattices, for both the GBM and one-factor mean reversion oil price models, are established according to the fiscal regime and cash flow model, costs in exploration, development and oil production, and royalty and tax determined in Chapter 7. Cash flows are calculated according to two cost cases: a low cost case and a high cost case. All the cost data are based on an onshore oil well of about 4,000 foot depth.

Cost data for both the low cost case and the high cost case are recorded in Table 7-4 and Table 7-5 as Cash Flow Data (1) and Cash Flow Data (2). For the low cost case, operating costs of $\$1.00/bbl$ oil produced for primary recovery and $\$3.00/bbl$ oil produced for water flooding with a yearly escalation of 1% are applied. The exploration and development cost data for the low cost case are based on academic data with adjustments on them: the sum of exploration drilling and development costs is \$751,545 for one production well and the cost for four water injection wells and facilities is \$3,141,180. For the high cost cases, the operating costs are adjusted as follows: $\$30.00/bbl$ oil produced for primary production and $\$50.00/bbl$ oil produced for water flooding with a yearly escalation rate of 3%. The exploration and development costs for high cost case are the most current data. That is, \$1,951,000 for one exploration well and \$1,617,000 for one development well. The cost for four water injection wells and facilities is \$14,272,000 for the high cost case. 4.97% annual escalation is applied to the water injection well and facility cost for both the low cost and high cost cases. For both the low cost and high cost cases, when the production rate is high, the variable cost is the controlling cost factor. Thus, the fixed cost is neglected. When the production rate is low, the fixed cost becomes the controlling cost factor and then the variable cost is neglected. Therefore, there are four minimum operating costs for primary and water flooding oil production: \$2,000/month for primary recovery and \$6,000/month for water flooding for the low cost case; \$6,000/month for primary production and \$18,000/month for water flooding for high cost case. 1% and 3% annual escalation rates are applied for the above minimum operating costs for the low cost and high cost cases respectively.

8.6.2 Establishment of Project Value Lattice

The determination of project value at each time T_i at each price state is a backward process starting from T_{100}. For the convenience of describing value calculation and determination process, the cash flows at different price states for different switching times are first defined.

Let $CF_{i,k}^{j}$ denote cash flow at T_i and at price state of k for water flooding switching at T_j $(j = 0, 1, 2, \dots, 27, 28;\ i = j + 1, j + 2, \dots, 99, 100;\ k = 0, 1, 2, \dots, i, i + 1 \text{ for any } i)$. For example, if $j = 3$, there will be cash flows for the oil production when water flooding starts at T_3, which are

$CF_{4,k}^{3} (k = 0, 1, \dots, 4)$,

$CF_{5,k}^{3} (k = 0, 1, \dots, 5)$,

…

$CF_{99,k}^{3}(k = 0, 1, \dots, 99)$,

$CF_{100,k}^{3}(k = 0, 1, \dots, 100)$.

And let $CF_{i,k}^{PP}$ denote cash flows for primary oil production only $(i = 1, 2, 3, \dots, 27, 28)$. Since there is no primary oil production after year seven, $CF_{i,k}^{PP}$ ends at T_{28}. Then

1) $V_{i,k}^{j} = CF_{i,k}^{j} + [q_{i,k}V_{i+1,k+1}^{j}(u) + (1 - q_{i,k})V_{i+1.k}^{j}(d)]e^{-r\Delta t}$, (8-32)

for any $i > j$ and $j = 0, 1, 2, \dots, 27, 28$, where

$q_{i,k}$: the probability of price up move from T_i to T_{i+1} at price state k ;

$(1 - q_{i,k})$: the probability of a downward price move from T_i to T_{i+1} at price state k ;

$V_{i,k}^{j}$: value of the project at T_i at price state k for the oil production starting water flooding switching at T_j ;

$V_{i+1,k+1}^{j}(u)$: value of the project at T_{i+1} starting water flooding at T_j when oil price moves up from price state k at T_i ;

$V_{i+1,k}^{j}(d)$: value of the project at T_{i+1} starting water flooding at T_j when oil price moves down from price state k at T_i ;

r : risk-free discount rate;

Δt : a quarter of year, or 0.25 year.

Since $q_{i,k}$ is the risk-neutral probability, values of the project at different times can be related to each other with a risk-free discount rate.

$$2)\ V_{j,k}^{j} = CF_{j,k}^{PP} - SWC(j) + \left[q_{j,k} \times V_{j+1,k+1}^{j}(u) + \left(1 - q_{j,k}\right) V_{j+1,k}^{j}(d)\right] e^{-r\Delta t}$$

$$(j = 0, 1, 2, \dots, 27, 28), \qquad (8-33)$$

where $V_{j,k}^{j}$ is the value of the project at T_j at price state k for oil production starting water flooding at T_j ; $SWC(j)$ is the switching cost at time T_j . $V_{j,k}^{j}$ captures the value of water flooding which starts at T_j at price state k. This value is compared with the project value $V_{j,k}^{PP}$, for which water flooding does not start at T_j at price state k, to determine whether or not water flooding should start at T_j at price state k. Following two equations are designed to perform the comparison:

$$3)\ V_{i,k}^{PP} = CF_{i,k}^{PP} + \left[q_{i,k} \times V_{i+1,k+1}(u) + (1 - q_{i,k}) V_{i+1,k}(d)\right] e^{-r\Delta t} \qquad (8-34)$$

$$4)\ V_{i,k} = Max\left(V_{i,k}^{j=i}, V_{i,k}^{PP}\right). \qquad (8-35)$$

Four steps need to follow using Equations (8-32) through (8-35) to calculate the maximum value of the project when water flooding option can be exercised at every price state at each of the possible switching times:

1) Start at T_{100}, for each water flooding switching option, $i.e.$, water flooding starts at T_0, T_1, T_2, … , T_{27}, T_{28}, calculate the project values for each price state all the way to one lag behind the water flooding switching time according to Eq. (8-32);
2) Calculate the project values at water flooding switching time according to Eq. (8-33) for each switching option;

3) The value lattice of water flooding switching at T_{28} is used as the maximum value-determining lattice. In this lattice, for each price state k, starting from T_{27}, calculate the value of $V_{27,k}^{PP}$ according to Eq. (8-34), which is the project value when water flooding happens at T_{28} but does not happen at T_{27}. $V_{27,k}^{PP}$ is compared with $V_{27,k}^{27}$, which is the value at T_{27} for the oil production starting water flooding at T_{27}. The maximum value is assigned to T_{27} according to Eq. (8-35), which is $V_{27,k}$.
4) The process continues until T_0 is reached and the maximum value of the project is calculated and the best switching time is determined at each price state.

Figure 8-6 illustrates the process of determining the maximum value at T_j for price state k. Figure 8-7 is the illustration of determining the maximum values at different price states for different water flooding switching times.

For each water flooding switching option, there is a 101×101 triangular matrix for cash flow and value calculations. Since there are 29 water flooding switching options, there are 29 production curves, 29 of the 101×101 cash flow and value lattices. Comparing and determining the maximum value need to be conducted among the 29 water flooding switching options based on the values of different price states. In order to meet the large computation need, a computer program is developed to facilitate the real options evaluation process.

8.6.3 Development of the Computer Program for Binomial Lattice Real Options Evaluation

In this study, a binomial lattice real options evaluation computer program is developed. The program is designed to fulfill seven functions: 1) generate quarterly oil production rates from the reservoir simulation results conducted by the University of Texas Chemical Flooding ($UTCHEM$) simulator, for the 29 water flooding switching options; 2) generate binomial lattices of probabilities and oil prices, for both the geometric Brownian motion (GBM) and one-factor mean reversion price models; 3) make a series of specially designed cash flow calculators to calculate cash flows for oil prices at different times and different price states for the 29 water flooding switching options; 4) generate cash flow binomial lattices for the 29 water flooding switching options; 5) generate binomial value lattices for the 29 water flooding switching cases; 6) conduct real options evaluations; 7) conduct base case analysis under deterministic oil prices for both the GBM and one-factor mean reversion price models.

The post-process results of oil production rates from $UTCHEM$ simulator cannot be used directly to calculate cash flows. The $UTCHEM$ simulation reports the daily oil production rates at about 40 day's frequency, according to the $UTCHEM\ INPUT$ file, in the early oil production time when oil production declines very fast, and longer time frequency when changes of the oil production rates become slower. The developed computer program in this study first generates density functions of oil production rates through linear interpolation of oil production rates from the $UTCHEM$ simulation results. Then the numerical integration of the density functions is used to obtain quarterly oil production rates for the 29 water flooding oil production curves. The quarterly oil production results are shown in Figure 8-8, which represents the daily oil production rates very well.

The developed real options evaluation computer program generates risk-neutral probabilities for both the GBM and one-factor mean reversion price models according to Equations (8-10) and (8-22), and the parameters in Table 8-1; and it generates price lattices for the two price models according to Equations (8-11), (8-12), (8-23) and (8-24), and the parameters in Table 8-1. Figure 8-9 shows the computer generated probability lattice for the one-factor mean reversion price model and Figure 8-10 shows the price lattice for the GBM price model, for the oil produced in the synthetic oil reservoir. Compared with Figure 8-5, to simplify computations, in Figure 8-9, the computer-generated probability lattice only includes the probabilities for price up movement (q_t). The probabilities for the price down movement, $(1 - q_t)$, are not included.

With the defined cash flow model, as described in Section 7.7.1 in Chapter 7, for each of the 29 water flooding switching cases, a cash flow calculator is created. Then according to the selected oil price model, the program automatically reads the oil price from each node of the price lattices, calculates cash flow for each oil price, and generates 29 cash flow lattices in 29 Excel spread sheets for the 29 oil production curves. Figure 8-11 shows one example of the generated cash flow lattices for the one-factor mean reversion oil price model.

With the cash flow lattices, the real options evaluation computer program automatically establishes the value lattices for the 29 water flooding switching options and then obtains the real options evaluation results according to Equations (8-32) through (8-35) following the four steps described in Section 8.6.2. The real options evaluation results are then exported in an Excel spread sheet which shows the project value at each price state for each of the 100 yearly quarters and highlights the time and price state when water flooding switching occurs.

For the base case analysis, the real options evaluation computer program automatically creates 29 Excel spread sheets of cash flows with the traditional net present value (NPV) evaluation results, each sheet representing one switching option, for the defined deterministic oil prices from both the GBM and one-factor mean reversion price models; generates a chart of the $NPVs$ for the 29 water flooding switching options; and obtains the highest project value under the deterministic oil prices and the corresponding switching time. The results from base cases are then compared with the real options evaluation results regarding switching time and project values.

Figure 8-12 shows the data flow for the real options evaluation computer program. With this program, it takes about one and half hours to complete a real options evaluation using an up-to-date computer with a quad-core processor and a $12GB\ RAM$, due to the magnitude of calculations for each evaluation case.

8. 7 RESULTS OF REAL OPTIONS EVALUATIONS AND ANALYSIS

In order to establish a theoretically sound, mathematically solid, specific industry needs oriented, and feasible and reliable method to conduct real options evaluations for the petroleum $E\&P$ industry, not only is one complete set of standard real options case used to test the method with the desirable results, but other cases are designed and studied in order to give insight on how this real options evaluation method should be applied to real world needs, to which aspects attention should be paid when using this method, and what conclusions are reached from using this method.

The cases that have been studied include: 1) Base Cases without Options; 2) High Cost Mean Reversion case; 3) High Cost GBM case; 4) High Cost Mean Reversion with Stop Options case; 5) Low Cost Mean Reversion case; 6) Low Cost GBM case; and 7) Oil Production Rate Cut case. The High Cost Mean Reversion case is the standard real options evaluation case in this study.

In the Base Cases without Options, the traditional NPV method is used to evaluate different water flooding switching opportunities. Throughout the project life, the deterministic oil prices are used for the two oil price models, $i.e.$, the GBM and one-factor mean reversion models. For the one-factor mean reversion price process, the constant long run oil price $\bar{p}$ of \$78.00/bbl, according to the WTI historical monthly oil prices from January 5, 2004 to May 28, 2010, is used as the deterministic price. For the GBM price model, the expected future price $E\left(P_{T_i}\right) = P_0 Exp(\mu T_i)$ is calculated as the deterministic price and applied to each time T_i, where $P_0 = \$76.05/\text{bbl}$; $\mu = 0.16247$

(annualized); $\sigma = 0.3217$ (annualized). P_0 is the WTI oil price at the third quarter of 2010. μ and σ are from the calibration results for the historical WTI monthly oil prices from January 4, 2000 to April 7, 2010. The net present values for the oil production from the 29 water flooding switching cases are thus calculated. The switching time with the highest NPV is the best switching time, which is also called the NPV optimization result, for the Base Cases without Options.

Since negative net income occurs when oil prices are low and/or oil production rate is low as well, an approximate stop option is included for the analysis. When net income is negative, it indicates that cost is higher than the benefit from operation, and that the operation should not be continued once the costs of stop and re-start the operation are taken into consideration. For simplicity, in this study, to include a stop option, the corresponding cash flow is set to zero once negative net income occurs. This way, since the costs to stop and re-start the operation are not included, the stop option just approximately represents the real cases.

As observed from the oil production decline curves, the oil production rates are very high at the beginning of primary and water flooding oil production: about 1,600 bbl/day for primary oil production and 2,600 bbl/day for water flooding oil production. A synthetic production profile is generated by cutting all of the oil production rates by a factor of (1/4.5), which results in 22% of the original oil production rates, to create the Oil Production Rate Cut case.

Figures 8-13 through 8-20 contain the analyzing charts for one of the Base Cases without Options with the high cost data, as described in 7.7.3, and mean reversion oil price model. Figures 8-13 and 8-14 show the gross revenue and net revenue for the 29 different water flooding switching opportunities. Both are very similar to the oil production quarterly rates, indicating that revenue is very closely related to the amount of oil produced. Figures 8-15 through 8-17 show the amount of depreciation, depletion, and amortization ($DD\&A$) for water flooding, total $DD\&A$, and operating costs at different water flooding switching times. These three figures demonstrate that water flooding and oil production rates have strong impacts on operating costs and $DD\&A$. Negative net income occurs after about 25 quarter years and cash flow becomes very small after about 40 quarter years, as shown in Figures 8-18 and 8-19. Figure 8-20 shows that the best water flooding switching time is at T_5 for this base case.

Table 8-2 contains the summary of the real options evaluation results, including water flooding switching times and the values of the project according to different water flooding switching times. Emphasis should be put on the corresponding changes in water flooding switching times and project values, not on the absolute project values, when changes are made for a specific factor for the evaluation purpose.

Figure 8-21 shows the real options evaluation results for the High Cost Mean Reversion case, the standard real options evaluation case. The High Cost Mean Reversion case shows that with the one-factor mean reversion oil price model and reasonable water flooding switching and operating costs, real options evaluation reaches different results from the base case NPV optimization results. For the High Cost Mean Reversion case, real options evaluation shows that water flooding does not start at T_0 or T_1. Water flooding starts at T_2 when oil price is high at T_2; but it does not starts at T_2 when oil price is low at T_2. The project value is \$2.7E+7 from the real options evaluation for the High Cost Mean Reversion case. The traditional NPV optimization method shows that for the base case with high cost and mean reversion price model, the best water flooding switching time is T_5 with a project value of \$1.1E+7, as shown in Table 8-2. The inclusion of stop options does increase the project value for the real options evaluation, but it does not change the best water flooding switching time.

Table 8-2 also shows that the real options evaluation results are very sensitive to oil price models, operating costs, and investment costs of wells and facilities. When GBM oil price model is used, no matter whether in high cost cases or low cost cases, no matter whether production rates are cut or not, the best water flooding switching time moves to T_{28}, which is the latest water flooding switching time, in all designed cases. As shown in Figure 8-22, with GBM oil price model, the expected oil prices increase dramatically with the increase of time. About 15 years from now (the third quarter of 2010), the expected oil price will reach about \$900/bbl, which is very unlikely to happen. The oil prices become the dominant factor that impacts cash flows. The later the water flooding switching happens, the later the high oil production rates occur because of the water flooding, and the higher the project values. The real options evaluation method does respond to this price model and moves the best water flooding switching time to T_{28} and it agrees with the base case NPV

optimization result on water flooding switching time. Figure 8-23 shows the results for real options evaluation for the High Cost GBM case. In this figure, because of the limited screen width, the water flooding switching time, which occurs in T_{28}, is unable to be seen.

When the one-factor mean reversion oil price model is used, the real options evaluation results are sensitive to the combining effects of initial investment for primary oil production, switching costs for water flooding, and the operating costs. In the Low Cost Mean Reversion and the Low Cost Mean Reversion with Stop Option cases, the best time of water flooding switching moves to T_0. In these cases, since the costs are so low, production rates and the time value of money become the controlling factors on project values. The more oil that is produced at an earlier time, the higher the project values. The real options evaluation results agree with those from the traditional NPV optimization method on water flooding switching time, as shown in Table 8-2.

Figure 8-24 shows the results of Low Cost Mean Reversion case. The values in the cells that are highlighted are the project values for which water flooding switching starts at the times as the cells can indicate. As it is shown, water flooding switching starts at T_0 in this case.

As shown in Table 8-2, for all the cases designed and studied, with the inclusion of the approximate stop options, the best switching time of water flooding does not change, compared with the corresponding cases without stop options, from the real options evaluation results. The results from the real options evaluation agree with those from the traditional NPV optimization method for those cases.

For the studied High Cost cases, the oil rate cut does not change the water flooding switching time from the real options evaluation results for both the GBM and one-factor mean reversion oil price models. However, with the traditional NPV optimization method, when one-factor mean reversion oil price model is applied, the oil rate cut moves the water switching time from T_5 to T_{28}, along with a negative resulting project NPV.

In summary, the established real options evaluation method can be used to identify the best time to switch from primary oil recovery to water flooding oil recovery, and thus, gives the maximum project value according to the best water flooding switching time. With the mean reversion oil price model, the real options evaluation finds that the best water flooding switching time is earlier than the traditional NPV optimizing method. The method also reveals that the best water flooding switching time can be much different when different oil price models are applied and different cost data are used. The method challenges the knowledge on oil price models, the sound judgment in selecting the appropriate oil price model, and the reasonable investment and operating costs in the project evaluation.

Table 8-1: Parameters Used in the One-factor Mean Reversion and GBM Price Models for the Oil Produced in the Synthetic Oil Reservoir

Parameters	Values*	
	Mean Reversion Oil Price Model	GBM Oil Price Model
P_0 , \$/bbl	76.05	76.05
Δt , year	0.25	0.25
η , Annualized	0.9235	
μ , Annualized		0.16247
σ, Annualized	0.3512	0.3217
$\bar{p}$, \$/bbl	78.00	
$\bar{x}$	4.289929	
u	1.191961	1.174509
d	0.838954	0.85142

* The parameters for the GBM price model are from the historical WTI monthly oil prices from January 4, 2000 to April 7, 2010; the parameters for the one-factor mean reversion price model are from the historical WTI monthly oil prices from January 4, 2000 to May 28, 2010; and

$$\bar{x} = Ln\bar{p} - \frac{\sigma^2}{2\eta},$$

$$u = e^{\sigma\sqrt{\Delta t}},$$

$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}.$$

Table 8-2: Real Options Evaluation Results

	Real Options				Base Case NPV Optimization			
	GBM		MR		GBM		MR	
	Time to Start Switch	Project Value	Time to Start Switch	Project Value	Time to Start Switch	Project Value	Time to Start Switch	Project Value
High Cost	T_{28}	\$5.9E+8	T_2	\$2.7E+7	T_{28}	\$2.4E+8	T_5	\$1.1E+7
Low Cost	T_{28}	\$7.1E+8	T_0	\$1.7E+8	T_{28}	\$3.1E+8	T_0	\$1.4E+8
High Cost, Net Income ≥ 0	T_{28}	\$6.0E+8	T_2	\$3.5E+7	T_{28}	\$2.4E+8	T_5	\$1.1E+7
Low Cost, Net Income ≥ 0	T_{28}	\$7.1E+8	T_0	\$1.7E+8	T_{28}	\$3.1E+8	T_0	\$1.4E+8
High Cost, Oil Rate Cut	T_{28}	\$1.4E+8	T_2	\$4.8E+6	T_{28}	4.37E+8	T_{28}	\$-8.7E+6

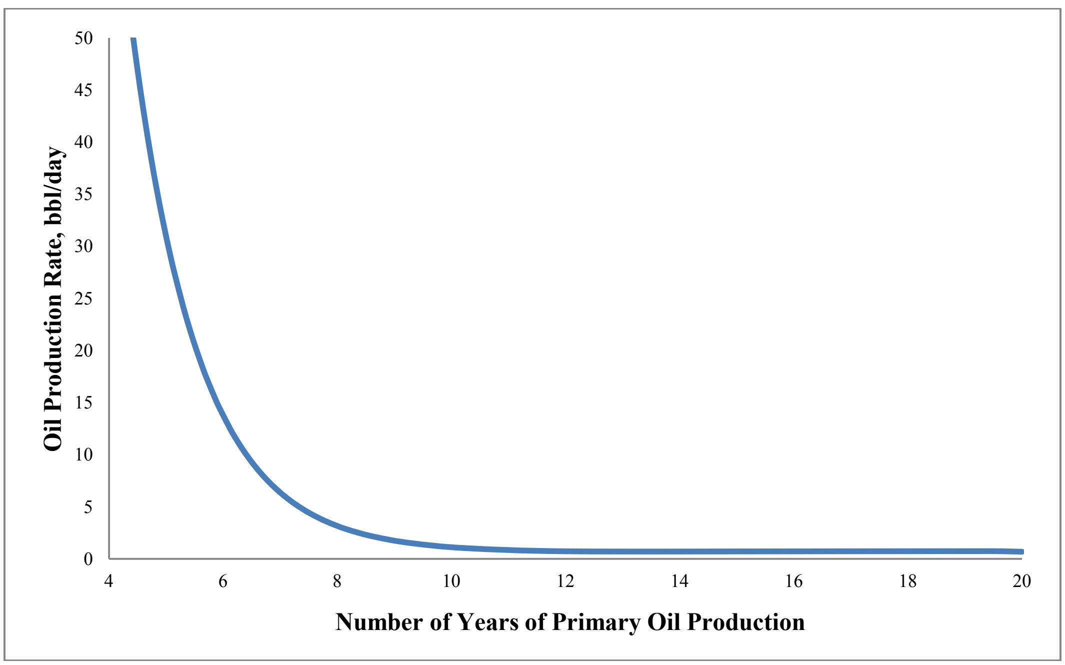


Figure 8-1: Primary Oil Production Rate Change with Production Time

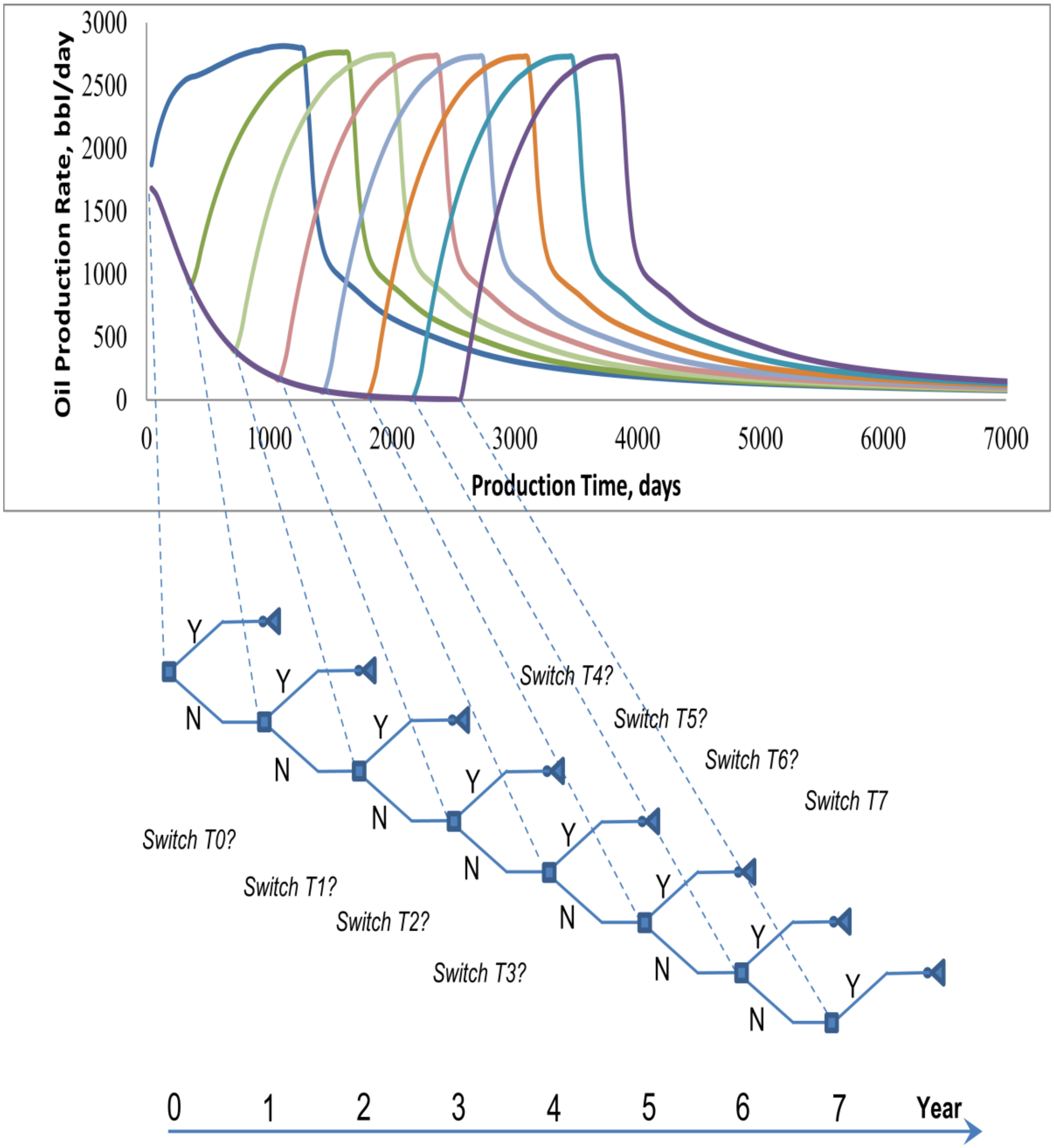


Figure 8-2: Decision Making Process of Water Flooding Switching

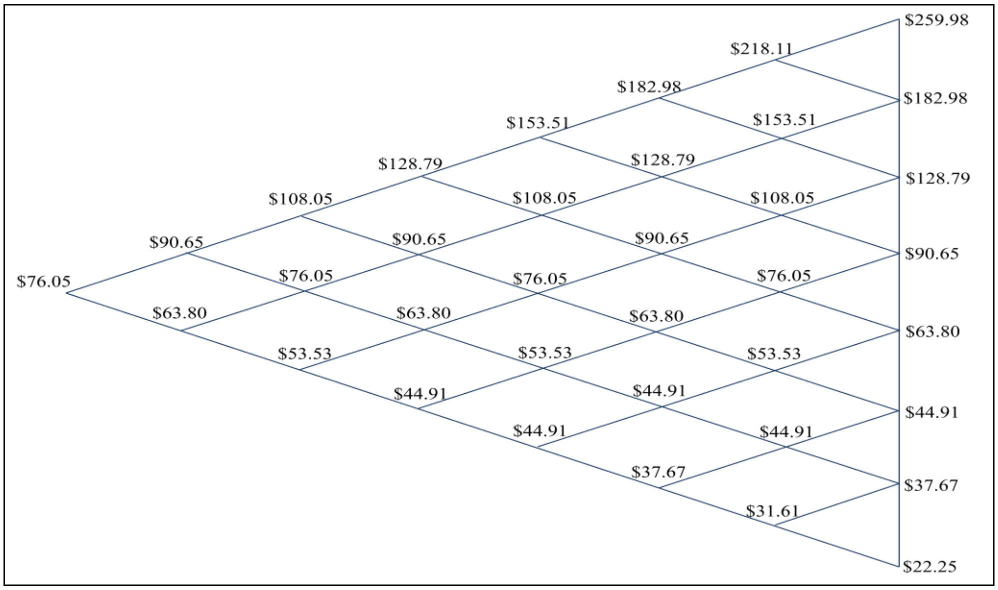


Figure 8-3: Illustration of Partial Binomial Lattice on Oil Prices (P) according to the One-factor Mean Reversion Price Model at the Three-month Time Step Starting from the Third Quarter of Year 2010 for the Oil Produced in the Synthetic Oil Reservoir

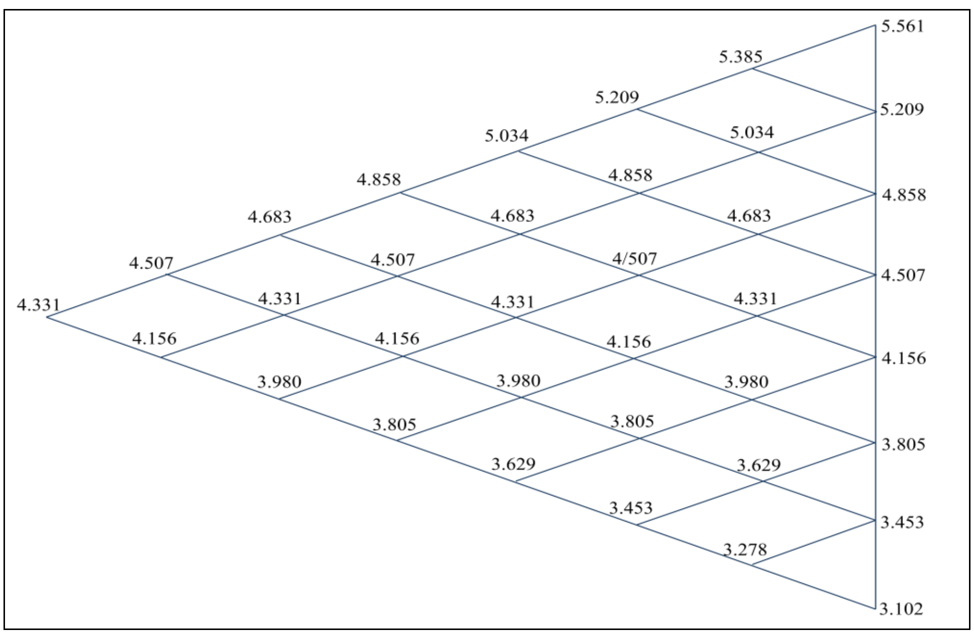


Figure 8-4: Illustration of Partial Binomial Lattice on the Logarithm of Oil Prices (LnP) according to the One-factor Mean Reversion Price Model at the Three-month Time Step Starting from the Third Quarter of Year 2010 for the Oil Produced in the Synthetic Oil Reservoir

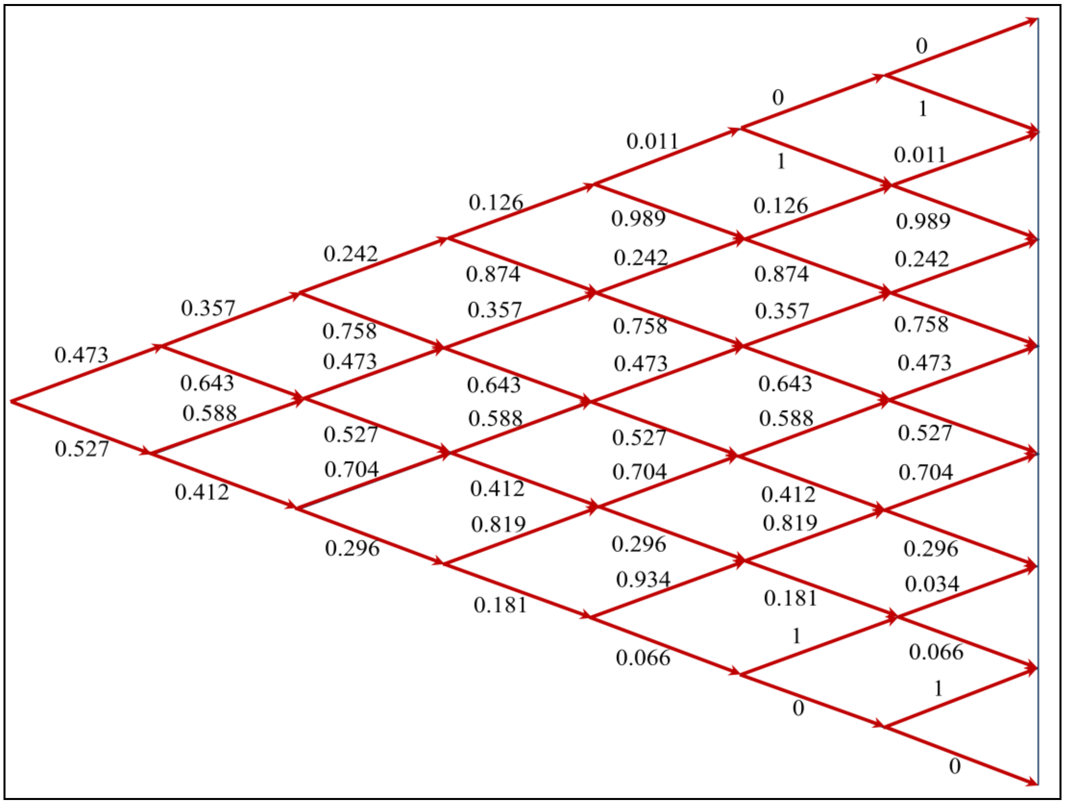
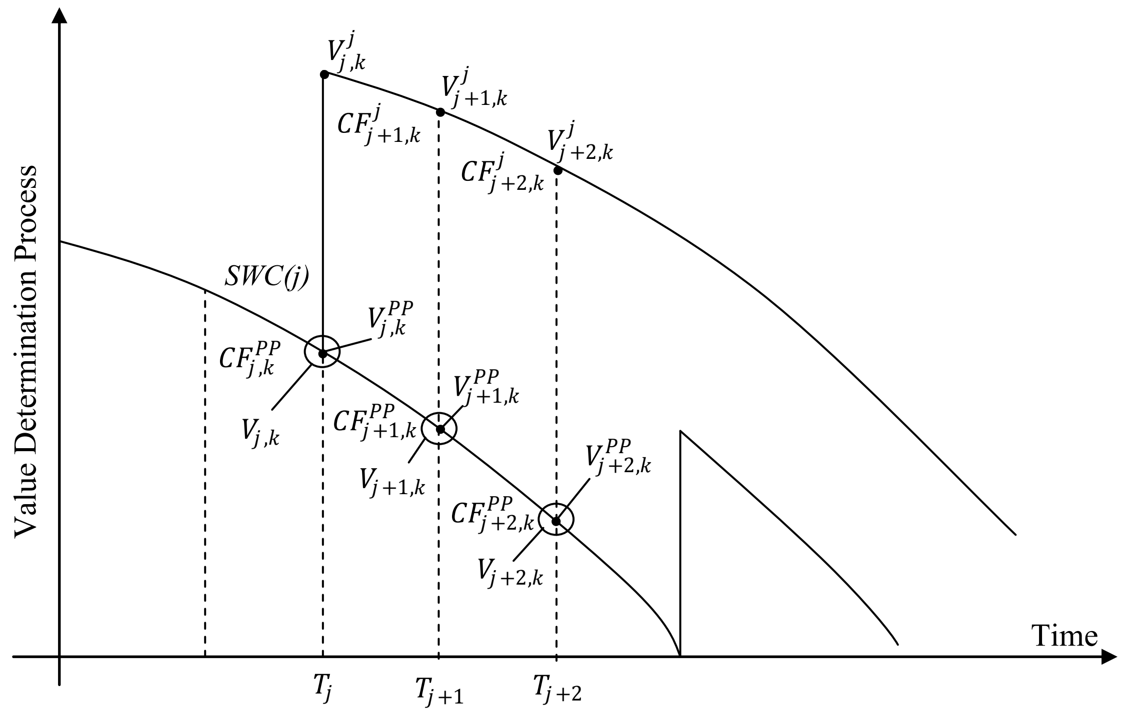


Figure 8-5: Illustration of Partial Binomial Probability Lattice according to the One-factor Mean Reversion Price Model at the Three-month Time Step Starting from the Third Quarter of Year 2010 for the Oil Produced in the Synthetic Oil Reservoir



$$V^{j}_{j,k} = CF^{PP}_{j,k} - SWC(j) + [V^{j}_{j+1,k+1}(u)q_{j,k} + V^{j}_{j+1,k}(d)\left(1 - q_{j,k}\right)]e^{-r\Delta t}$$
$$V^{j}_{j+1,k} = CF^{j}_{j+1,k} + [V^{j}_{j+2,k+1}(u)q_{j+1,k} + V^{j}_{j+2,k}(d)\left(1 - q_{j+1,k}\right)]e^{-r\Delta t}$$
$$V_{j,k} = Max(V^{PP}_{j,k}, V^{j}_{j,k})$$

Figure 8-6: Illustration of the Process to Determine the Maximum Value at T_j for Price State k

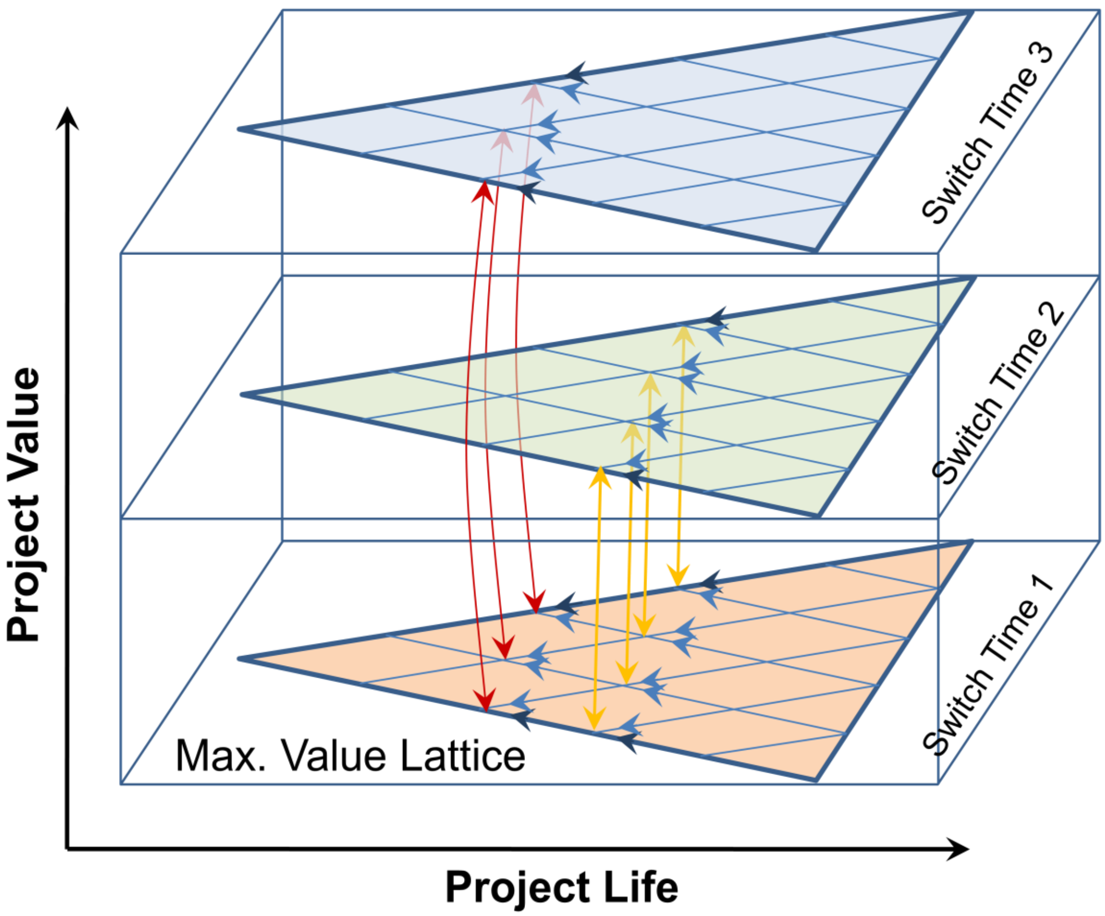


Figure 8-7: Illustration of Determining the Maximum Values at Different Price States for Different Water Flooding Switching Times

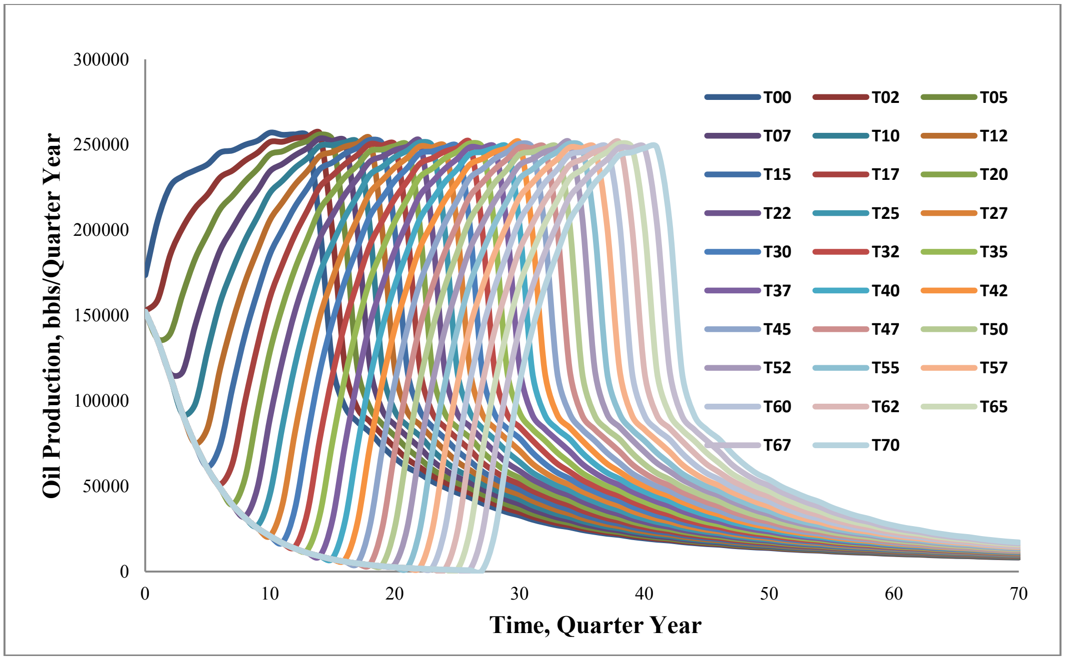
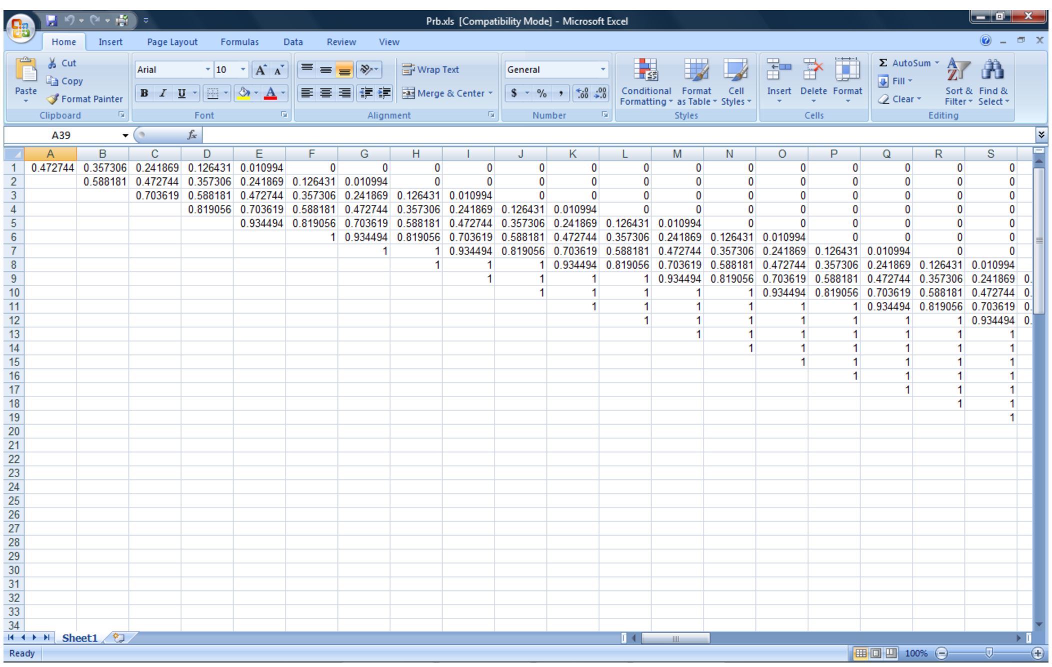
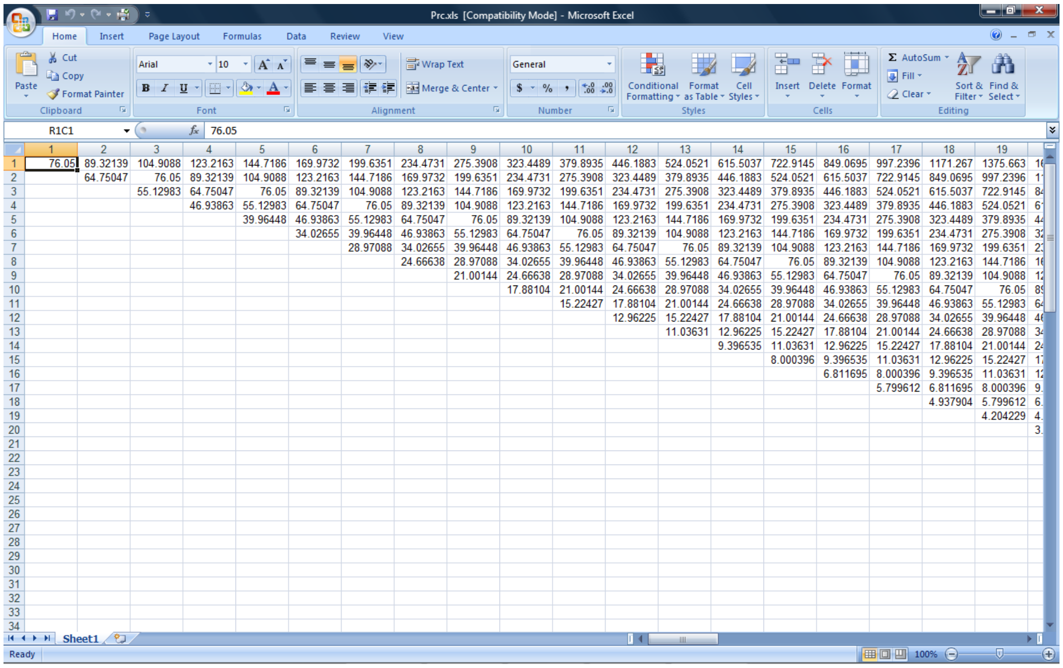


Figure 8-8: Quarterly Oil Production Rates for the 29 Water Flooding Switching Options for the Oil Produced in the Synthetic Oil Reservoir



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1	0.472744	0.357306	0.241869	0.126431	0.010994	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0.588181	0.472744	0.357306	0.241869	0.126431	0.010994	0	0	0	0	0	0	0	0	0	0	0	0
3			0.703619	0.588181	0.472744	0.357306	0.241869	0.126431	0.010994	0	0	0	0	0	0	0	0	0	0
4				0.819056	0.703619	0.588181	0.472744	0.357306	0.241869	0.126431	0.010994	0	0	0	0	0	0	0	0
5					0.934494	0.819056	0.703619	0.588181	0.472744	0.357306	0.241869	0.126431	0.010994	0	0	0	0	0	0
6						1	0.934494	0.819056	0.703619	0.588181	0.472744	0.357306	0.241869	0.126431	0.010994	0	0	0	0
7							1	1	0.934494	0.819056	0.703619	0.588181	0.472744	0.357306	0.241869	0.126431	0.010994	0	0
8								1	1	1	0.934494	0.819056	0.703619	0.588181	0.472744	0.357306	0.241869	0.126431	0.010994
9									1	1	1	1	0.934494	0.819056	0.703619	0.588181	0.472744	0.357306	0.241869
10										1	1	1	1	1	0.934494	0.819056	0.703619	0.588181	0.472744
11											1	1	1	1	1	1	0.934494	0.819056	0.703619
12												1	1	1	1	1	1	1	0.934494
13													1	1	1	1	1	1	1
14														1	1	1	1	1	1
15															1	1	1	1	1
16																1	1	1	1
17																	1	1	1
18																		1	1
19																			1

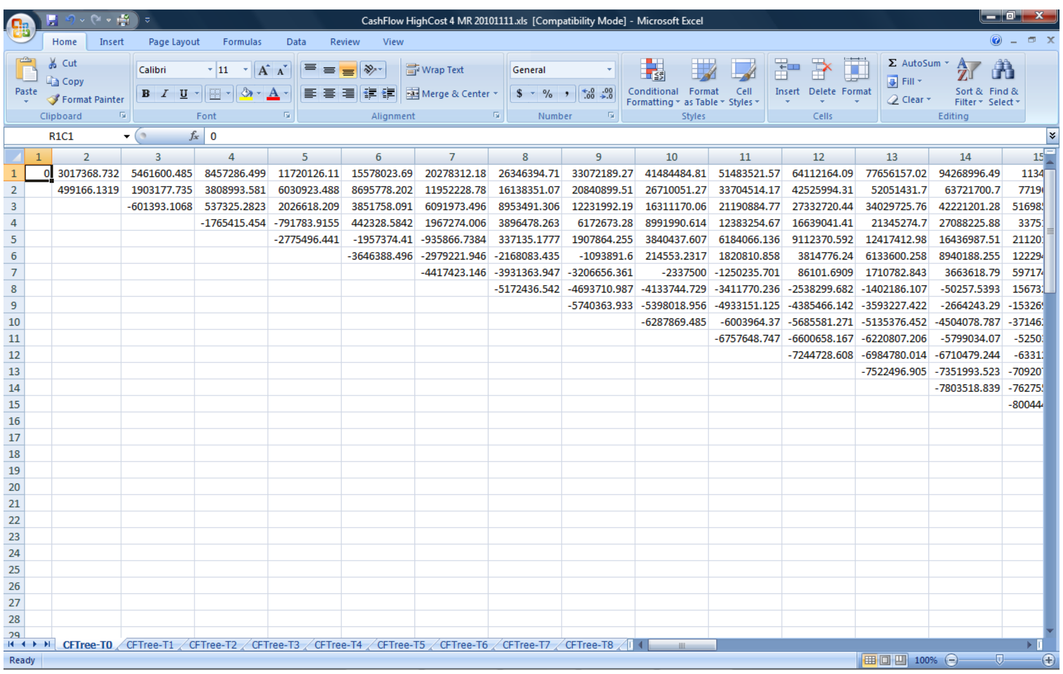
Figure 8-9: Computer Generated Probability (q_t) Lattice for the One-factor Mean Reversion Price Model for the Oil Produced in the Synthetic Oil Reservoir



	1	2	3	4	5	6	7	8	9	10
1	76.05	89.32139	104.9088	123.2163	144.7186	169.9732	199.6351	234.4731	275.3908	323.4489
2		64.75047	76.05	89.32139	104.9088	123.2163	144.7186	169.9732	199.6351	234.4731
3			55.12983	64.75047	76.05	89.32139	104.9088	123.2163	144.7186	169.9732
4				46.93863	55.12983	64.75047	76.05	89.32139	104.9088	123.2163
5					39.96448	46.93863	55.12983	64.75047	76.05	89.32139
6						34.02655	39.96448	46.93863	55.12983	64.75047
7							28.97088	34.02655	39.96448	46.93863
8								24.66638	28.97088	34.02655
9									21.00144	24.66638
10										17.88104

	11	12	13	14	15	16	17	18	19
1	379.8935	446.1883	524.0521	615.5037	722.9145	849.0695	997.2396	1171.267	1375.663
2	275.3908	323.4489	379.8935	446.1883	524.0521	615.5037	722.9145	849.0695	997.2396
3	199.6351	234.4731	275.3908	323.4489	379.8935	446.1883	524.0521	615.5037	722.9145
4	144.7186	169.9732	199.6351	234.4731	275.3908	323.4489	379.8935	446.1883	524.0521
5	104.9088	123.2163	144.7186	169.9732	199.6351	234.4731	275.3908	323.4489	379.8935
6	76.05	89.32139	104.9088	123.2163	144.7186	169.9732	199.6351	234.4731	275.3908
7	55.12983	64.75047	76.05	89.32139	104.9088	123.2163	144.7186	169.9732	199.6351
8	39.96448	46.93863	55.12983	64.75047	76.05	89.32139	104.9088	123.2163	144.7186
9	28.97088	34.02655	39.96448	46.93863	55.12983	64.75047	76.05	89.32139	104.9088
10	21.00144	24.66638	28.97088	34.02655	39.96448	46.93863	55.12983	64.75047	76.05
11	15.22427	17.88104	21.00144	24.66638	28.97088	34.02655	39.96448	46.93863	55.12983
12		12.96225	15.22427	17.88104	21.00144	24.66638	28.97088	34.02655	39.96448
13			11.03631	12.96225	15.22427	17.88104	21.00144	24.66638	28.97088
14				9.396535	11.03631	12.96225	15.22427	17.88104	21.00144
15					8.000396	9.396535	11.03631	12.96225	15.22427
16						6.811695	8.000396	9.396535	11.03631
17							5.799612	6.811695	8.000396
18								4.937904	5.799612
19									4.204229

Figure 8-10: Computer Generated Price Lattice for the GBM Price Model for the Oil Produced in the Synthetic Oil Reservoir



	1	2	3	4	5	6	7
1	0	3017368.732	5461600.485	8457286.499	11720126.11	15578023.69	20278312.18
2		499166.1319	1903177.735	3808993.581	6030923.488	8695778.202	11952228.78
3			-601393.1068	537325.2823	2026618.209	3851758.091	6091973.496
4				-1765415.454	-791783.9155	442328.5842	1967274.006
5					-2775496.441	-1957374.41	-935866.7384
6						-3646388.496	-2979221.946
7							-4417423.146

	8	9	10	11	12	13	14
1	26346394.71	33072189.27	41484484.81	51483521.57	64112164.09	77656157.02	94268996.49
2	16138351.07	20840899.51	26710051.27	33704514.17	42525994.31	52051431.7	63721700.7
3	8953491.306	12231992.19	16311170.06	21190884.77	27332720.44	34029725.76	42221201.28
4	3896478.263	6172673.28	8991990.614	12383254.67	16639041.41	21345274.7	27088225.88
5	337135.1777	1907864.255	3840437.607	6184066.136	9112370.592	12417412.98	16436987.51
6	-2168083.435	-1093891.6	214553.2317	1820810.858	3814776.24	6133600.258	8940188.255
7	-3931363.947	-3206656.361	-2337500	-1250235.701	86101.6909	1710782.843	3663618.79
8	-5172436.542	-4693710.987	-4133744.729	-3411770.236	-2538299.682	-1402186.107	-50257.5393
9		-5740363.933	-5398018.956	-4933151.125	-4385466.142	-3593227.422	-2664243.29
10			-6287869.485	-6003964.37	-5685581.271	-5135376.452	-4504078.787
11				-6757648.747	-6600658.167	-6220807.206	-5799034.07
12					-7244728.608	-6984780.014	-6710479.244
13						-7522496.905	-7351993.523
14							-7803518.839

Figure 8-11: An Example of Computer Generated Cash Flow Lattice for One-factor Mean Reversion Price Model for the Oil Produced in the Synthetic Oil Reservoir

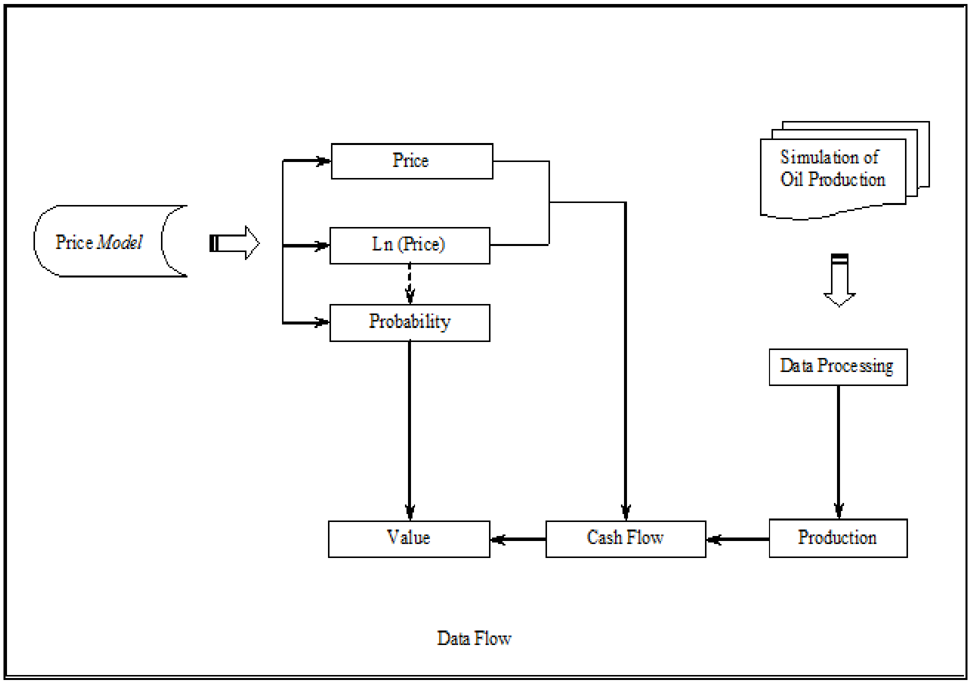


Figure 8-12: Data Flow for Real Options Evaluation Program

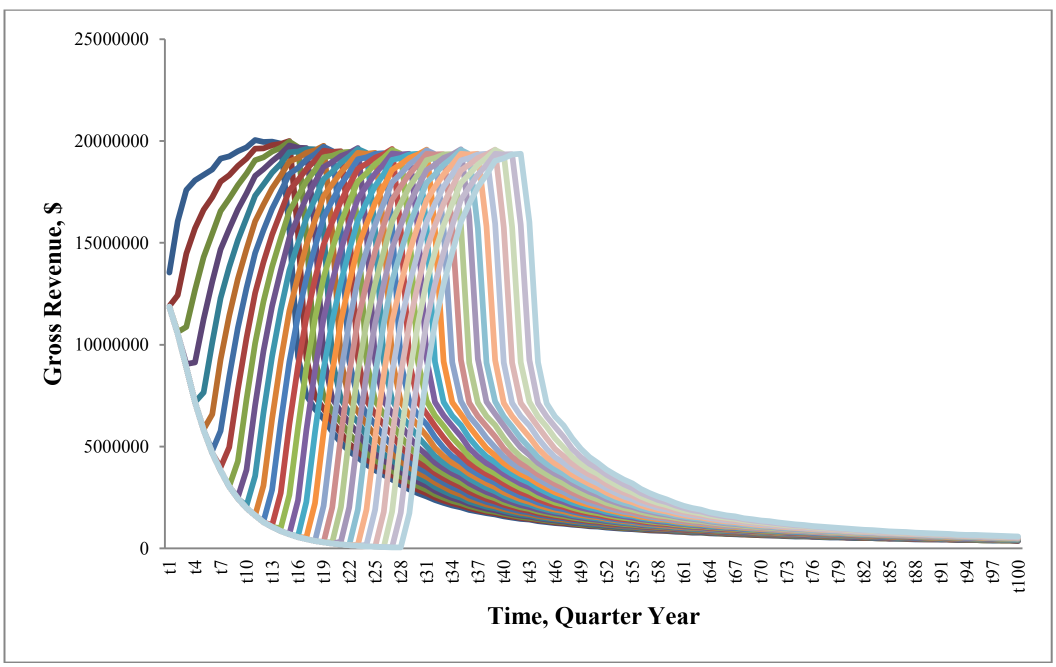


Figure 8-13: Gross Revenue for the Base Case with High Cost and Mean Reversion Price Model at Different Water Flooding Switching Times

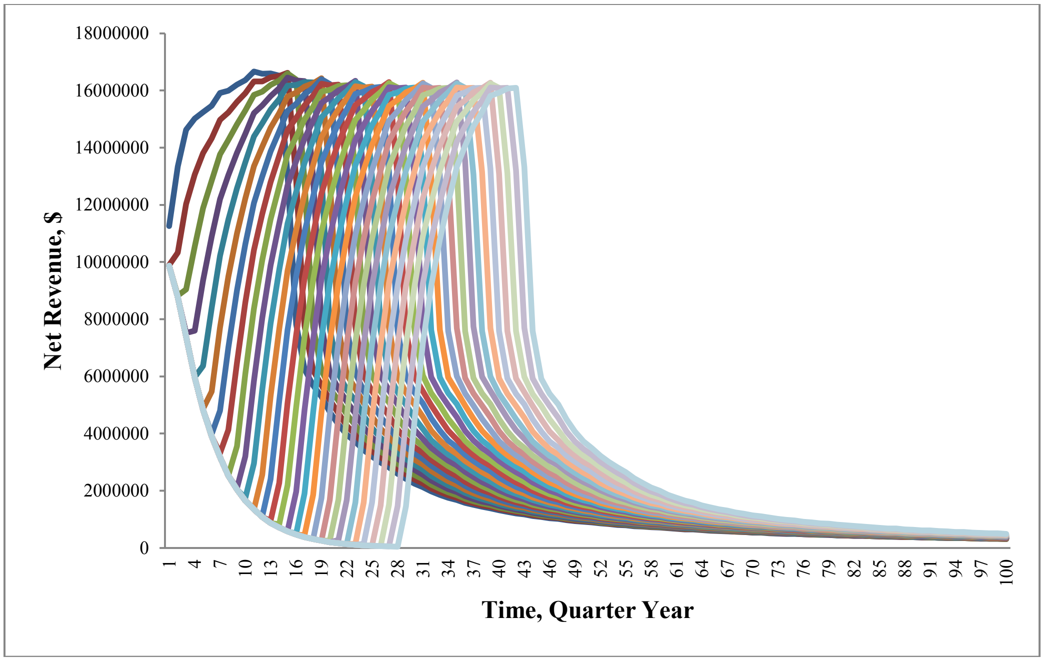


Figure 8-14: Net Revenue for the Base Case with High Cost and Mean Reversion Price Model at Different Water Flooding Switching Times

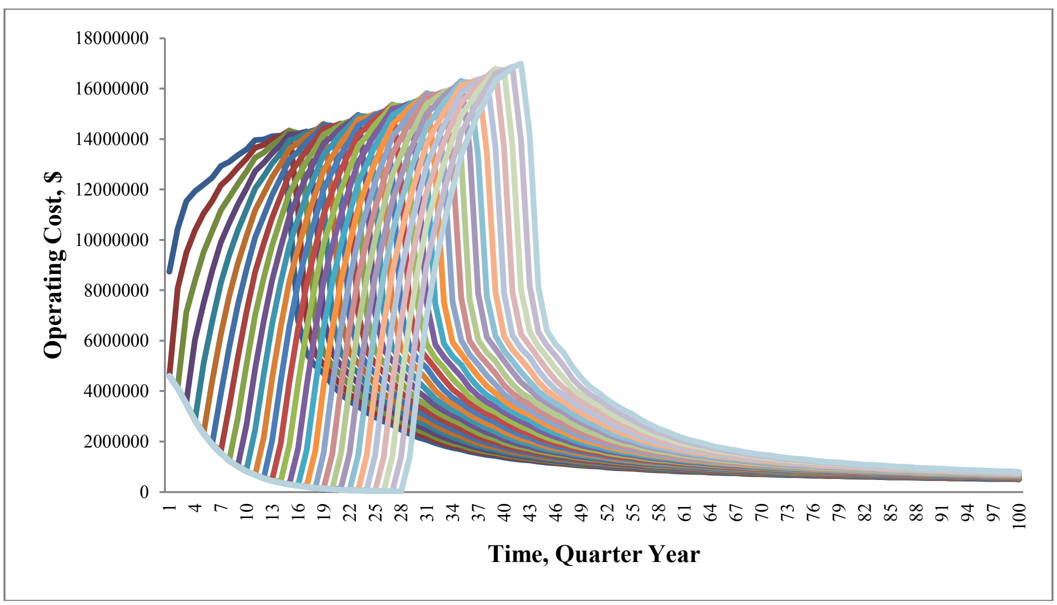


Figure 8-15: Operating Cost for the Base Case with High Cost and Mean Reversion Price Model at Different Water Flooding Switching Times

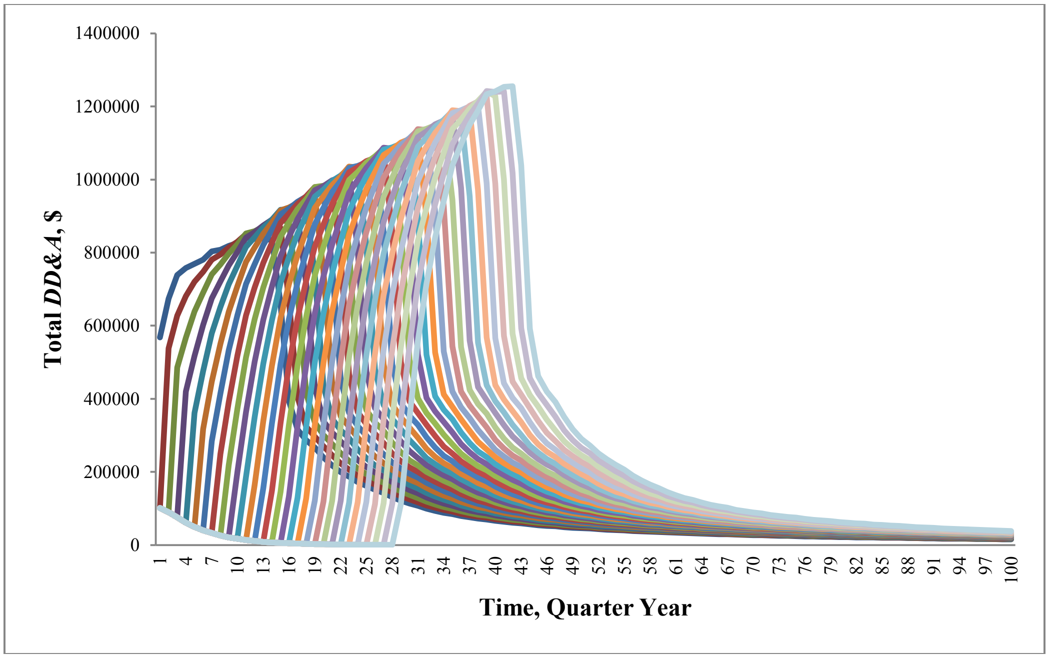


Figure 8-16: Total $DD\&A$ for the Base Case with High Cost and Mean Reversion Price Model at Different Water Flooding Switching Times

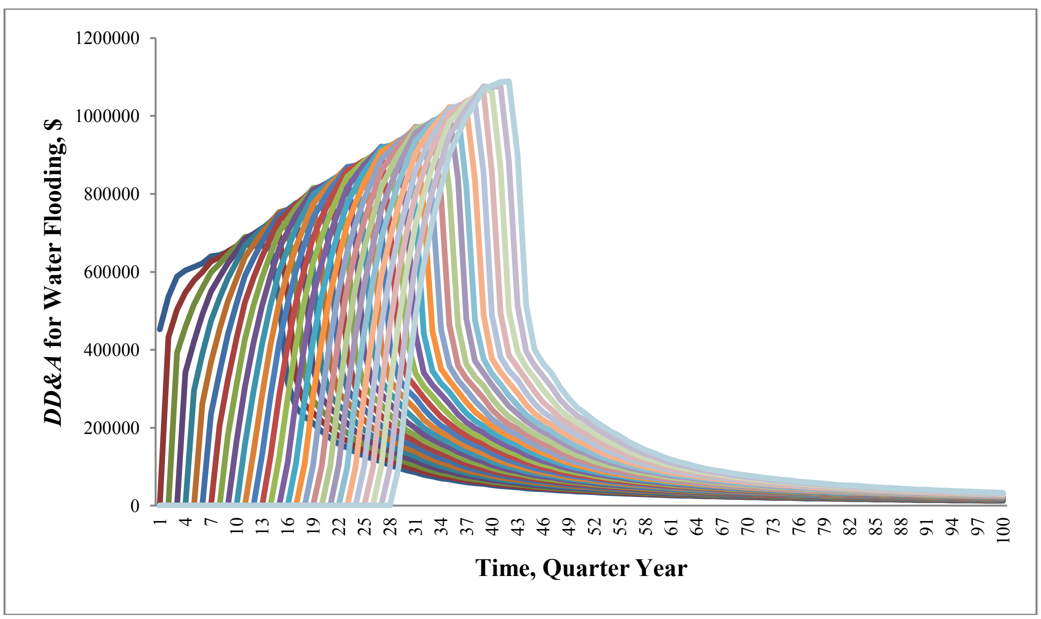


Figure 8-17: $DD\&A$ for Water Flooding for the Base Case with High Cost and Mean Reversion Price Model at Different Water Flooding Switching Times

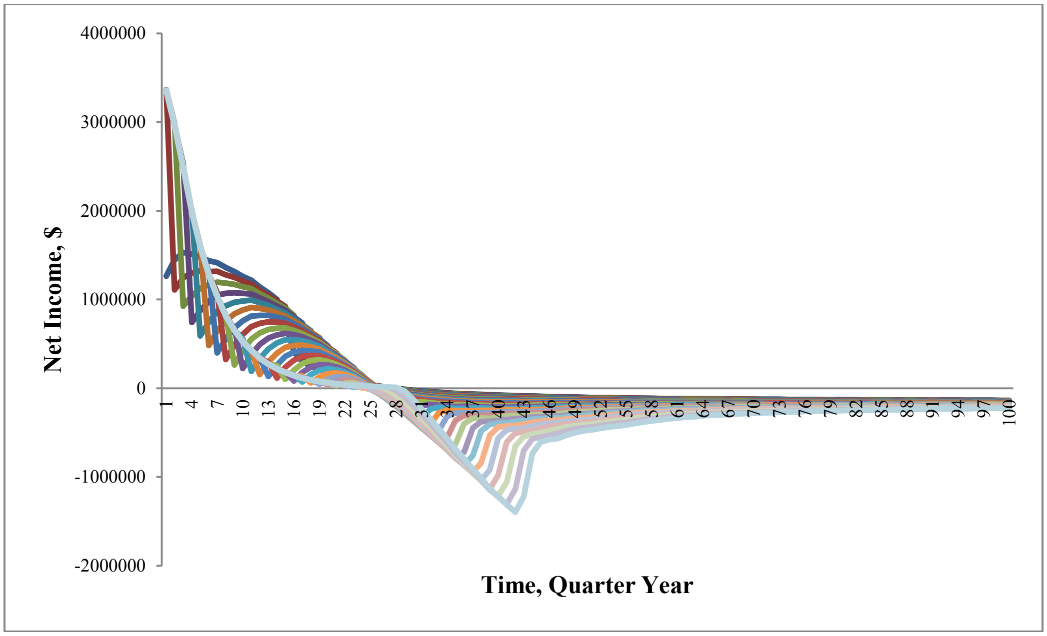


Figure 8-18: Net Income for the Base Case with High Cost and Mean Reversion Price Model at Different Water Flooding Switching Times

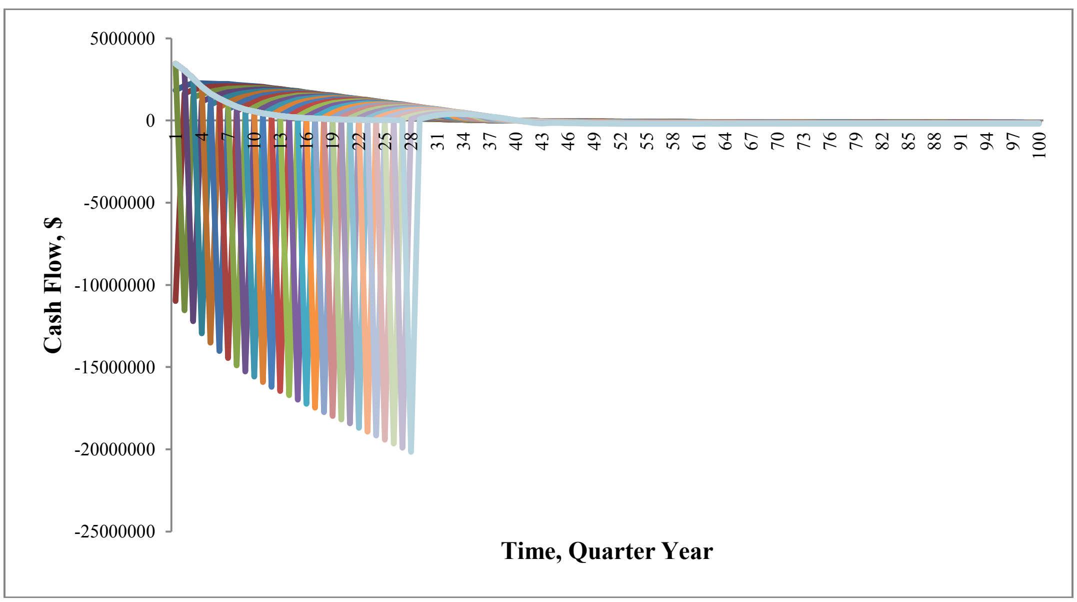


Figure 8-19: Cash Flow for the Base Case with High Cost and Mean Reversion Price Model at Different Water Flooding Switching Times

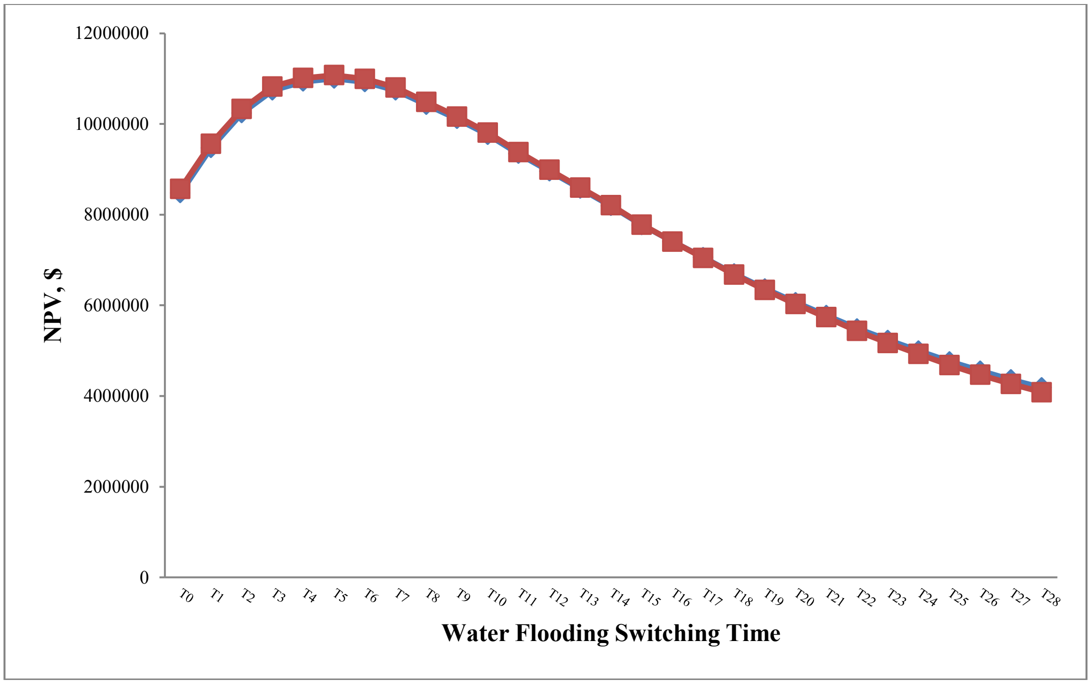
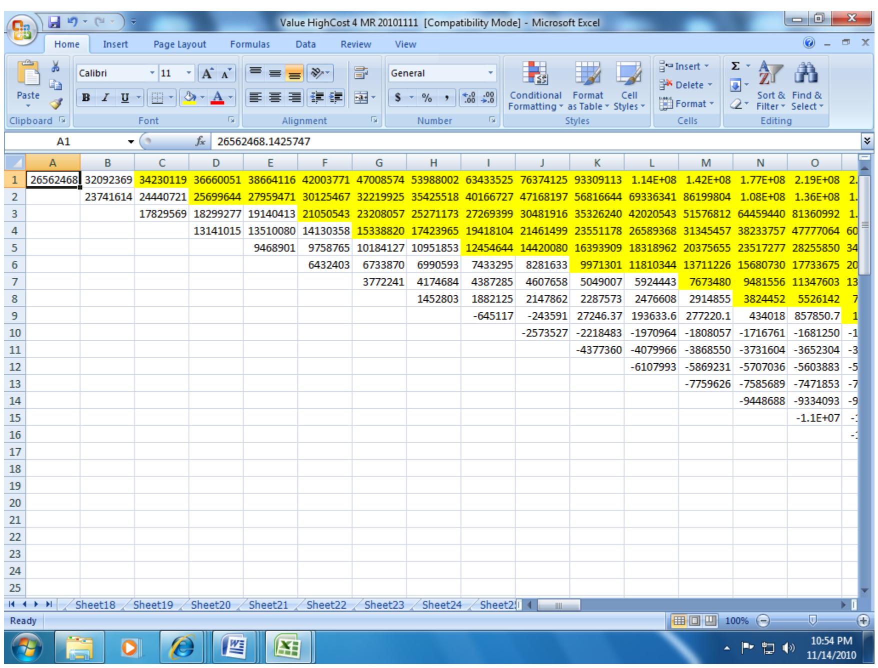


Figure 8-20: NPV of the Base Case with High Cost and Mean Reversion Price Model at Different Water Flooding Switching Times



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	26562468	32092369	34230119	36660051	38664116	42003771	47008574	53988002	63433525	76374125	93309113	1.14E+08	1.42E+08	1.77E+08	2.19E+08
2		23741614	24440721	25699644	27959471	30125467	32219925	35425518	40166727	47168197	56816644	69336341	86199804	1.08E+08	1.36E+08
3			17829569	18299277	19140413	21050543	23208057	25271173	27269399	30481916	35326240	42020543	51576812	64459440	81360992
4				13141015	13510080	14130358	15338820	17423965	19418104	21461499	23551178	26589368	31345457	38233757	47777064
5					9468901	9758765	10184127	10951853	12454644	14420080	16393909	18318962	20375655	23517277	28255850
6						6432403	6733870	6990593	7433295	8281633	9971301	11810344	13711226	15680730	17733675
7							3772241	4174684	4387285	4607658	5049007	5924443	7673480	9481556	11347603
8								1452803	1882125	2147862	2287573	2476608	2914855	3824452	5526142
9									-645117	-243591	27246.37	193633.6	277220.1	434018	857850.7
10										-2573527	-2218483	-1970964	-1808057	-1716761	-1681250
11											-4377360	-4079966	-3868550	-3731604	-3652304
12												-6107993	-5869231	-5707036	-5603883
13													-7759626	-7585689	-7471853
14														-9448688	-9334093
15															-1.1E+07

Figure 8-21: Real Options Evaluation Results for the High Cost Mean Reversion Case

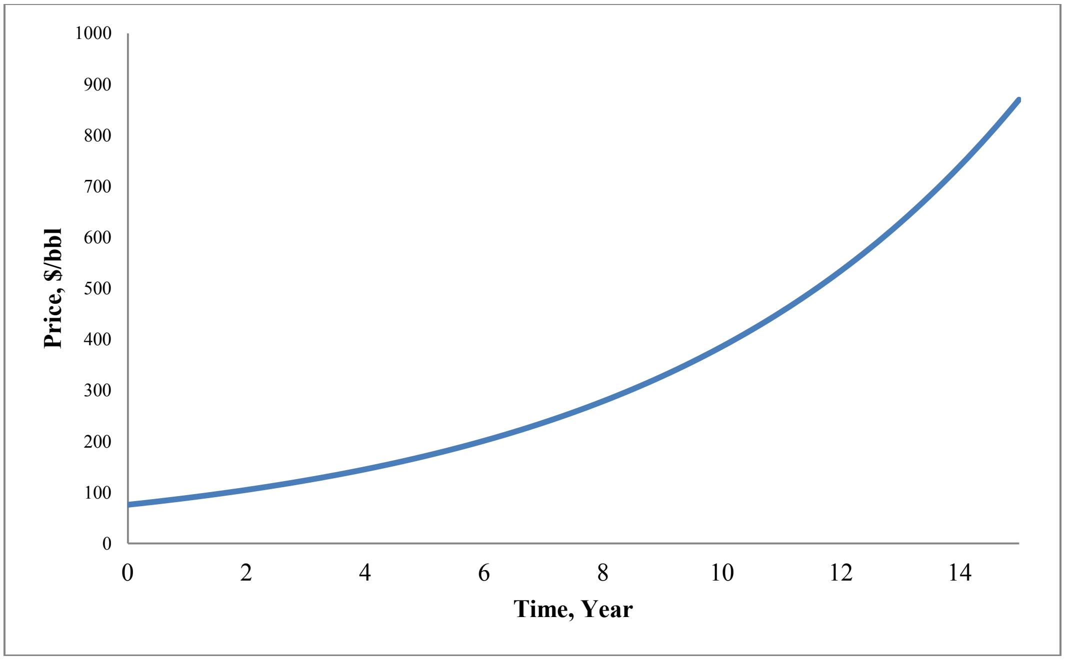
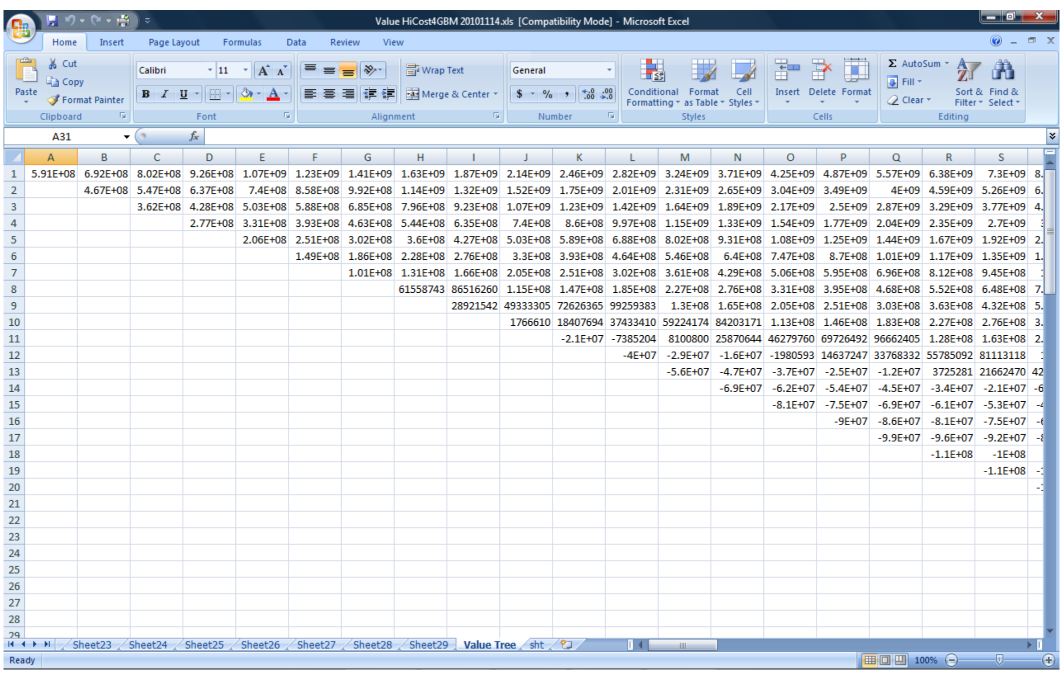
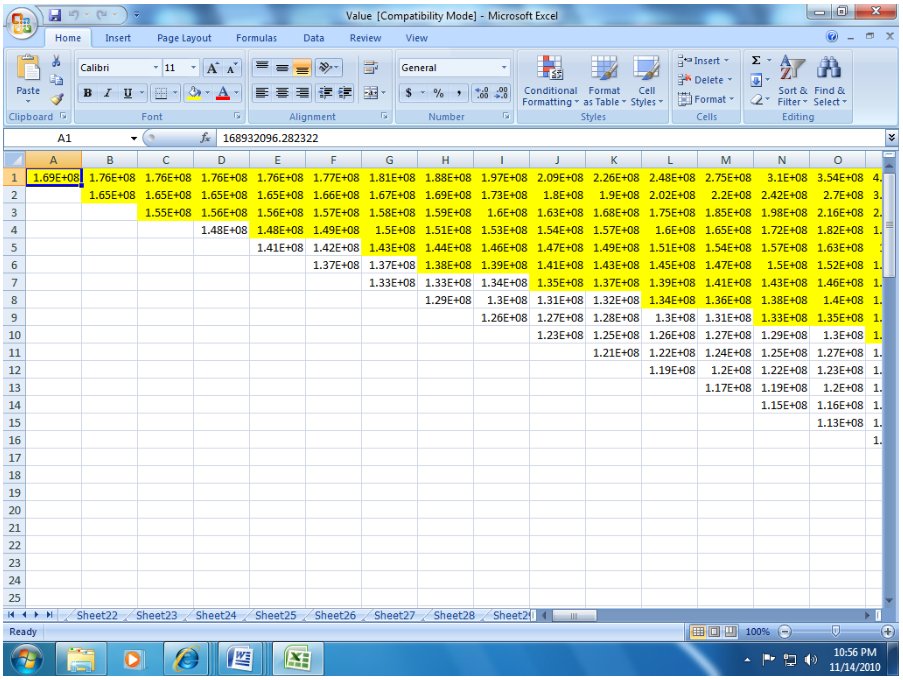


Figure 8-22: Expected Oil Prices from the GBM Oil Price Model in Next Fifteen Years Starting from the Third Quarter of Year 2010



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1	5.91E+08	6.92E+08	8.02E+08	9.26E+08	1.07E+09	1.23E+09	1.41E+09	1.63E+09	1.87E+09	2.14E+09	2.46E+09	2.82E+09	3.24E+09	3.71E+09	4.25E+09	4.87E+09	5.57E+09	6.38E+09	7.3E+09
2		4.67E+08	5.47E+08	6.37E+08	7.4E+08	8.58E+08	9.92E+08	1.14E+09	1.32E+09	1.52E+09	1.75E+09	2.01E+09	2.31E+09	2.65E+09	3.04E+09	3.49E+09	4E+09	4.59E+09	5.26E+09
3			3.62E+08	4.28E+08	5.03E+08	5.88E+08	6.85E+08	7.96E+08	9.23E+08	1.07E+09	1.23E+09	1.42E+09	1.64E+09	1.89E+09	2.17E+09	2.5E+09	2.87E+09	3.29E+09	3.77E+09
4				2.77E+08	3.31E+08	3.93E+08	4.63E+08	5.44E+08	6.35E+08	7.4E+08	8.6E+08	9.97E+08	1.15E+09	1.33E+09	1.54E+09	1.77E+09	2.04E+09	2.35E+09	2.7E+09
5					2.06E+08	2.51E+08	3.02E+08	3.6E+08	4.27E+08	5.03E+08	5.89E+08	6.88E+08	8.02E+08	9.31E+08	1.08E+09	1.25E+09	1.44E+09	1.67E+09	1.92E+09
6						1.49E+08	1.86E+08	2.28E+08	2.76E+08	3.3E+08	3.93E+08	4.64E+08	5.46E+08	6.4E+08	7.47E+08	8.7E+08	1.01E+09	1.17E+09	1.35E+09
7							1.01E+08	1.31E+08	1.66E+08	2.05E+08	2.51E+08	3.02E+08	3.61E+08	4.29E+08	5.06E+08	5.95E+08	6.96E+08	8.12E+08	9.45E+08
8								61558743	86516260	1.15E+08	1.47E+08	1.85E+08	2.27E+08	2.76E+08	3.31E+08	3.95E+08	4.68E+08	5.52E+08	6.48E+08
9									28921542	49333305	72626365	99259383	1.3E+08	1.65E+08	2.05E+08	2.51E+08	3.03E+08	3.63E+08	4.32E+08
10										1766610	18407694	37433410	59224174	84203171	1.13E+08	1.46E+08	1.83E+08	2.27E+08	2.76E+08
11											-2.1E+07	-7385204	8100800	25870644	46279760	69726492	96662405	1.28E+08	1.63E+08
12												-4E+07	-2.9E+07	-1.6E+07	-1980593	14637247	33768332	55785092	81113118
13													-5.6E+07	-4.7E+07	-3.7E+07	-2.5E+07	-1.2E+07	3725281	21662470
14														-6.9E+07	-6.2E+07	-5.4E+07	-4.5E+07	-3.4E+07	-2.1E+07
15															-8.1E+07	-7.5E+07	-6.9E+07	-6.1E+07	-5.3E+07
16																-9E+07	-8.6E+07	-8.1E+07	-7.5E+07
17																	-9.9E+07	-9.6E+07	-9.2E+07
18																		-1.1E+08	-1E+08
19																			-1.1E+08

Figure 8-23: Real Options Evaluation Results for the High Cost GBM Case



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	
1	1.69E+08	1.76E+08	1.76E+08	1.76E+08	1.76E+08	1.77E+08	1.81E+08	1.88E+08	1.97E+08	2.09E+08	2.26E+08	2.48E+08	2.75E+08	3.1E+08	3.54E+08	4.
2		1.65E+08	1.65E+08	1.65E+08	1.65E+08	1.66E+08	1.67E+08	1.69E+08	1.73E+08	1.8E+08	1.9E+08	2.02E+08	2.2E+08	2.42E+08	2.7E+08	3.
3			1.55E+08	1.56E+08	1.56E+08	1.57E+08	1.58E+08	1.59E+08	1.6E+08	1.63E+08	1.68E+08	1.75E+08	1.85E+08	1.98E+08	2.16E+08	2.
4				1.48E+08	1.48E+08	1.49E+08	1.5E+08	1.51E+08	1.53E+08	1.54E+08	1.57E+08	1.6E+08	1.65E+08	1.72E+08	1.82E+08	1.
5					1.41E+08	1.42E+08	1.43E+08	1.44E+08	1.46E+08	1.47E+08	1.49E+08	1.51E+08	1.54E+08	1.57E+08	1.63E+08	1
6						1.37E+08	1.37E+08	1.38E+08	1.39E+08	1.41E+08	1.43E+08	1.45E+08	1.47E+08	1.5E+08	1.52E+08	1.
7							1.33E+08	1.33E+08	1.34E+08	1.35E+08	1.37E+08	1.39E+08	1.41E+08	1.43E+08	1.46E+08	1.
8								1.29E+08	1.3E+08	1.31E+08	1.32E+08	1.34E+08	1.36E+08	1.38E+08	1.4E+08	1.
9									1.26E+08	1.27E+08	1.28E+08	1.3E+08	1.31E+08	1.33E+08	1.35E+08	1.
10										1.23E+08	1.25E+08	1.26E+08	1.27E+08	1.29E+08	1.3E+08	1.
11											1.21E+08	1.22E+08	1.24E+08	1.25E+08	1.27E+08	1.
12												1.19E+08	1.2E+08	1.22E+08	1.23E+08	1.
13													1.17E+08	1.19E+08	1.2E+08	1.
14														1.15E+08	1.16E+08	1.
15															1.13E+08	1.
16																1.

Figure 8-24: Real Options Evaluation Results for the Low Cost Mean Reversion Case

CHAPTER 9: CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

The following conclusions are reached from the studies on the geometric Brownian motion (GBM) and one-factor mean reversion oil price models with West Texas Intermediate (WTI) historical oil prices, and from the establishment of binomial lattice real options evaluation method for the water flooding switching time based on the reservoir simulation results, successful efforts accounting rules, concessionary fiscal regime cash flow model, and binomial lattice risk-neutral evaluations:

1. The results of parameter calibrations for the GBM price model for the historical WTI oil prices reveal that the assumptions of constant drift rate and constant volatility for the GBM price model do not hold for the historical WTI daily, weekly, and monthly oil price data.
2. The simulation results for the GBM and one-factor mean reversion price models show that the one-factor mean reversion model is a better model to fit the historical WTI oil prices than the GBM model.
3. Applying the one-factor mean reversion price model to the historical WTI oil prices reveals that the evolution of the historical WTI oil prices from January 2, 1986 to May 28, 2010 can be classified into three price regimes: 1) B4-2K regime (1986 to 1999) with a long run price of \$19.50/bbl; 2) 2K-2.3K price regime (2000-2003) with a long run price of \$29.24/bbl; and 3) AF-2.3K price regime (2004-2010) with a long run price of about \$77.00/bbl. Extending

parameters to the future oil prices involves the risk that the future oil prices may not be in the same price regime as the one from which the mean reversion parameters are calibrated.

4. The calibrated long run prices and mean reversion rates for the historical WTI oil prices reveal that in the AF-2.3K price regime, the world economy has increased its tolerance to higher oil prices and to the higher price fluctuation from its long run price compared with that in the B4-2K and 2K-2.3K price regimes.
5. The established binomial lattice real options evaluation method can be successfully used to identify the best time to switch from primary oil recovery to water flooding when stochastic oil price models are included.
6. The real options evaluation method reveals that the best switching time for water flooding can be much different when different oil price models are applied and different cost data are used.
7. With the one-factor mean reversion oil price model and the most updated cost data, the real options evaluation method finds that the best water flooding switching time is earlier than the traditional net present value (NPV) optimizing method.
8. The real options evaluation results reveal that most of time water flooding should start when oil prices are high, and should not start when oil prices are low.

With the established real options evaluation framework, the following recommendations for future work are proposed:

1. Apply the established real options evaluation method to real industry cases to guide the process of determining the switching time from primary to water flooding oil recovery, with the oil production profile and complete cost data from petroleum $E\&P$ companies.
2. The established real options evaluation framework should be extended to identify the switching time from secondary to tertiary oil recovery such as chemical flooding. As shown in Chapter 6, the time of water flooding oil production can last 20 years from water cut of 97% to 99%, indicating that only one to three percent of oil is produced from the reservoir compared to 97% to 99% of water taken from underground. During those 20 years, when is the best time to switch to tertiary recovery? Since tertiary oil recovery is much expensive than water flooding, it is very important to conduct thorough research on the switching cost and the operating cost for the tertiary recovery.
3. The framework should be used to compare different enhanced oil recovery methods such as chemical and CO_2 flooding. It can also be used to compare the conventional energy production options with the alternative energy production options.

4. The framework should also be extended to natural gas production. The study should include the stochastic gas price process and the correlation between oil and gas prices.
5. Since the results from real options evaluations are very sensitive to cost data, a stochastic cost process should also be included in the evaluation framework. Future studies should include cost-price correlations.

Appendix A: MDM INPUT File

NX, NUMBER OF X DIVISIONS ?

41.00

NY, NUMBER OF Y DIVISIONS ?

41.0

NZ, NUMBER OF Z DIVISIONS ?

3.0

NV, NUMBER OF VARIOGRAMS (IN THE NESTED MODEL) ?

1.0000

NR, NUMBER OF REALIZATIONS, <1 AS 1 EXCEPT O-FORM. ?

1.0000

NS, NUMBER FOR STARTING RANDOM NUMBER (SEED) ?

87654

MO, OUTPUT (INPUT) OPTIONS, SUM OF ALL OPTIONS ?

5130

DX, GRID SIZE IN X DIRECTION, CONSISTENT UNIT ?

64.39

DY, GRID SIZE IN Y DIRECTION, SAME UNIT AS DX ?

64.39

DZ, GRID SIZE IN Z DIRECTION, SAME UNIT AS DX ?

20.

#1: P OF P-NORMAL FOR THIS TERM OF VARIOGRAM, 0=LOG-N. ?

0.

#1: MEAN VALUE FOR THIS TERM, CONSISTENT UNIT AS S.D. ?

172

#1: CORRELATION LENGTH, X MAJOR AXIS, SAME UNIT AS DX ?

128.

#1: CO.MODEL,-1=EXP,-2=DEXP,-3=SPH,-5=INPUTACF,+=POWER ?

-3.

#1: RATIO OF CORRELATION LENGTHES IN MAJOR AXES, LX/LY ?

1.0

#1: RATIO OF CORRELATION LENGTHES IN MAJOR AXES, LX/LZ ?

10

#1: VDP FOR THIS TERM,SAME UNIT AS MEAN ?

0.70

Appendix B: Survey on the Costs of Petroleum Exploration and Production ($E\&P$) for Real Options PhD Studies

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October 2010

Table 1: General Field Information

Onshore Successful Vertical Oil Well with Artificial Lift in a Sandstone or Carbonate Reservoir Five Spots Production (One Producer, Four Injectors)	
Reservoir Top Depth, feet	4300
Initial Pressure, psi	4000
Bottom Hole Pressure for Production, psi	500
Reservoir Size, Acres	160
Net Pay, feet	60
Water Production Rate, bbl/day	2000 -3000
Oil Production Rate, bbl/day	15 - 2800

Table 2: Investment Costs for Financial Accounting Purposes (Successful Accounting)

Acquisition cost	
Lease Bonus – 160 acres	
Costs on Seismic Studies for Exploration Purposes	
Costs on Geological and Geophysical (G&G) for Exploration Purposes	
Successful Exploratory Drilling Costs – One Well	
Completion Costs – One Well	
Surface Facility Costs for Production	
Development Costs	
Water Injection Well Costs, per well	
Water Injection Well Costs, four wells	
Fixed Costs for Water Injection Facilities, per well	
Fixed Costs for Water Injection Facilities, four wells together	

Table 3: Investment Cost Data for Tax Accounting Purposes

Leasehold Costs for Depletion	
Tangible Drilling Costs for Depreciation	
Intangible Drilling Costs	

Table 4: Operating Costs -5 Spots (One Producer, Four Injectors)

Fixed Operating Costs without Water Injection	
Fixed Operating Costs With Water Injection	
Variable Operating Costs Without Water Injection, $/bbl oil produced	
Variable Operating Costs with Water Injection, $/bbl oil produced	
Variable Operating Costs with Water Injection, $/bbl water produced	
Other Operating Costs	

Table 5: Time Needed for Conducting the Activities

Seismic Studies for Exploration Purposes, months	
Geological and Geophysical (G&G) for Exploration Purposes, months	
Successful Exploratory Drilling, months	
Completion, months	
Surface Facility Costs for Production, months	
One Water Injection Well, months	
Four Water Injection Wells, months	
Other Water Injection Facilities, months	

*Cost data are based on 2010 prices, or otherwise the specified years

Appendix C: Acquisition, Exploration, Development Costs from Various Sources and the Cost Data Used in Cash Flow Model

	Industry Data (Based on 2010 Prices) *	Academic Data (Based on 2010 Prices) *	Government Statistics		Cash Flow Data (1)	Cash Flow Data (2)
			API JAS 2002	API Office (Based on 2008 Statistics)		
Acquisition Cost (Lease Bonus -160 Acres)		$160,000			$160,000	$160,000
Cost on Seismic Studies for Exploration Purposes		$50,000			$50,000	
Cost on Geological and Geophysical (G&G) Studies for Exploration Purposes		$10,000			$10,000	
Successful Exploratory Drilling Cost, 4370 ft, Cost/Well	$ 325,000	$ 232,000			$451,545	
U. S. Onshore Exploration Oil Well Average Cost, Average Depth of 4279 ft, Cost/Well			$ 296,000	$ 1,951,000		$1,951,000
Cost for Completing Exploration Well, Cost/Well	$ 175,000	$ 283,000			$300,000	
Surface Facility Cost for Primary Oil Production, 5-Spot Well Configuration, Cost/Well	$75,000	$ 50,000			$112,500	
U. S. Onshore Development Oil Well Average Cost, Average Depth 4396 ft, Cost/Well			$ 281,000	$1,617,000		$1,617,000
Water Injection Well Cost (Including Completion), 5-Spot Well Configuration, Cost/ Well	$500,000	$ 480,000			$751,545	$3,568,000
Cost for Water Injection Facilities, Four Wells for 5-Spot Well Configuration		$60,000			$135,000	

Sources: 1) Industry data: from Texas American Resource Company (Taylor, 2010) and (Klein, 2010)
2) Academic data: from Petroleum Engineering Department, the University of Texas at Austin, Austin, Texas (Bommer, 2010)
3) API JAS 2002: from API's 2002 Joint Association Survey on Drilling Costs (JAS) (American Petroleum Institute, 2003)
4) API Office: from Energy Market Statistics, American Petroleum Institute (Narayanan, 2010)

* based on onshore oil well cost data

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Vita

Liying Xu was born in Yushan, Jiangxi Province in P. R. China on March 3, 1962, the daughter of Chang-rong Xu and Xiang-lan Li. She entered Zhejiang University in China in September 1979 and graduated with a B.S. degree in Chemical Engineering in 1983. Afterwards, she taught in Changzhou Light Industry College, Jiangsu, China for one year. In 1984, she entered the graduate program of the Research Institute of Petroleum Processing (RIPP) in Beijing, China. She completed her M.S. degree in Petroleum Processing Engineering in 1987. She was then employed in 1987 by the R&D Department in the Research Institute of Petroleum Processing (RIPP), SINOPEC, as a research engineer and senior research engineer for ten years. During those years, she conducted research in the areas of economic evaluation for new R&D projects, the optimization of petroleum processing schemes for petroleum refineries across the country, and the long-term planning of petroleum companies. With the outcome from her research, she had six publications and wrote more than ten technical reports. She received Outstanding Research Achievement awards many times from RIPP and SINOPEC: the Outstanding Research Paper award from SINOPEC for ethylene feedstock optimization and the First Place Outstanding Research Paper award from the Administration of Chemical Industry, China. In 1997, she was invited to the Energy and Mineral Resources (EMR) program at the University of Texas at Austin, Austin, Texas as a visiting scholar. She was then admitted to the Ph.D. program and Energy and Mineral Resource (EMR) program in the Petroleum Engineering department at the

University of Texas at Austin. She received her M.A. degree from the EMR program in May 2006. During the time of her graduate study, she undertook many research projects at the Bureau of Economic Geology (BEG) and in the Center for Petroleum Asset Risk Management (CPARM) as a research assistant at the University of Texas at Austin. She also worked as an intern in the Lower Colorado River Authority (LCRA), Austin, Texas for about one year in the area of Fuel and Energy Risk Management. As a Ph.D. student, Liying received many awards including the American Association of University Women (AAUW) fellowship for Ph.D. candidates, 2009 Washington State University Summer Doctoral Fellows, Washington State University Advance Excellence in Science and Engineering award, and SPE Star academic fellowship in Mid-Continent North America Region.

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